

Early Stopping Criterion for Message-Passing Decoding of LDLC

Jin Xu *, Can Duan *, Danfeng Zhao **, Yang Wang **, Feng Xie **

* Wireless Technology preresearch Team, ZTE Corporation, Shenzhen, 518055, China

** College of Information and Communication Engineering, Harbin Engineering University, Harbin, 150001, China

xu.jin7@zte.com.cn, duan.can@zte.com.cn

Abstract—An early stopping criterion is proposed for low-density lattice codes (LDLC) to reduce the number of decoding iterations. The stopping criterion is based on a new metric which is used to predict the convergence of the iterative decoding algorithm for LDLC. Simulation results demonstrate that the proposed criterion can decrease the average iteration number considerably while the decoding performance degradation is within 0.2dB in the low symbol error rate region. Besides, the proposed criterion can provide a flexible tradeoff between performance and complexity.

Keywords—low-density lattice codes; iterative decoding; stopping criterion; message passing; check equations

I. INTRODUCTION

Low-density lattice codes (LDLC) have been recently proposed in [1]. The LDLC are designed directly in the Euclidean space and can approach the additive white Gaussian noise (AWGN) channel capacity with an iterative decoder whose complexity is linear in block length. All these characteristics make the LDLC a powerful coding scheme for the AWGN channel [2, 3].

Currently, the iterative decoder for the LDLC can be divided into two categories: quantized decoders [1] and parametric decoders [4-7]. The quantized decoder is based on the quantization of the passing messages, which results in large storage requirement and computational complexity. In order to decrease the storage requirement, a parametric decoder is proposed firstly in [4] and the passing messages are represented as mixtures of Gaussians in this decoder. Then a more efficient parametric decoder is presented in which two new Gaussian mixture reduction algorithms are used to control the number of Gaussians in mixtures [5]. Simulation results show that the computational complexity of the decoder can be reduced significantly. Following this direction, in [6] and [7] the passing messages are approximated into single Gaussian functions which can be described by just two parameters, the mean and variance. Consequently, both the memory and computational complexity are further reduced. In [8] a max-product algorithm for LDLC is derived directly in the Euclidean space and an efficient implementation based on the parametric approach is given. The max-product algorithm can improve the performance for small dimensional LDLC.

Besides, the parametric decoder is extended to the complex case in [9].

However, all the aforementioned iterative decoders of the LDLC do not have an early stopping criterion and the iterative algorithms can not terminate until the maximum iteration number is reached. Considering that the early stopping criterion can efficiently decrease the average number of iterations, reduce the processing time and save the power consumption, we propose an early stopping criterion for the LDLC decoders in the paper. The average iteration number can be reduced considerably by this stopping criterion in the low symbol error rate (SER) region.

The rest of the paper is organized as follows. Section II reviews the definition of the LDLC. Then, the proposed early stopping criterion is presented in Section III. In Section IV, simulation results are provided and we conclude the paper in Section V.

II. LOW DENSITY LATTICE CODES

The LDLC are lattices. According to the lattice theory, an n dimensional lattice is a discrete set of points in R^n and can be defined by an n dimensional square matrix $\mathbf{G}$ which is called a generator matrix. Given the generator matrix $\mathbf{G}$, a lattice point $\mathbf{x}$ can be written as

$$\mathbf{x} = \mathbf{G}\mathbf{b} \quad (1)$$

where $\mathbf{b}$ is an n dimensional vector of integers.

In LDLC the inverse of the generator matrix is referred to as the parity check matrix $\mathbf{H} = \mathbf{G}^{-1}$ and $\mathbf{H}$ is sparse. In this case, for a lattice point $\mathbf{x}$ we have

$$\mathbf{H}\mathbf{x} = \mathbf{b} \in \mathbb{Z}^n \quad (2)$$

Similarly to low-density parity-check codes (LDPC), the number of non-zero elements in a row or column of $\mathbf{H}$ is called a row degree or column degree. When all the row degrees and column degrees are equal to a common degree d and the d non-zero values in each row and column of $\mathbf{H}$ are the same except for a possible change of order and sign, the LDLC defined by $\mathbf{H}$ is called “Latin square LDLC”. And the

sequence of these d non-zero values is referred to as the generating sequence of the Latin square LDLC.

An example of the parity check matrix of a Latin square LDLC with $n=6$ and $d=3$ is given below and the corresponding bipartite graph is shown in Fig.1.

$$\mathbf{H} = \begin{bmatrix} 0 & -0.8 & 0 & -0.5 & 1 & 0 \\ 0.8 & 0 & 0 & 1 & 0 & -0.5 \\ 0 & 0.5 & 1 & 0 & 0.8 & 0 \\ 0 & 0 & -0.5 & -0.8 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0.5 & 0.8 \\ 0.5 & -1 & -0.8 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

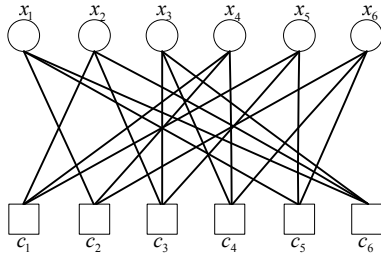


Figure 1. The bipartite graph of an LDLC

In this paper we use Latin square LDLC and we normalize the determinant of $\mathbf{H}$ to have $|\det(\mathbf{H})| = 1$.

III. STOPPING CRITERION FOR LDLC DECODING

In the decoding of LDPC the iterative algorithm can be terminated if the estimated codeword satisfies all parity checks. Similarly to LDPC, there exist parity check matrix $\mathbf{H}$ and check equations in LDLC. Therefore, the check equations can be used as an early stopping criterion in theory. However, the check equations in LDLC are calculated in the real number field and they can rarely be satisfied due to the quantization error, Gaussian approximation and limited operational precision even if the estimated integer information vector is correct. So the check equations cannot be used directly in LDLC. In this section we propose an early stopping criterion for LDLC decoding and a new metric base on the check equations is employed to judge whether further iterations are necessary.

Assume that a LDLC codeword $\mathbf{x} = \mathbf{G}\mathbf{b}$ was transmitted, where $\mathbf{G}$ is the generator matrix of the LDLC and $\mathbf{b}$ is an integer information vector. In the decoder the iterative message-passing algorithm is used to estimate the transmitted codeword and the corresponding integer vector. We denote the estimated codeword at the end of t th iteration by $\hat{\mathbf{x}}^{(t)}$ and the estimated integer vector can be obtained as

$$\hat{\mathbf{b}}^{(t)} = \lfloor \mathbf{H}\hat{\mathbf{x}}^{(t)} \rfloor \quad (4)$$

where $\lfloor \cdot \rfloor$ represents the element-wise rounding operation.

According to [1], the estimated codeword can converge to the correct lattice point for the decodable codeword and the distance between $\mathbf{H}\hat{\mathbf{x}}^{(t)}$ and $\mathbf{b}$ decreases gradually as the iteration number increases. In order to exploiting the check equations to avoid the unnecessary iterations, we define the distance from $\mathbf{H}\hat{\mathbf{x}}^{(t)}$ to its nearest integer vector as the metric to check the tendency to converge to the correct lattice point.

$$D^{(t)} = \frac{1}{n} \sum_{i=1}^n \left| \left(\mathbf{H}\hat{\mathbf{x}}^{(t)} \right)_i - \left(\lfloor \mathbf{H}\hat{\mathbf{x}}^{(t)} \rfloor \right)_i \right| \quad (5)$$

where $(\cdot)_i$ represents the i th element of a vector. Considering the property of the rounding operation the $D^{(t)}$ satisfies

$$0 \leq D^{(t)} \leq 0.5 \quad (6)$$

and the value is independent of the codeword length.

Fig. 2 plots the $D^{(t)}$ of a LDLC with dimension $n=100$ as a function of the iteration number. The results show the evolution of the $D^{(t)}$ for a decodable and undecodable codeword. We can observe that the $D^{(t)}$ has a tendency to converge to zero for the decodable codeword and changes irregularly for the undecodable codeword.

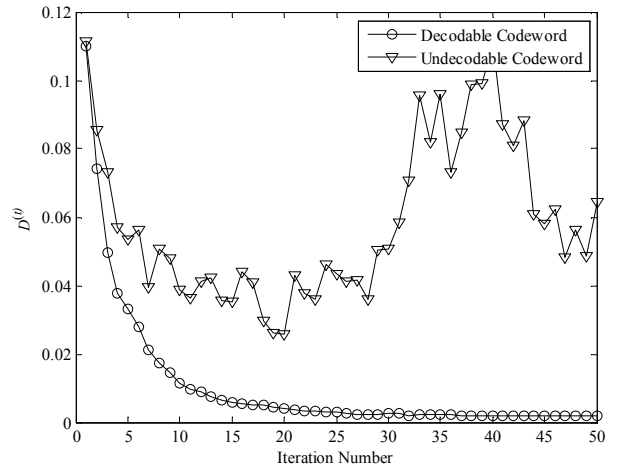


Figure 2. Metric $D^{(t)}$ of a LDLC with dimension $n=100$

Based on this observation, we propose an early stopping criterion for the LDLC decoding and introduce two parameters in the proposed stopping criterion: a distance threshold α and a depth factor T . If the $D^{(t)}$ is lower than the distance threshold α and decreases continually for $T-1$ times, the iterative decoding of LDLC terminates. The stopping condition can be represented as follows.

$$\begin{cases} D^{(t)} < \alpha \\ D^{(t)} > D^{(t+1)} > \dots > D^{(t+T-1)} \end{cases} \quad (7)$$

Obviously, the proposed early stopping criterion only requires $T+1$ real-number memory units to store T distances and the distance threshold α . Besides, the equation (5) should be calculated at each iteration and the computational complexity added by the equation is $O(n \cdot d)$. Considering the high computational complexity of the LDLC decoding algorithm at each iteration and the decrease of the iteration number due to early stopping, the overall complexity of the decoding algorithm can be actually reduced by the early stopping criterion.

IV. SIMULATION RESULTS

To evaluate the performance of the proposed early stopping criterion we perform numerous Monte Carlo simulations in the AWGN channel using a Latin square LDLC with $n = 100$ and $d = 5$. The generating sequence of the Latin square LDLC is $\{1/2.31, 1/3.17, 1/5.11, 1/7.33, 1/11.71\}$. Additionally, the iterative decoding algorithm in [5] is employed in the decoder and parameters of the algorithm are the same as those in [5]. The SER and average iteration number of the decoder with the early stopping criterion are simulated for different values of stopping condition parameters and compared to that of the original decoder in [5]. The maximum iteration number in the decoder is set to be 50.

The SER performance of the iterative decoding algorithm without and with the early stopping criterion is given in Fig.3. The parameter α for the stopping criterion is set to be 0.05. The results show that the SER performance gets better when the depth factor T is set to be larger. Compared to the performance of the original decoding algorithm, the performance degradation due to the early stopping is within 0.2dB as $T = 13$. The performance loss is due to the fact that some codewords may converge to the incorrect lattice point.

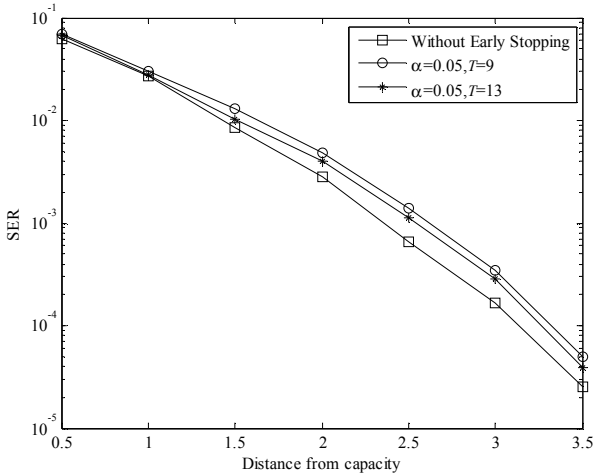


Figure 3. SER performance of the LDLC

Fig.4 depicts the average iteration number of the iterative decoding algorithm with the early stopping criterion when $\alpha = 0.05$. It can be observed that the smaller the value of T , the less the value of the average iteration number. In addition, the reduction of the average iteration number is more

considerable in the low SER region. For example, the average iteration number can be reduced by 54.6% when $T = 13$ and SER is in the order of 10^{-5} .

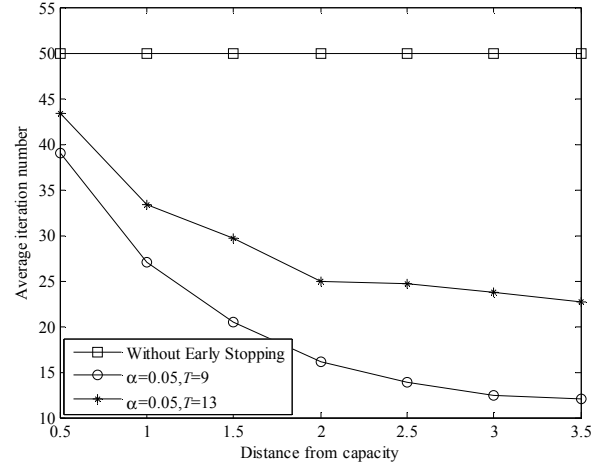


Figure 4. Average iteration number of the iterative decoding algorithm

Fig.5 and Fig.6 show the SER performance and average iteration number of the iterative decoding algorithm with the early stopping criterion when the distance threshold has a smaller value, $\alpha = 0.01$. The results indicate that the average iteration number can also be reduced significantly by the early stopping criterion while the SER performance is close to that of the iterative decoding algorithm without early stopping. Besides, compared with the results in Fig.3 and Fig.4, the average iteration number for $\alpha = 0.05$ is smaller when the same SER performance is obtained.

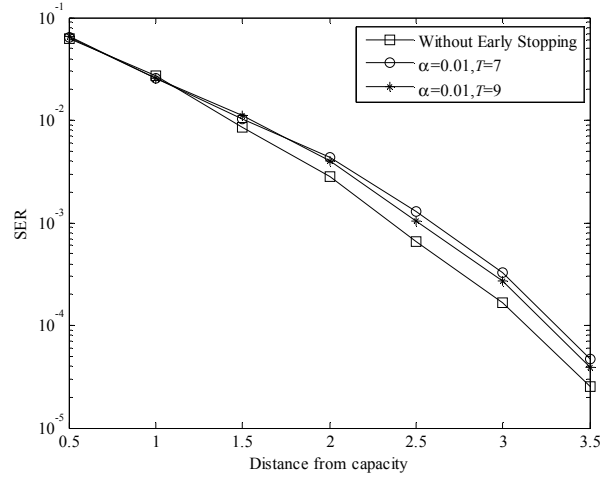


Figure 5. SER performance of the LDLC

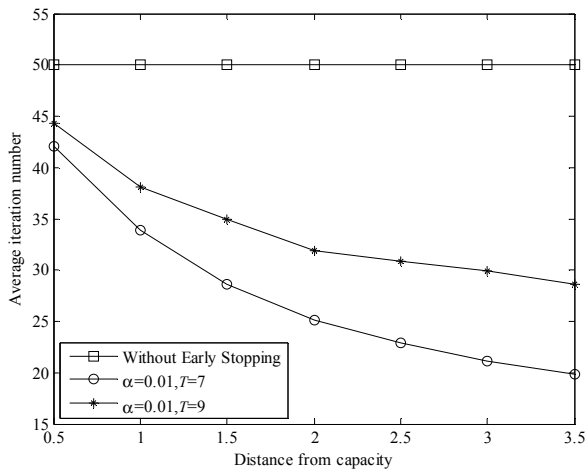


Figure 6. Average iteration number of the iterative decoding algorithm

V. CONCLUSIONS

In this paper we propose an early stopping criterion for the iterative decoding of LDLC. A new metric and two parameters are adopted to predict the convergence of the decoder and avoid the unnecessary iterations. Monte Carlo simulation results show that the average iteration number can be reduced considerably by the early stopping criterion while the performance loss compared with the original iterative decoder is limited to 0.2 dB. Additionally the proposed early stopping criterion can be combined with all the iterative decoding algorithms for LDLC.

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REFERENCES

- [1] N Sommer, M Feder and O Shalvi, "Low-density lattice codes," IEEE Transactions on Information Theory, vol. 54, no. 4, pp. 1561-1585, 2008.
- [2] N S Ferdinand, M Nokleby and B Aazhang, "Low density lattice codes for the relay channel" in Proceedings of IEEE International Conference on Communications, Budapest, 2013, pp. 3035 - 3040.
- [3] Y Wang and A Burr, "Physical layer network coding via low density lattice codes," in Proceedings of European Conference on Networks & Communications, Bologna, 2014, pp. 1 - 5.
- [4] B Kurkoski and J Dauwels, "Message-passing decoding of lattices using Gaussian mixtures," in Proceedings of IEEE International Symposium on Information Theory, Toronto, ON, 2008, pp. 2489 - 2493.
- [5] Y Yona and M Feder, "Efficient parametric decoder of low density lattice codes," in Proceedings of IEEE International Symposium on Information Theory, Seoul, 2009, pp. 744 - 748.

- [6] B Kurkoski and J Dauwels, "Reduced-memory decoding of low-density lattice codes," IEEE Communications Letters, vol. 14, no. 7, pp. 659-661, 2010.
- [7] Y Li and Z Bie, "Message-passing decoding algorithm of low-density lattice codes with Gaussian approximation," in Proceedings of IEEE International Conference on Wireless Information Technology & Systems, Maui, HI, 2012, pp. 1 - 4.
- [8] Y Yona and M Feder, "Max-product algorithm for low density lattice codes," in Proceedings of IEEE International Symposium on Information Theory, Cambridge, MA, 2012, pp. 1727 - 1731.
- [9] Y Yona and M Feder, "Complex low density lattice codes," in Proceedings of IEEE International Symposium on Information Theory, Austin, TX, 2010, pp. 1027 - 1031.



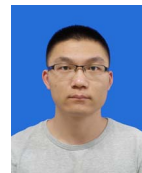
Jin Xu was born in the Hubei Province, P. R. China, in 1978. He received the Ph. D degree in engineering from the Wuhan University, P. R. China in 2007. He is currently with ZTE Corp., where he is a senior engineer in the wireless pre-research team. His research interests include channel coding, network coding, multiple-access communications as well as their application in cellular mobile networks.



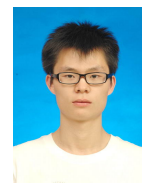
Can Duan was born in Hebei Province, China, received the MS degree in Information engineering from Nanjing University of Posts and Telecommunications, in 2010. She joined ZTE Corp. in 2010. Currently, she is a senior researcher in the wireless pre-research team. Her research interests include channel coding and multiple-access communications as well as their application in cellular mobile networks.



Dan-feng Zhao received the B.S. and Ph.D. degrees in college of information and communication engineering from Harbin Engineering University, Harbin, China, in 1983, and 2002. He has been a professor and an assistant dean of the college of information and communication engineering, Harbin Engineering University in 2002 and from 2002 to 2012, respectively. His current research interests are in areas of modern communication systems, communications and signal processing.



Yang Wang received the B.S. degree in communication engineering from Harbin Engineering University, China, in 2010. Currently, he is working toward a Ph.D. degree with the college of information and communication engineering, Harbin Engineering University. His current research interests include MIMO systems and advanced coding and modulation.



Feng Xie received the B.S. degree in communication engineering from Harbin Engineering University, China, in 2013. Currently, he is working toward a M.S. degree with the college of information and communication engineering, Harbin Engineering University. His current research interests include signal processing technology and advanced coding and modulation.