

A Low-Complexity PS based Radix IFFT method for PAPR Reduction in OFDM systems

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Abstract— We have proposed one of the distortion-less PAPR reduction methods which employs the Permutation Sequence (PS) in the frequency domain with embedded side information. However, the proposed PS method has high computation complexity. To improve the computation complexity for the previous proposed PS method, this paper proposes a low-complexity PS method which can keep the better peak-to-average power ratio (PAPR). In the proposed PS method, the signals at the middle stages of an N-point radix IFFT using decimation in frequency (DIF) is considered for PS processing. This paper shows that the proposed method has a lower multiplicative complexity than conventional PS method with keeping the PAPR reduction performance. The proposed method is called PS based radix IFFT, where DIF-IFFT is assigned through different stages of the IFFT transform. This proposed method can reduce the multiplicative complexity relatively with keeping the similar PAPR reduction to other methods such as C-PTS and C-PS. The proposed method also can reduce the side information as compared with the C-PTS. This paper presents the various computer simulation results to verify the effectiveness of proposed method.

Keywords— Peak-to-Average Power Ratio (PAPR), Permutation Sequences (PS), Partial Transmit Sequence (PTS), decimation in frequency (DIF).

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is very attractive technique for achieving high speed transmission in wireless channels. However, there are some drawbacks of using OFDM. One of drawbacks of using OFDM is its high PAPR (peak to Average Power Ratio) in the time domain signal which would lead the significant degradation of BER (Bit Error Rate) performance and undesirable spectrum regrowth due to the non-linear amplifier located at the transmitter, as in [1] and [2].

To reduce the occurrence of larger PAPR, various methods have been proposed up to today. These methods can be categorized into two methods as distortion and distortion-less methods. The distortion methods include the clipping and filtering method [3, 4]. Although these methods can improve

the PAPR performance relatively, these methods lead the degradation of BER performance due to the clipping distortion. The distortion-less methods as proposed in [5]-[10] include the coding, tone reservation (TR), tone injection (TI), active constellation extension (ACE), and multiple signal representation methods such as partial transmit sequence (PTS), selected mapping (SLM) and Permutation Sequences (PS). Among these distortions-less methods, the PS method is considered as one of promising methods which can achieve better PAPR performance with smaller side information, as in [10]. However, the PS method requires multiple inverse fast Fourier transforms (IFFTs) processing which lead high computational complexity to achieve better PAPR performance. To realize the PS method in the actual hardware, it is required to reduce the computational complexity with keeping better PAPR performance.

To solve the above problem on the PS method, this paper proposes a low complexity PS based radix IFFT method. In the proposed method, the PS processing is done in the middle stages of an N-point radix IFFT. We employ decimation-in-frequency (DIF) which has lower multiplicative complexity for the final stages than the decimation-in-time (DIT) as proposed in [11]. The proposed PS based radix IFFT method can achieve a smaller number of transform computations as compared with the conventional PS method (C-PS) in [10] with keeping almost the same PAPR performance.

This paper is organized as follows. In section II, we present the basic concept of OFDM system. Section III introduces the conventional PTS (C-PTS) and C-PS methods. Section IV proposes a low complexity PS based radix IFFT method. Section V presents the various computer simulation results to verify the effectiveness of proposed PS based radix IFFT method, and finally we draw some conclusions in Section VI.

II. PAPR IN OFDM SYSTEMS

An OFDM symbol consists of large number of narrow band sub-carriers which are modulated by constellations mapping. This mapping can be achieved from the phase-shift keying (PSK) or quadrature amplitude modulation (QAM). After

serial to parallel (S/P) conversion for the high-speed binary serial input stream and constellation mapping, a new parallel signal sequence $\{X_0, X_1, \dots, X_{N-1}\}$ is obtained, where X_i is discrete complex-valued modulated signal. The parallel signal is assigned to N orthogonal sub-carriers. Finally, the modulated signals are added together to form an OFDM symbol in the frequency domain. The frequency domain signal is converted to the time domain signal as given in the following equation.

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, 0 \leq n \leq N-1 \quad (1)$$

The PAPR of an OFDM signal is defined as the ratio of the peak to the average power of the signal, which is given by,

$$PAPR(x) = 10 \log_{10} \frac{\max_{0 \leq n \leq N-1} \{|x_n|^2\}}{E\{|X|^2\}} \quad (dB) \quad (2)$$

where x_n is the time domain signal at n -th sampling time and $E\{\cdot\}$ denotes the expected value.

The PAPR performance is usually evaluated by the complementary cumulative distribution function (CCDF) which is given by,

$$\begin{aligned} CCDF(N, PAPR_0) &= \Pr\{PAPR > PAPR_0\} \\ &= 1 - (1 - e^{-PAPR_0})^N \end{aligned} \quad (3)$$

III. CONVENTIONAL PTS AND CONVENTIONAL PS METHODS

A. Conventional PTS Method

In PTS method, the input data sequence of N subcarriers is firstly partitioned into V disjoint sub-blocks where all subcarriers which are occupied by the other sub-blocks are set to zero. The frequency domain input sequence is given by,

$$X_k = \sum_{i=1}^V X_k^{(i)} \quad (4)$$

Let $b_i = e^{j\phi_i}$ be the set of weighting factors with $\phi_i \in [0, 2\pi]$ which are applied to the sub-blocks $X_k^{(i)}$. The substitute frequency-domain signal are given by,

$$\tilde{X} = \sum_{i=1}^V b_i X_k^{(i)} \quad (5)$$

The time domain signal can be obtained by converting the frequency domain signal given in (5) by IFFT, which is given by,

$$\begin{aligned} \tilde{x} &= IFFT \left\{ \sum_{i=1}^V b_i X_k^{(i)} \right\} = \sum_{i=1}^V b_i \cdot IFFT \{X_k^{(i)}\} \\ &= \sum_{i=1}^V b_i x_n^{(i)} \end{aligned} \quad (6)$$

In the practical application of PTS, a set of phase weighting factors is usually selected for generating phase weighting

sequences. Assuming W predetermined phase weighting factors, we can set phase weighting factor for the first sub-block to one fixed phase and $(V-1)$ sub-blocks to be optimized. To select the optimal phase weighting sequence for each input data sequence, W^{V-1} possible combinations should be checked, and then the candidate sequence with the minimum PAPR is selected.

For the C-PTS, the optimum PAPR performance can be found after searching W^{V-1} alternative combinations if the number of sub-blocks is V and the number of predetermined phase weighting factors is W . In the processing for the optimization of phase weighting sequence, a large numbers of complex multiplications are required and the computational complexity becomes very large.

At the receiver, the side information about phase weighting factors which is selected at the transmitter is required for the correct demodulation of data information. The required side information for the C-PTS when the number of clusters is V and the number of predetermined phase weighting factors is W is given by $\log_2 W^{V-1}$ bits.

B. Conventional PS Method

In this section, we introduce the Permutation Sequences method which was proposed by authors in [10]. Figure 1 shows the structure of frequency domain OFDM symbol for the PS method when performing the permutation only for the first $M/2$ subcarriers. The OFDM symbol consists of $(M-3)$ data sub-carriers, two dummy sub-carriers and one parity sub-carrier which can be expressed by,

$$X_0 = [D_1, D_2, P, X_0, X_1, \dots, X_{M/2-3}, X_{M/2-2}, \dots, X_{(M-4)}] \quad (7)$$

where D is the dummy subcarriers with power of 0 (Null subcarriers) and P is the parity check subcarrier with power of α of which power is taken as the same as the averaged power of data subcarriers. The dummy (null subcarriers) and parity subcarriers will be used for the detection of location for the first data subcarrier X_0 at the receiver for the correct demodulation of data information. After the insertion of zero padding, two null subcarriers and one parity sub-carrier, the optimization of PAPR performance can be achieved by circular rotation of subcarriers only for the first part of $M/2$ subcarriers so as to improve the PAPR performance as shown in Fig. 1.

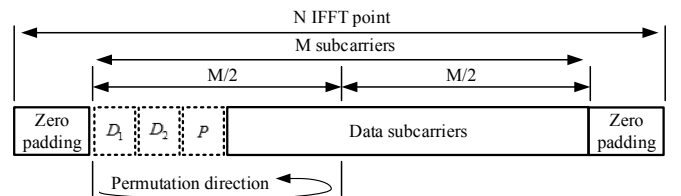


Figure 1. Structure of frequency domain OFDM symbol for PS method.

In the PS method, the first half of $M/2$ subcarriers are simply rotated in the frequency domain so as to achieve the

better PAPR performance. The number of rotation of subcarriers performed at the transmitter can be detected by using the dummy and parity check subcarriers at the receiver. When the number of rotation for the first $M/2$ subcarriers is 6 ($L=6$) as an example, the resultant ordering of subcarriers is given by,

$$X_{L=6} = [X_2, X_3, \dots, X_{M/2-3}, D_1, D_2, P, X_0, X_1, \dots, X_{(M-4)}] \quad (8)$$

Figure 2 shows the above example of PS method with $L=6$.

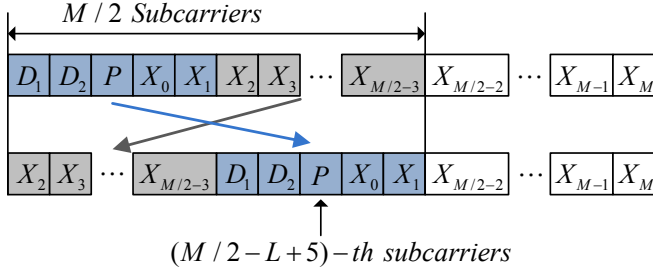


Figure 2. Example of permutation sequence when $L=6$.

When $L=6$, the location of parity check subcarrier P becomes $(M/2-L+5)$ -th subcarrier in the OFDM symbol. In this case, the sign of parity check subcarrier P is set by the following equation in order to detect the number of rotation for the subcarriers with higher accuracy at the receiver.

$$P = (-1)^{(M/2-L+5)} \cdot \alpha \quad (9)$$

The permutation for the rotation of the first part of $M/2$ subcarriers is taken from 0 to $M/2-1$. In the proposed method, the best PAPR performance will be selected from among the rotation of subcarriers from 0 to $(M/2-1)$. Then the selected ordering of subcarriers with the best PAPR performance will be converted to the time domain signal by using IFFT.

IV. PROPOSAL OF LOW-COMPLEXITY PS METHOD

A. PS Based Radix IFFT Technique

The discrete Fourier transform (DFT) of an N -point sequence x_n can be directly computed as

$$X_k = \sum_{n=0}^{N-1} x_n T_N^{nk} \quad (10)$$

Since the IDFT can be computed by taking the complex conjugate of the input and output sequences while using the same DFT parameters, as in [11], we need only consider the DFT calculation. Thus, we only use the corresponding FFT computation in the following.

An FFT algorithm recursively converts the DFT computation to $r \times N/r$ -point DFTs recurring through $m = \log_r N$ stages. The value corresponds to a radix- r IFFT algorithm. The DIF radix- r IDFT of (10) is given by

$$X(rk + k_0) = \sum_{n=0}^{N/r-1} \left(\sum_{i=0}^{r-1} x(n + \frac{N}{r}i) T_r^{ik_0} \right) T_{N/r}^{nk} \quad (11)$$

where $k_0 = 0, 1, \dots, r-1$ is the index of the butterfly outputs.

As we consider the inputs to stage ν for PS processing, symbols and indices are represented with subscript ν : x_ν and n_ν for an input x and time index n , respectively, and X_ν and k_ν for an output X and frequency index k , respectively. Considering the form of (11), the butterfly outputs at stage ν are given by

$$X_\nu^\eta(rk_\nu + k_0) = \sum_{n=0}^{N/r-1} \left(\sum_{i=0}^{r-1} x_\nu(n_\nu + \frac{N}{r}i) T_r^{ik_0} \right) T_{N/r}^{n_\nu k_0} \quad (12)$$

where $k_\nu = 0, 1, \dots, (N/r^\nu)-1$, $n_\nu = 0, 1, \dots, (N/r^\nu)-1$ and $\eta = 0, 1, \dots, r^{\nu-1}$ denotes a particular $(N/r^{\nu-1})$ -point DFT at stage ν . Figure 3 shows the recursive reduction of the η -th $(N/r^{\nu-1})$ -point DFT to (N/r^ν) -point DFTs with DIF radix- r at stage ν . Hence, there are $r^{\nu-1}N/r^{\nu-1}$ -point DFTs at stage ν . It is assumed that the input sequence is in normal order, and the output is in digit-reversed order.

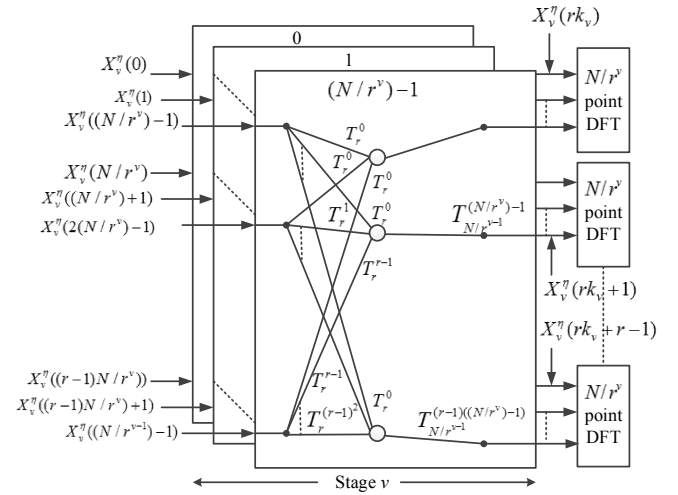


Figure 3. Recursive reduction of the η -th $(N/r^{\nu-1})$ -point DFT to (N/r^ν) -point DFTs with DIF radix- r at stage ν .

In the proposed PS based radix method, OFDM signal is generated without two dummy sub-carriers (D) and one parity (P) sub-carriers. From this fact, the side information about the location of PS processing is required at the receiver in order to recover the received signals correctly which is the same as the C-PTS method. Figure 4 shows an example of proposed PS based radix method. From the figure, the inputs $x_\nu^\eta(n_\nu + (N/r^\nu)i)$ at stage ν are used for PS Processing and then the remaining $(m-\nu)$ stages are used to compute the multiple transforms.

As a consequence, if stage ν is used to obtain the PS processing, the data is recovered at $(m-\nu)$ stage at the

receiver since the value of ν is known at the receiver. Figure 5 shows the structure of transmitter for the proposed PS based radix method.

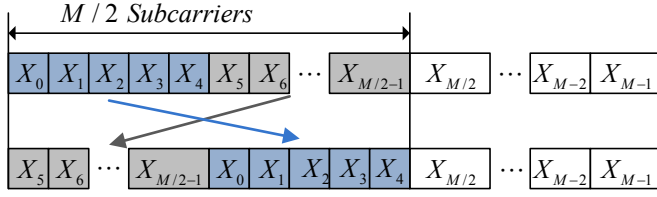


Figure 4. Example of proposed PS based radix method when $L=6$.

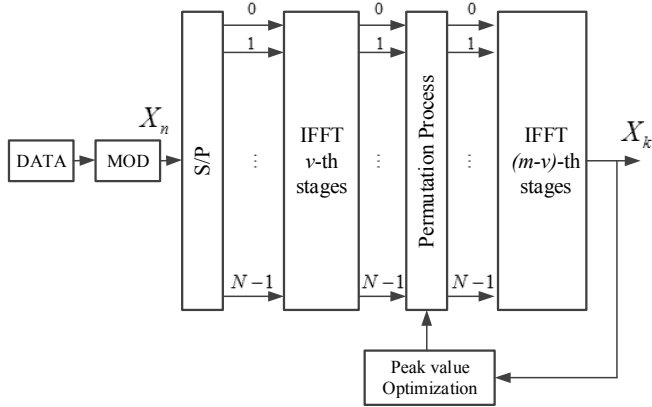


Figure 5. Structure of transmitter for the proposed PS method.

B. Computational Complexity Analysis

In this section, we evaluate the multiplicative complexity for the DIF algorithm as a number of complex multiplications by twiddle factors $T_{N/r^{v-1}}^{n_r k_0}$ and $T_r^{i k_0}$ as shown in Fig. 3. The twiddle factors $T_r^{i k_0}$ are trivial (± 1 and $\pm j$), if we consider only radix-2 and radix-4 and so they do not introduce any multiplicative complexity. By using (12) and Figure 3, the number of twiddle factors $T_{N/r^{v-1}}^{n_r k_0}$ and $T_r^{i k_0}$ at stage ν for the DIF algorithm can be obtained by the following equation.

$$\alpha_v^{DIF} = r^{v-1} \left(\frac{N}{r^v} - 1 \right) [(r-1) + (r-2) + (r-1)^2 - 1] \quad (13)$$

In (13), since it is clearly that $\alpha_{v+1}^{DIF} < \alpha_v^{DIF}$ is satisfied, α_v^{DIF} could decrease through successive stages. In order to compare the multiplicative complexity between two different PAPR techniques, we define the reduction ratios by the following equation.

$$R^{mul} = 1 - (M_{total}^1 / M_{total}^2) \quad (14)$$

where the overall multiplicative complexity for the PS based radix IFFT technique using DIF is given by,

$$M_{total} = \sum_{i=1}^{v-1} \alpha_v + (L-1) \cdot \sum_{i=v}^m \alpha_v \quad (15)$$

From the above considerations, it can be observed that higher radix algorithms have more twiddle factors per stage but fewer nontrivial multiplications compared to lower radix algorithms. From the above facts, the multiplicative complexity for the proposed PS based radix method can be reduced for the remaining stages by using a high radix algorithm with keeping the same PAPR reduction per stage.

V. PERFORMANCE RESULTS

This section presents extensive simulations results to verify the effectiveness of proposed PS based radix method. The simulation parameters to be used in the following evaluations are shown in Table I.

TABLE 1. SIMULATION PARAMETERS

Modulation	64QAM
Demodulation	Coherent
Allocated bandwidth	5MHz
Number of IFFT points	256
Number of sub-carriers	256
Number of cluster (V)	4
Number of discrete phase (W)	4
Symbol duration	56.1 μ s
Guard interval	5.61 μ s

Figure 6 shows the PAPR performance for the conventional OFDM, C-PTS, C-PS and proposed PS based radix methods. From the figure it can be observed that the PAPR reduction performance for the proposed PS based radix method can achieve the same performance as that for the conventional PTS method and PS method when the middle stages of IFFT are $(m-\nu)_{radix-2} = 2, 3, 4$ and 5, respectively.

Table II shows the comparisons for the multiplication complexity for the conventional PS and proposed PS based radix methods. From the table, it can be concluded that the proposed PS based radix-2 method can reduce the multiplication complexity by 64.12% at $(m-\nu)_{radix-2} = 4$ and can reduce the multiplication complexity up to 90.96% at $(m-\nu)_{radix-2} = 2$ without decreasing of PAPR performance when comparing with the C-PS method.

Figure 7 shows the multiplication complexity for the conventional PS and proposed PS based radix methods when changing the number of subcarriers $N=256, 512$ and 1024. From the figure, it can be observed that the proposed PS based radix method at stage $(m-\nu)_{radix-2} = 2$ can achieve better reduction of multiplication complexity than that for the C-PS method when increasing the number of subcarriers.

TABLE 2. MULTIPLICATION COMPLEXITY FOR PROPOSED PS METHOD OVER CONVENTIONAL PS METHOD

	Multiplication Complexity				
	(m-v=6)	(m-v=5)	(m-v=4)	(m-v=3)	(m-v=2)
Conventional PS method	0%	0%	0%	0%	0%
Proposed PS based radix	32.64%	48.63%	64.12%	78.57%	90.96%

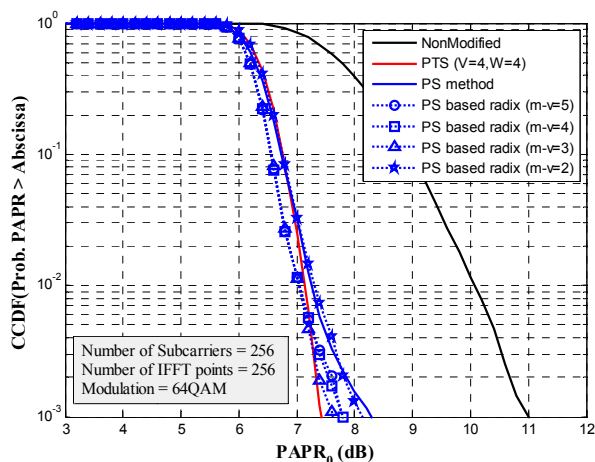


Figure 6. Comparisons of PAPR reduction among C-PTS, PS and PS Based radix-2 methods.

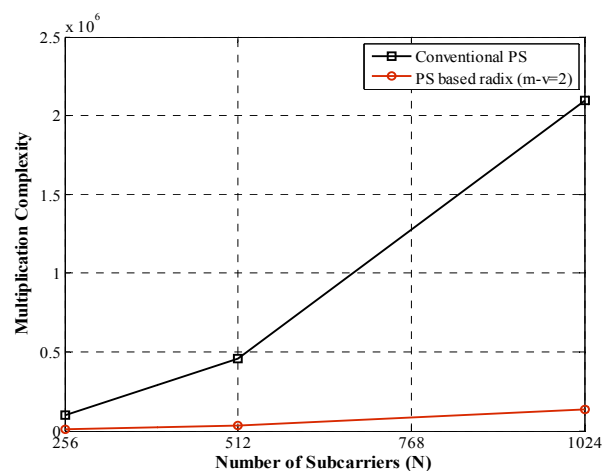


Figure 7. Comparisons of multiplication complexity between C-PS and proposed PS methods when changing the number of subcarriers.

VI. CONCLUSIONS

This paper proposed the PS based radix method which can reduce the computation complexity with keeping the better PAPR performance. The features of proposed method are to perform the PS processing in the middle stages of an N-point radix IFFT. From the computer simulation results, this paper confirmed that the proposed method shows the same PAPR performance and lower multiplication complexity as compared with C-PS method.

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