

CF based Performance Analysis of Chirp-UWB System over Multipath Channels

Changhui Wang¹, Zhiquan Bai^{1*}, Shen Gao¹, Peng Gong², and Kyungsup Kwak³

¹School of Information Science and Engineering, Shandong University, Jinan, China

²National Key Laboratory of Mechatronic Engineering and Control, Beijing Institute of Technology, Beijing, China

³Graduate School of Information Technology and Telecommunication, INHA University, Incheon, Korea

*Corresponding author: zqbai@sdu.edu.cn

Abstract— Since the energy of chirp waveform distributes evenly in time domain, chirp waveform has the advantage of low peak-to-average power ratio. In this paper, we first employ chirp function to generate the ultra-wideband (UWB) pulse for narrow-band interference (NBI) suppression, then the simplified characteristic function (CF) method is proposed to evaluate the NBI suppression performance of the direct sequence spread spectrum binary phase shift keying (DS-BPSK) chirp pulse based UWB (Chirp-UWB) system over multipath channels. Simulation results and theoretical analysis show that the proposed CF based performance analysis of Chirp-UWB system not only superiors to the traditional Gaussian pulse based UWB system, but also outperforms the conventional approximate analysis method in accuracy and complexity.

Keywords— UWB; Chirp waveform; NBI; multipath channel; characteristic function (CF)

I. INTRODUCTION

Ultra-wideband (UWB) [1] technology is a good candidate for high data rate and short range wireless communications and has attracted considerable interests since FCC approved the spectrum occupation for UWB technology in 2002. FCC regulates that the signal whose absolute bandwidth is greater than 500 MHz or relative bandwidth is greater than 0.2 is ultra wideband signal. Recently, FCC regulates that UWB systems can exploit the frequencies from 3.1 GHz to 10.6GHz with the transmit power spectral density (PSD) less than -41.3 dBm/MHz in order to reduce the interferences between UWB system and the existing narrowband (NB) systems. Actually, there always exist some NB systems [2] with PSD much larger than UWB system, such as IEEE 802.11, GPS, and Bluetooth and so on. These NB systems may affect the communication quality of UWB system, it is necessary to reduce the NB interference of UWB system.

The concept of chirp pulse based UWB (Chirp-UWB) [3] is that the system spectrum of the chirp spread spectrum (CSS) technology based UWB system satisfies FCC regulation and also generates the desired power spectrum to avoid the interference with NB systems. Chirp-UWB system has remarkable superiority such as low complexity, long transmission distance, avoidance of high peak to average ratio and so on. There are some literatures about the performance analysis of UWB systems under additive white Gaussian noise

(AWGN) [4]–[8], but the detailed research about the performance analysis of Chirp-UWB system under multipath channel environment is not so much. Therefore, it is necessary to study the performance of Chirp-UWB system considering IEEE 802.11 NB interference (NBI) under multipath channel environment.

In this paper, we mainly study the chirp pulse design and also study the performance analysis of the Chirp-UWB system under multipath channel environment with NBI. In the performance analysis, we utilize the deterministic characteristic function (CF) based analysis method, and deduce the bit error rate (BER) formula of the Chirp-UWB system. Simulation results show that the accuracy of the presented CF based theoretical system performance analysis is better than the traditional Gaussian approximation based analysis method.

The organization of this paper is as follows. Chirp theory is presented in Section II. Section III introduces Chirp-UWB system model and the CF based performance analysis under NBI is given. Simulation results and discussion of the proposed system performance analysis are given in section IV. Finally, section V draws the conclusions.

II. CHIRP THEORY

A chirp pulse function in time domain can be written as [9],

$$c_{linear}(t) = a(t) \cos(2\pi f_0 t + \pi \mu t^2 + \varphi_0), \quad (1)$$

where $a(t)$ is the envelope of the chirp signal, f_0 is the centre frequency, μ is the chirp rate and φ_0 is the initial phase. In addition, $a(t)$ is zero outside its time interval T .

The chirp pulse's phase is as follows,

$$\theta(t) = 2\pi f_0 t + \pi \mu t^2 + \varphi_0. \quad (2)$$

The chirp rate is an important parameter which denotes the rate of the change of the instantaneous frequency $f_w(t)$, and it is given as,

$$\mu(t) = \frac{df_w(t)}{dt} = \frac{1}{2\pi} \frac{d^2\theta(t)}{dt^2}. \quad (3)$$

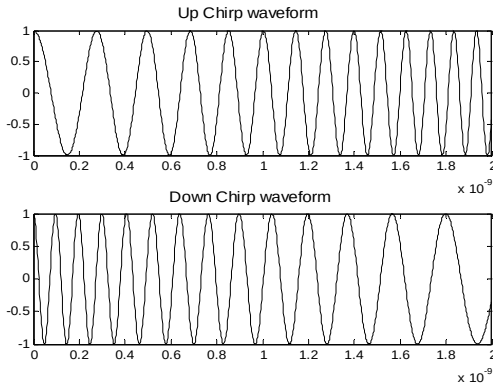


Figure 1. Up-chirp and down-chirp waveform

The chirp waveforms with $\mu(t) > 0$ are called up-chirps and those with $\mu(t) < 0$ are called down-chirps, as shown in Figure 1.

Here, we use the range of the instantaneous frequency to describe the bandwidth B and it can be expressed as,

$$B = |\mu(t)| \cdot T. \quad (4)$$

The chirp rate of the linear chirp waveform is a constant, which means that all the frequency bands have the same weights. Hence, we consider the design of the nonlinear chirp waveform for the NBI suppression, which comes from the results that the nonlinear chirp pulse has the best NBI suppression performance if its chirp rate achieves the highest at the NBI frequency band.

Similarly to the above linear chirp waveform, the expression of the adaptive *atan* chirp waveform is given by,

$$c(t) = \cos \left(2\pi f_i t + \left(-\frac{T_{fra}}{\alpha} \right) \cdot \frac{2\pi(f_i - f_i)}{\arctan(\alpha)} \cdot \left[\beta \cdot \arctan \beta - \frac{\ln(1 + \beta^2)}{2} \right] \right), \quad (5)$$

where f_i is the centre frequency of the interference signal, $f_i = 3.1\text{GHz}$, $0 \leq t \leq T$, $T_{fra} = (f_i - f_i)/\mu$ which corresponds to the centre frequency of NBI. For convenience, we define $\beta = (-t/T_{fra} + 1)\alpha$ and α is a very important parameter which identifies the instantaneous frequency.

The chirp rate has the highest value at the NBI frequency band, that is, the nonlinear chirp waveform through NBI band with the fastest speed and the NBI band has the small weight compared with other bands, and then the NBI can be excellently suppressed. In this way, the nonlinear chirp waveform can achieve the best interference suppression performance.

Due to the characteristic which the inequality weights during the whole frequency bands, we get that the nonlinear chirp waveforms have better characteristics than the traditional UWB pulses. So we can use chirp waveforms to substitute the traditional UWB pulses. On the receiver side, we apply the CF method to evaluate the NBI suppression performance of the Chirp-UWB system over multipath channels. This method can get more accurate performance.

III. DS-BPSK CHIRP-UWB SYSTEM MODEL AND CF BASED PERFORMANCE ANALYSIS

A. DS-BPSK Chirp-UWB System Model

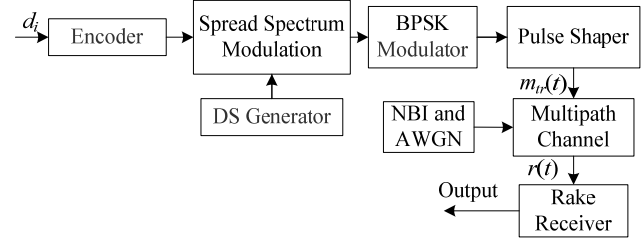


Figure 2. DS-BPSK Chirp-UWB System Model

In this paper, the system model is given in Figure 2. For a single user direct sequence spread spectrum binary phase shift keying (DS-BPSK) Chirp-UWB system, the transmitted signal is written as,

$$m_{tr}(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_c-1} d_i c_j p_{tr}(t - iT_b - jT_c), \quad (6)$$

where T_b is the duration occupied by one data bit, and T_c is the chip duration time of a signal waveform. $p_{tr}(t)$ is the transmitted waveform which is the chirp waveform here. N_c is the number of the pulses transmitted in one bit. c_j is the j -th DS code and its length is N_c . d_i is the i -th transmitted data bit. In general, we have $d_i \in \{\pm 1\}$, $c_j \in \{\pm 1\}$ and the bit interval is $T_b = N_c T_c$.

Considering the multipath channel, we can simplify its expression as,

$$h(t) = \sum_{l=0}^L \alpha_l \delta(t - \tau_l), \quad (7)$$

where L is the total number of multipath, α_l is the gain of the l -th path, τ_l is the delay of the l -th path.

IEEE 802.11a system [10] is one of the main interference sources in UWB spectrum for its high PSD compared with UWB signals. According to the characteristics of IEEE 802.11a system, it can be modelled as many different forms. Generally speaking, an NBI signal of IEEE 802.11a system with a single tone can be modelled as,

$$i(t) = \sqrt{2P} \cos(2\pi f_i t + \phi), \quad (8)$$

where P is the average power, f_i is the frequency of the interference signal and ϕ is the initial phase of the interference signal.

B. CF Based Performance Analysis

In the receiver side of the DS-BPSK Chirp-UWB system, the received signal with NBI is given as,

$$r(t) = r_{UWB}(t) + r_i(t) + n(t) = \sum_{l=1}^L \alpha_l m_{tr}(t - \tau_l) + \beta i(t - \varepsilon) + n(t), \quad (9)$$

where $r_{UWB}(t)$, $r_i(t)$ and $n(t)$ denote the received UWB multipath signal, the NBI signal and also the Gaussian noise

with $n(t) \sim N(0, \sigma_n^2)$, respectively. β is the corresponding path gain and ε is the corresponding path delay. $m_{tr}(t)$ is the transmitted signal. α_l is the transmission gain of the l -th path, τ_l is the delay of the l -th path.

The output decision variables of Rake receiver can be expressed as,

$$y_m = \sum_{f=1}^F w_f \int_{iT_b}^{(i+1)T_b} r(t) v(t - iT_b - \tau_f) dt = \sum_{f=1}^F w_f (S_{UWB} + I + n), \quad (10)$$

where $v(t) = \sum_{j=0}^{N_c-1} c_j p_{tr}(t - jT_c)$ is the template signal, F is the total correlator number of the Rake receiver, w_f is the weight factor of the f -th correlator, S_{UWB} is the useful signal component, I is the interference component, n is the AWGN component. Considering the output of the f -th correlator during the i -th symbol interval, the output components are S_{UWB} , I , and n , respectively.

The CF [11] of AWGN component with $n \sim N(0, \sigma_n^2)$ is,

$$\begin{aligned} \Phi_n(\psi) &= E[e^{j\psi n}] = \int_{-\infty}^{\infty} f(n) e^{j\psi n} dn \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_n} \exp(-\frac{n^2}{2\sigma_n^2}) e^{j\psi n} dn = \exp(-\sigma_n^2 \psi^2 / 2) \end{aligned} \quad (11)$$

S_{UWB} can be expressed by the following formula,

$$\begin{aligned} S_{UWB} &= \int_{iT_b}^{(i+1)T_b} r_{UWB}(t) v(t - iT_b - \tau_f) dt \\ &= \alpha_l d_i N_c E_p + \sum_{l=1, l \neq f}^L \alpha_l d_l R(\tau_l + \tau_f) \end{aligned} \quad (12)$$

where E_p is the energy of single transmitted pulse.

Therefore, the CF of S_{UWB} can be given as,

$$\Phi_{S_{UWB}}(\psi) = \cos(\alpha_l d_i N_c E_p) \cdot \cos(\alpha_l d_l R(\tau_l + \tau_f)). \quad (13)$$

With a single frequency interference case, the interference component I can be expressed as,

$$\begin{aligned} I &= \int_{iT_b}^{(i+1)T_b} r_l(t) v(t - iT_b - \tau_f) dt \\ &= \beta \sqrt{2P} \operatorname{Re}(e^{j2\pi f_l \varepsilon} \sum_{j=0}^{N_c-1} c(j) e^{j2\pi f_l (\frac{T_b}{2} + iT_b)}) \\ &\quad \cdot \int_{-T_b/2}^{+T_b/2} e^{-j2\pi f_l v} p_{tr}(v + \frac{T_b}{2} - jT_c - \tau_f) dv \end{aligned} \quad (14)$$

We define $w_i = 2\pi f_i$. Then, we can set,

$$P(w_i) = \int_{-T_b/2}^{+T_b/2} e^{-jw_i v} p_{tr}(v + \frac{T_b}{2} - jT_c - \tau_f) dv. \quad (15)$$

Utilizing chirp pulse with the time interval $[-\frac{T_c}{2}, \frac{T_c}{2}]$ and changing the integral interval into $(-\infty, \infty)$, we can see that $P(w_i)$ can be viewed as the Fourier transform of $p_{tr}(v + \frac{T_b}{2} - jT_c - \tau_f)$,

$$P(w_i) = \sqrt{\pi/\mu} \exp\left\{-j \frac{(w_0 - w)^2}{2\mu}\right\} \cdot \{[C(x_2) - C(x_1)] + j[S(x_2) - S(x_1)]\} e^{-jw_i(jT_c + \tau_f - \frac{T_b}{2})}, \quad (16)$$

$$\text{where } C(x) = \int_0^x \cos\left(\frac{\pi y^2}{2}\right) dy, \quad S(x) = \int_0^x \sin\left(\frac{\pi y^2}{2}\right) dy,$$

$$x_1 = \frac{-\mu T/2 + (w_0 - w)}{\sqrt{\mu\pi}}, \text{ and } x_2 = \frac{\mu T/2 + (w_0 - w)}{\sqrt{\mu\pi}}.$$

The interference component I is expressed as,

$$\begin{aligned} I &= \beta \sqrt{2P} \sum_{j=0}^{N_c-1} R_j \sqrt{\pi/\mu} \exp\left\{-j \frac{(w_0 - w)^2}{2\mu}\right\} \\ &\quad \cdot \{[C(x_2) - C(x_1)] + j[S(x_2) - S(x_1)]\}, \end{aligned} \quad (17)$$

where $R_j = c(j) \cos w_i(T_b + iT_b - jT_c - \tau_f + \varepsilon)$ and its CF [12] is,

$$\begin{aligned} \Phi_{R_j|c, \varepsilon}(\psi) &= E[e^{j\psi R_j} | c_j = c, \varepsilon = \varepsilon] \\ &= \cos[c \cos w_i(T_b + iT_b - jT_c - \tau_f + \varepsilon)]. \end{aligned} \quad (18)$$

Assume c_j is uniform distribution during $[0, N_c - 1]$, then the CF of I on the condition of ε is,

$$\begin{aligned} \Phi_{I|\varepsilon}(\psi) &= \frac{1}{N_1} \sum_{c=0}^{N_1-1} \prod_{j=0}^{N_c-1} \Phi_{R_j|c, \varepsilon} (\beta \sqrt{2P} \sqrt{\pi/\mu} \exp\{-j \frac{(w_0 - w)^2}{2\mu}\}) \\ &\quad \cdot \{[C(x_2) - C(x_1)] + j[S(x_2) - S(x_1)]\} \psi \end{aligned} \quad (19)$$

If NBI is synchronized with UWB, we have $\varepsilon = 0$ and the CF of I can be obtained as,

$$\begin{aligned} \Phi_I(\psi) &= \frac{1}{N_1} \sum_{c=0}^{N_1-1} \prod_{j=0}^{N_c-1} \Phi_{R_j|c} (\beta \sqrt{2P} \sqrt{\pi/\mu} \exp\{-j \frac{(w_0 - w)^2}{2\mu}\}) \\ &\quad \cdot \{[C(x_2) - C(x_1)] + j[S(x_2) - S(x_1)]\} \psi \end{aligned} \quad (20)$$

According to the above analysis of the decision variables and the CF of the interference signal, we can get the system average error probability with NBI which can be expressed as,

$$\begin{aligned} P_e &= P(y_m^{(1)} < 0) + P(y_m^{(0)} > 0) \\ &= P(\sum_{f=1}^F w_f (S_{UWB}^{(1)} + I_{CDMA} + n) < 0) + P(\sum_{f=1}^F w_f (S_{UWB}^{(0)} + I_{CDMA} + n) < 0). \end{aligned} \quad (21)$$

Because of the relationship between the cumulative distribution function and the CF, the average error probability can be obtained by the above two formulas,

$$P_e = \frac{1}{2} - \frac{1}{2\pi} \int_0^\infty \Phi_{S_{UWB}}(\psi) \Phi_I(\psi) \Phi_n(\psi) d\psi, \quad (22)$$

where $\Phi_{S_{UWB}}(\psi)$ is the CF of the useful signal part S_{UWB} , $\Phi_I(\psi)$ is the CF of the interference signal part I , $\Phi_n(\psi)$ is the CF of AWGN.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we present the performance analysis of the CF based DS-BPSK Chirp-UWB system. The comparison

with the traditional Gaussian approximation method is provided. With Gaussian approximation analysis, the signal to interference plus noise power ratio can be expressed as,

$$SINR = \frac{S_{UWB}^2}{N_0 + \sigma_n^2}, \quad (23)$$

where S_{UWB} is the useful signal component, σ_n^2 is the variance of AWGN, N_0 is the variance of NBI. The Gaussian pulse based average system error probability can be expressed as,

$$P_e = Q(\sqrt{SINR}) = Q\left(\frac{S_{UWB}}{\sqrt{N_0 + \sigma_n^2}}\right). \quad (24)$$

The detailed system parameters are showed as follows. The number of data symbols is set to be 10^5 , the bandwidth of the chirp pulse is from 3.1 GHz to 10.6 GHz, and its duration time is 10ns. We also consider the multipath environment with multipath resolution 5ns and the signal to interference ratio (SIR) is set as $SIR=-20$ dB. The interference carrier frequency is $f_i=5.3$ GHz with bandwidth $B_i=0.1$ GHz. The simulation is carried out in the CM1 and CM4 multipath channel model.

Figure 3 reveals the BER performance with different pulse waveforms. We obtain that the system with linear chirp pulse waveform can save 2.7 dB E_b/N_0 than the system applying Gaussian pulse with the same BER at 0.058. Furthermore, the system utilizing nonlinear chirp pulse waveform can save 2.5 dB E_b/N_0 than the system with linear chirp pulse waveform. We find that the chirp pulse based DS-BPSK UWB system have better performance than the traditional Gaussian pulse based UWB system.

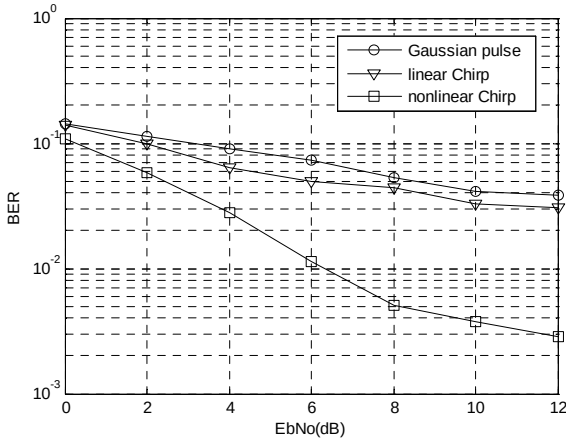


Figure 3. BER performance using different pulse waveforms

The BER performance under different multipath channel models is shown in Figure 4 and Figure 5. In CM1 channel model, we see that with the same BER at 10^{-2} , the system utilizing deterministic CF analysis method can have 2.0 dB E_b/N_0 advantage near the actual simulation than the system applying traditional Gaussian approximation method. Furthermore, in CM4 channel model, it is shown that the system with deterministic CF analysis method can have 1.0dB E_b/N_0 advantage near the actual simulation than the system applying traditional Gaussian approximation method with the same BER at 0.05. In CM4 channel, there are more multipath

that the characteristics of NBI is more close to the Gaussian distribution, so the performance of the two analysis methods and the actual performance don't vary that much. Compared with the above two Figure, we obtain that the proposed method presents more accurate performance than the Gaussian approximate method in CM1 channel. In more complex channel environment, the deterministic CF analysis method also obtains accurate system performance.

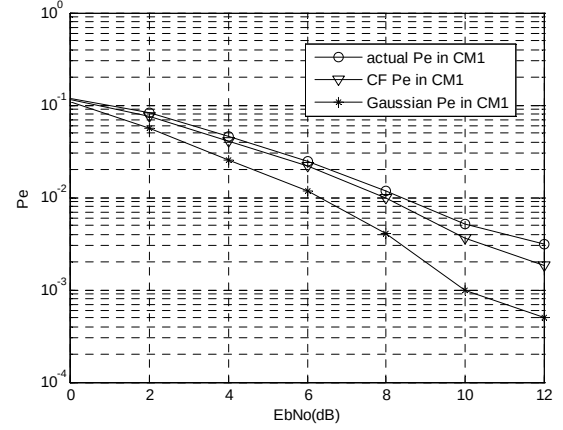


Figure 4. System Performance of the two analysis methods under CM1 channel

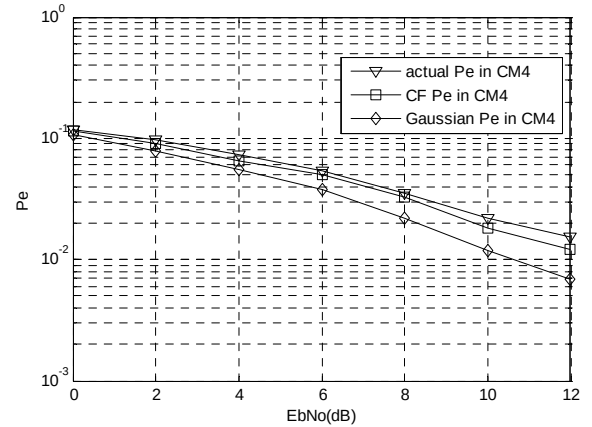


Figure 5. System Performance of the two analysis methods under CM4 channel

Comparing the simulation results based on the CF and Gaussian approximation method, we can see that the accuracy of the system performance with deterministic CF analysis method is better than that with Gaussian approximation method. Therefore, the Chirp-UWB system can be used in the complex UWB communication environment to avoid NBI and the proposed method can improve the accuracy of the performance analysis.

V. CONCLUSIONS

Considering the characteristics of the chirp waveform, we analyse the Chirp-UWB system under multipath channel environment. In order to alleviate the narrowband interferences within UWB spectrum, we also introduce the

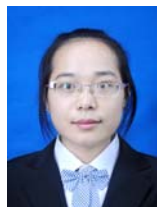
chirp pulse design with NBI suppression. At last, the deterministic CF based performance analysis of the Chirp-UWB system has been presented under multipath channels and it is shown that the deterministic CF analysis method is more accurate than Gaussian approximation method.

ACKNOWLEDGMENT

This research is supported by National Science Foundation of China (No.61301147) and Beijing Natural Science Foundation (No.4132055).

REFERENCES

- [1] M. Z. Win and R. A. Scholtz, "Impulse radio: How it works," *IEEE Commun. Lett.*, vol. 2, pp. 36-38, Feb. 1998.
- [2] X. L. Chu and R. D. Murch, "The Effect of NBI on UWB Time-Hopping Systems," *IEEE Trans. on Wireless Commun.*, vol.3, pp. 1431-1436, Sept. 2004.
- [3] A. Springer, W. Gugler, M. Huemer, L. Reindl, C. C. W. Ruppel and R. Weigel, "Spread Spectrum Communications Using Chirp Signals," *EUROCOMM 2000*, pp. 166-170, May. 2000.
- [4] M. M. Hamalainen, V. Hovinen, R. Tesi, J. H. J. Iinatti and M. Latva-aho, "On the UWB system coexistence with GSM900 UMTSWCDMA and GPS," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1712-1721, Dec. 2002.
- [5] L. Q. Yang and G. B. Giannakis, "A general model and SINR analysis of low duty-cycle UWB access through multipath with narrowband interference and rake reception," *IEEE Trans. Commun.*, vol. 4, no. 4, pp. 1818-1833, Jul. 2006.
- [6] M. M. Hamalainen, R. Tesi and J. Iinatti, "On the UWB system performance studies in AWGN channel with interference in UMTS band," *IEEE Conf. Ultra Wideband Syst. Technol.*, pp. 321-325, May. 2002.
- [7] B. Firoozbakhsh, T. G. Pratt and N. Jayant, "Analysis of IEEE 802.11a interference on UWB systems," *IEEE Conf. Ultra Wideband Syst. Technol.*, pp. 473-477, Nov. 2003.
- [8] B. Hu and N. C. Beaulieu, "Performance of an ultra-wideband communication system in the presence of narrowband BPSK- and QPSK-modulated OFDM interference," *IEEE Trans. Commun.*, vol. 54, no. 10, pp. 1720-1724, Oct. 2006.
- [9] X. T. Li, Z. Q. Bai and K. S. Kwak, "NBI Suppression UWB System based on Novel Nonlinear Chirp Pulses," *The 9th international Symposium Communication and Information Technology 2009 (ISCIT2009)*, pp. 1167-1170, Sept. 2009.
- [10] V. D. Chakravarthy, A. K. Shaw, M. A. Temole, "Cognitive Radio-An Adaptive Waveform with Spectral Sharing Capability," *IEEE Wireless Communications and Networking Conference*, vol. 2, pp. 724-729, Mar. 2005.
- [11] A. S. H. Mahmoud, "New Quadrature-Based Approximations for the Characteristic Function and the Distribution Function of Sums of Lognormal Random Variables," *IEEE Trans. Vehicular Technology*, vol. 59, no. 7, pp. 3364-3372, Sept. 2010.
- [12] M. De Gregorio, G. Tiberi, A. Monorchio, R. Mittra, "Solution of wide band scattering problems using the characteristic basis function method," *IET Microwaves, Antennas & Propagation*, vol. 6, pp. 60-66, Jan. 2012.



Changhui Wang was born in Shandong province in 1989. She got her BS degree from Jinan University, Jinan, China in 2011. She is currently a master student of School of Information Science and Engineering, Shandong University, Jinan, China. Her research interests include ultra wideband wireless communication and complementary code technology.



research interests include cooperative communication and MIMO system, ultra wideband technology, cognitive radio and etc.

Zhiquan Bai received his M.S. degree from Shandong University, Jinan, China in 2003, and Ph.D degree with honor from INHA University under the Grant of Korean Government IT Scholarship, Korea in 2007. He was a research fellow in UWB Wireless Communications Research Center, INHA University, Korea from 2007 to 2008. After that, he has been an associate professor in School of Information Science and Engineering, Shandong University, China. His



Shen Gao was born in 1989 in Shandong province. Now she is a M.S. student in School of Information Science and Engineering, Shandong University, Jinan, China. Her research interests include narrowband interference suppression in ultra wideband system and orthogonal complementary code.



security, and the next generation wireless systems such as 3GPP LTE, UWB, MIMO, Cognitive radio etc.

Peng Gong received the BS degree in Mechanical Engineering from Beijing Institute of Technology, Beijing, China, in 2004, and the MS and Ph.D. degrees from the Inha University, Korea, in 2006 and 2010, respectively. In July 2010, he joined the School of Mechanical Engineering, Beijing Institute of Technology, China. His research interests include link/system level performance evaluation and radio resource management in wireless systems, information



University, Korea, as a professor. He is the director of UWB Wireless Communications Research Center in Inha University and Inha Fellow Professor (IFP). His research interests include multiple access communication systems, mobile communication systems, UWB radio systems, and ad hoc networks, as well as high-performance wireless Internet.

Kyungsup Kwak received his BS degree from Inha University, Incheon, Korea, in 1977; MS degree from the University of Southern California in 1981; and PhD degree from the University of California at San Diego in 1988. From 1988 to 1989, he was a member of Hughes Network Systems, San Diego, California. From 1989 to 1990, he was with the IBM Network Analysis Center, North Carolina. Since then, he has been with the School of Information and Communication, Inha