

A novel multi-carrier waveform with high spectral efficiency: semi-orthogonal frequency division multiplexing

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Abstract—In this paper, semi-orthogonal frequency multiplexing (SOFDM) is proposed, which has doubled spectral efficiency over OFDM by halving the subcarrier spacing. SOFDM is orthogonal in real field but non-orthogonal in complex field. To cancel the inter carrier interference caused by non-orthogonality, linear equalization algorithms are introduced based on inter carrier cross-correlation matrix. However SOFDM still suffer a performance loss due to the white noise power amplification effect of linear equalization. To further improve the performance, by use of the special property of inter carrier cross-correlation matrix, complementary SOFDM is provided, which is orthogonal in both real and complex field. It has identical spectral efficiency with OFDM but obtain a diversity gain in fading channel. And a HARQ-style flexible complementary SOFDM is introduced, which can obtain not only performance gain but also spectral efficiency gain.

Keywords—semi-orthogonal frequency division multiplexing; inter carrier cross-correlation; inter carrier interference; zero forcing; minimum mean square error

I. INTRODUCTION

Nowadays, the exponential increase of the data traffic demand justifies the design of a novel 5th Generation (5G) radio access technology (RAT) aiming at 10 Gbps upper data rate and sub-ms latency [1]. In addition to the data rates, the number of devices will also grow exponentially. To meet these requirements, worldwide wireless communication researchers both in academic area [2] [3] and industry area start to study new technologies.

New waveform, as one enabling technology, is one of the hottest topic in 5G research. Waveform can be divided into two categories: single carrier waveform and multiple carrier (MC) waveform. The latter one attracts more attention for numerous advantages. The most popular MC waveform is certainly orthogonal frequency division multiplexing (OFDM) since it is widely used in current wireless communication systems such as 3GPP LTE and WiMAX. OFDM has many advantages [4] [5] but needs perfect synchronization in time and frequency domain and it has a relative large side lobe which will make OFDM less attractive to some application such as cognitive radio [6]. Researchers continue to seek for new waveform to further increase spectral efficiency or suppress side lobe to fit

to flexible access spectrum. One important waveform is filter bank multicarrier (FBMC) [7]-[10]. Instead of sinc-pulse like OFDM, its subcarriers have more choices according to filter design to reduce side lobe levels [11]. FBMC has a potential to improve spectral efficiency by enabling transmission without cyclic prefix and with reduced guard bands [12]. Conventional FBMC operates based on real-valued modulation such as offset-QAM or PAM because it is only orthogonal in real field. Recently, A QAM-based FBMC scheme has been proposed [13]. This system utilizes two different filters to even and odd subcarrier symbols so that subcarriers become orthogonal in complex plane. However, both OQAM-FBMC and QAM-FBMC have the same spectral efficiency [14]. In ideal environments such as AWGN channel, when CP is unnecessary, they still have the identical spectral efficiency with OFDM [15] [16]. In order to further increase spectral efficiency, we propose a new MC waveform which has a halved subcarrier spacing than OFDM.. As a result, the spectral efficiency is almost doubled over OFDM but the subcarriers become non-orthogonal in complex field. In fact, as we will prove later, the orthogonality only exists in even or odd subcarriers. However, these subcarriers are still orthogonal in real plane which has been noticed in first studies about multicarrier systems [17] [18]. That's why we call it semi orthogonal frequency division multiplexing (SOFDM).

The paper is organized as follows. In section II, SOFDM concept and equalization algorithm in frequency domain at the receiver side is introduced. In section III, complementary SOFDM is introduced. And simulation results are presented in section IV. Section V concludes the paper.

II. SOFDM

A. System Model

The SOFDM signal can be obtained as:

$$x(t) = \frac{1}{\sqrt{T}} \sum_l \sum_{k=1}^K s_k(l) \cdot p_T(t-l \cdot T) \cdot e^{j \cdot 2\pi f_k t} \quad (1)$$

where T is the duration time of one symbol, l and k are symbol and subcarrier index respectively, K is the total subcarrier number, $s_k(l)$ is subcarrier data symbol

and $j = \sqrt{-1}$. The $p_T(t)$ is prototype filter at transmitter side and is a rectangular pulse of width T and the subcarrier spacing is $1/2T$. The subcarrier waveform of SOFDM can be expressed as:

$$g_k(t) \triangleq \frac{1}{\sqrt{T}} p_T(t) e^{j2\pi f_k t} = \frac{1}{\sqrt{T}} e^{j2\pi f_k t} (0 < t < T) \quad (2)$$

where $f_k = \frac{k}{2T}$. Then inter carrier cross-correlation is defined as:

$$\rho_{f_{k_1}, f_{k_2}} = \int_0^T g_{k_1}(t) \cdot g_{k_2}^*(t) dt = \frac{1}{T} \int_0^T e^{j2\pi(f_{k_1} - f_{k_2})t} dt \quad (3)$$

where k_1 and k_2 are subcarrier index. After calculation, it can be written as:

$$\rho_{\Delta f} = \frac{\sin(\pi \Delta f T)}{\pi \Delta f T} + j \cdot \frac{1 - \cos(\pi \Delta f T)}{\pi \Delta f T} \quad (4)$$

where $\Delta f \triangleq f_{k_1} - f_{k_2}$. Its value is shown in Fig. 1.

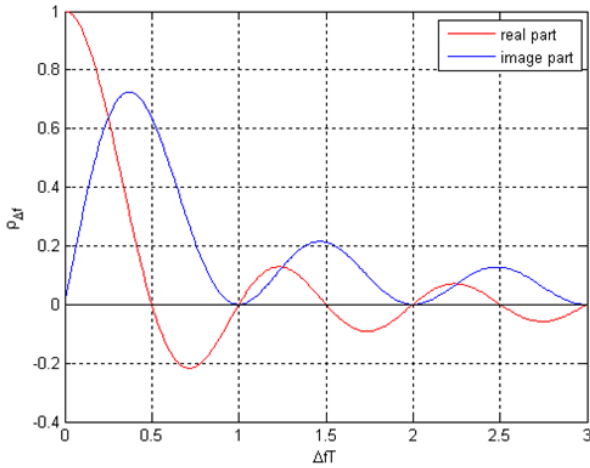


Fig. 1. Inter carrier cross-correlation

It can be seen that subcarriers of SOFDM are only orthogonal in real plane and non-orthogonal in complex plain since the subcarrier spacing is $1/2T$. And also we can see that the inter carrier cross-correlation is 0 or pure image-valued between two different subcarriers.

B. Transceiver

The baseband equivalent of SOFDM transceiver is depicted in Fig. 2. The normalized factor $1/\sqrt{T}$ is ignored in it for

simplicity. The $p_R(t)$ is prototype filter at receiver side. In this paper, we assume $p_R(t) = p_T(t)$.

The transmitted signal can be expressed as (1). The received signal in AWGN channel is obtained as:

$$y(t) = x(t) + n(t) \quad (5)$$

where $n(t)$ is zero mean additive white noise with variance of σ^2 . After the de-filter, the signal of l -th MC symbol is a vector and can be expressed as:

$$\mathbf{s}'(l) = [s'_1(l) \ s'_2(l) \ \dots \ s'_K(l)]^T \quad (6)$$

where $(\cdot)^T$ is transpose operator and the element $s'_k(l)$ of the vector $\mathbf{s}'(l)$ is obtained as:

$$\begin{aligned} s'_k(l) &= \frac{1}{\sqrt{T}} \int_0^T y(t) \cdot p_R(t) \cdot e^{-j2\pi f_k t} dt \\ &= s_k(l) + \sum_{\substack{i=1 \\ i \neq k}}^K \rho_{f_i, f_k} \cdot s_i(l) + n'_k(l) \end{aligned} \quad (7)$$

In (7), the second part is inter carrier interference and the last part is additive white noise which is obtained as:

$$n'_k(l) = \frac{1}{\sqrt{T}} \int_0^T n_l(t) \cdot e^{j2\pi f_k t} dt \quad (8)$$

where $n_l(t)$ is additive white noise in l -th MC symbol. So (6) can be re-written as:

$$\mathbf{s}'(l) = \mathbf{R} \cdot \mathbf{s}(l) + \mathbf{n}'(l) \quad (9)$$

$$\mathbf{n}'(l) \triangleq [n'_1(l) \ n'_2(l) \ \dots \ n'_K(l)]^T \quad (10)$$

where $\mathbf{R}$ is inter carrier cross-correlation matrix of size $K \times K$, and its element $R_{m,n} \triangleq \rho_{f_m, f_n}$, m and n is row and column index respectively, $1 \leq m, n \leq K$. According to (9), the linear zero forcing (ZF) estimation of $\mathbf{s}(l)$ can be obtained as:

$$\hat{\mathbf{s}}_{ZF}(l) = \mathbf{G}_{ZF} \cdot \mathbf{s}'(l) \quad (11)$$

$$\mathbf{G}_{ZF} \triangleq (\mathbf{R}^H \cdot \mathbf{R})^{-1} \cdot \mathbf{R}^H \quad (12)$$

where $\mathbf{G}_{ZF}$ is ZF equalization matrix, $(\cdot)^{-1}$ is inverse operator and $(\cdot)^H$ is Hermitian transpose operator. And the

linear minimum mean square error (MMSE) estimation can be obtained as:

$$\hat{\mathbf{s}}_{\text{MMSE}}(l) = \mathbf{G}_{\text{MMSE}} \cdot \mathbf{s}'(l) \quad (13)$$

$$\mathbf{G}_{\text{MMSE}} = \arg \min_{\mathbf{G}} \left(E \left(\left\| \hat{\mathbf{s}}(l) - \mathbf{s}(l) \right\|^2 \right) \right) \quad (14)$$

where $E(\cdot)$ is mathematical expectation function and $\mathbf{G}_{\text{MMSE}}$ is linear MMSE equalization matrix. The solution of (12) can be expressed as:

$$\mathbf{G}_{\text{MMSE}} = (\mathbf{R}^H \cdot \mathbf{R} + \sigma^2 \cdot \mathbf{R})^{-1} \cdot \mathbf{R}^H \quad (15)$$

Please note:

$$E(\mathbf{n}'(l) \cdot \mathbf{n}'^H(l)) = \sigma^2 \cdot \mathbf{R} \quad (16)$$

It should be noted that both linear ZF and MMSE equalization algorithm require $\mathbf{R}$ is a full-rank matrix. However, this condition is not always satisfied for all the possible value of total subcarrier number K . If $\mathbf{R}$ is not a full-rank matrix, to obtain a linear equalization, we have to set m mute subcarriers, $K - m \leq I$, where I is the rank of the matrix $\mathbf{R}$. As we have pointed out in section I, the SOFDM is orthogonal in real field. So if subcarrier data symbols are all real-valued, $\hat{\mathbf{s}}(l)$ can be simply obtained as:

$$\hat{\mathbf{s}}(l) = \text{Re}(\mathbf{s}(l)) \quad (17)$$

where $\text{Re}(\cdot)$ denotes real part. For complex-valued subcarrier symbols, the equalization is always necessary but the linear equalization will amplify the additive white noise power so that SOFDM will still suffer performance loss compared to OFDM which is ICI free. So we will propose a special scheme of SOFDM to improve the performance based on the property of SOFDM's inter carrier correlation matrix.

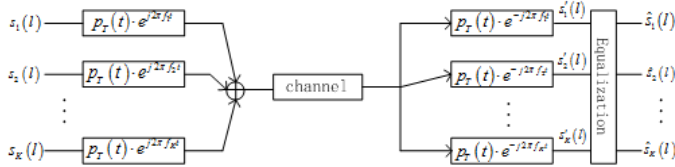


Fig. 2. SOFDM transceiver

III. COMPLEMENTARY SOFDM

The property of $\mathbf{R}$ is very important to SOFDM. One useful property is:

$$\mathbf{R} + \mathbf{R}^T = \mathbf{R} + \mathbf{R}^* = 2 \cdot \mathbf{I} \quad (18)$$

where $\mathbf{I}$ is the unit matrix. This implies that if we can design a multi-carrier system whose inter carrier cross-correlation matrix is $\mathbf{R}^T$ and combine its received signal with that of

SOFDM, the inter carrier interference (ICI) will be completely removed. We call this system complementary SOFDM. It has two parts: original part and complementary part. The former one is described in sector II which has an inter carrier correlation matrix of $\mathbf{R}$ and the latter one has an inter carrier correlation matrix of $\mathbf{R}^T$. These two parts occupy orthogonal resources (in time or frequency domain) and carry the identical data symbols in each subcarrier. There are a lot of methods to construct the commentary part. We will introduce one simple design in this section.

A. Transceiver

The only difference between original part and complementary part of complementary SOFDM is prototype filters. For complementary part, the prototype-filter is:

$$p_k^c(t) = \begin{cases} (-1)^{k+1}, & 0 < t < T \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

where k is subcarrier index, $1 \leq k \leq K$, K is total subcarrier number. Please note that prototype filter is identical at both transmitter and receiver side. The subcarrier waveform can be obtained as:

$$g_k^c(t) \triangleq \frac{(-1)^{k+1}}{\sqrt{T}} e^{j2\pi f_k t} \quad (0 < t < T) \quad (20)$$

Then the inter carrier cross-correlation is:

$$\rho_{f_{k_1}, f_{k_2}}^c = \frac{(-1)^{k_1+k_2}}{T} \int_0^T e^{j2\pi(f_{k_1}-f_{k_2})t} dt \quad (21)$$

where k_1 and k_2 are subcarrier index. The relationship between $\rho_{f_{k_1}, f_{k_2}}$ and $\rho_{f_{k_1}, f_{k_2}}^c$ can be expressed as:

$$\rho_{f_{k_1}, f_{k_2}}^c = \begin{cases} \rho_{f_{k_1}, f_{k_2}} = 0, & k_1 + k_2 \in \text{even} \\ -\rho_{f_{k_1}, f_{k_2}} = \rho_{f_{k_2}, f_{k_1}}, & \text{otherwise} \end{cases} \quad (22)$$

Then we have:

$$\mathbf{R}^c = \mathbf{R}^T \quad (23)$$

where $\mathbf{R}^c$ is inter carrier cross-correlation matrix of complementary part, its element $R_{m,n}^c \triangleq \rho_{f_m, f_n}^c$, m and n is row and column index, $1 \leq m, n \leq K$. The signal of complementary part is:

$$x^c(t) = \frac{1}{\sqrt{T}} \sum_l \sum_{k=1}^K s_k(l) \cdot p_k^c(t-l \cdot T) \cdot e^{j2\pi f_k t} \quad (24)$$

Similar to (9), after de-filter in AWGN channel, the signal of complementary part can be written as:

$$\mathbf{s}'^c(l) = \mathbf{R}^T \cdot \mathbf{s}(l) + \mathbf{n}'^c(l) \quad (25)$$

where $\mathbf{n}'^c(l)$ is white noise in l -th symbol of complementary part of SOFDM after is de-filter. According to (9), (18), (23) and (25), we can have:

$$\mathbf{s}'(l) + \mathbf{s}'^c(l) = 2 \cdot \mathbf{s}(l) + \mathbf{n}'(l) + \mathbf{n}'^c(l) \quad (26)$$

Then we obtained an estimation of $\mathbf{s}(l)$ without ICI. So equalization becomes unnecessary for complementary SOFDM.

Based on (26), we can design the complementary SOFDM as in Fig. 3. It is a group-wise system, each group is divided into original and complementary part and there are L multi-carrier symbols in each part ($L \geq 1$). Two parts carry the identical subcarrier data symbol $\mathbf{s}(l)$ where l is MC symbol index ($1 \leq l \leq L$). The group index is ignored for simplicity. And “symbol” in Fig. 3 stands for “multi-carrier symbol”.

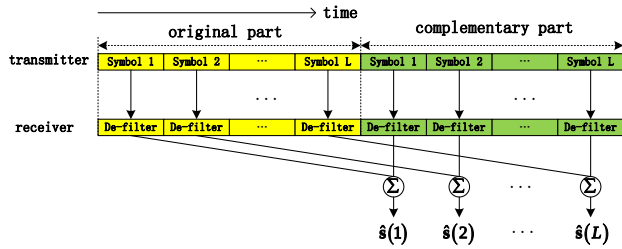


Fig. 3. Complementary SOFDM

Compared with SOFDM, complementary SOFDM can perfectly cancel ICI without loss of performance but it doesn't have spectral efficiency gain over OFDM since it needs doubled time resource than SOFDM. However, time diversity gain can be expected in this system if original and complementary parts experiences independent channel fading.

However, as we have pointed out in section II, without complementary part we can still detect subcarrier data symbols based on the signal of original part by use of equalization. Complementary part can be transmitted only when the corresponding original part's detection fails. An example has been shown in Fig. 4. Four MC symbols are transmitted. The original parts are transmitted first and the receiver try detecting without complementary part. In this case, only the 4-th MC symbol is successfully detected. So only the complementary part of the first 3 MC symbols are transmitted. As a result, the total throughput is increased by saving transmission time of the complementary part of the 4-th MC symbol. Obviously, in this HARQ-style scheme, extra signaling is needed. The receiver should feedback the detection result of each MC symbol's original part to transmitter.

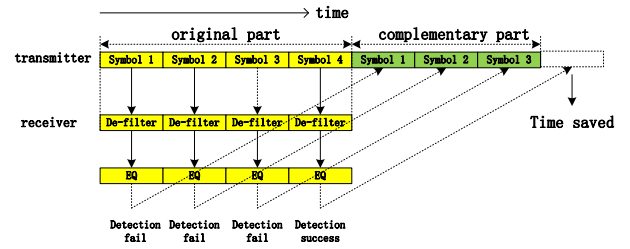


Fig. 4. An example of flexible complementary SOFDM

IV. NUMERICAL RESULTS

In this section we will investigate the performance and throughput. Ideal channel estimation is assumed in simulations.

A. SOFDM

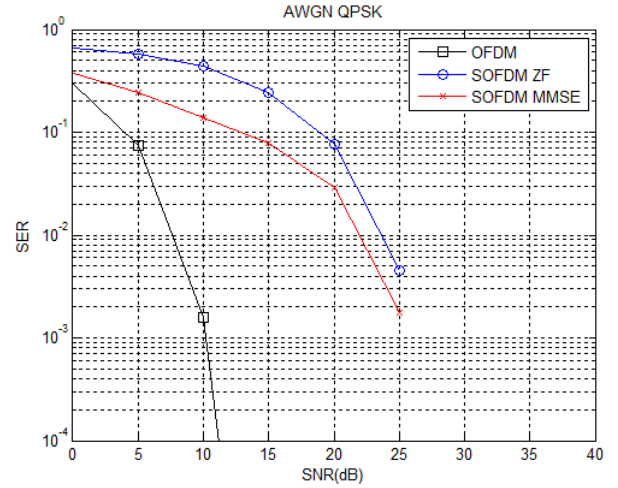


Fig. 5. Symbol error rate (SER) performance of OFDM and SOFDM of total 4 subcarriers with QPSK modulation in AWGN channel

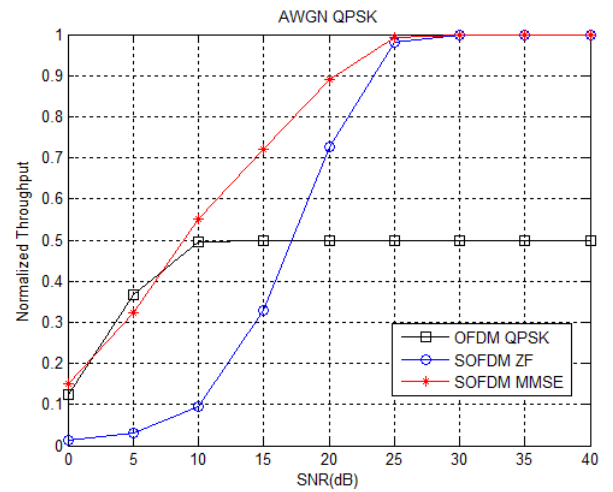


Fig. 6. Normalized throughput performance of OFDM and SOFDM of total 4 subcarriers with QPSK modulation in AWGN channel

It can be seen from Fig. 5 that SOFDM performance is worse than OFDM since ICI is introduced in SOFDM by non-orthogonality. And MMSE performance is better than ZF for SOFDM. This is because MMSE can suppress the effect of white noise power amplification of ZF. In Fig. 6, given the same total subcarrier, normalized throughput is evaluated for SOFDM and OFDM. OFDM throughput is higher than SOFDM in low SNR cases but in high SNR cases, SOFDM throughput is beyond OFDM for halved bandwidth over OFDM.

B. Complementary SOFDM

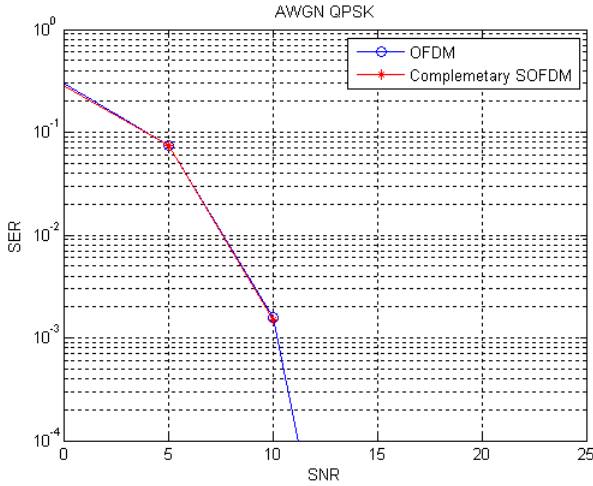


Fig. 7. Symbol error rate (SER) performance of SOFDM and complementary SOFDM of total 4 subcarriers with QPSK modulation in AWGN channel

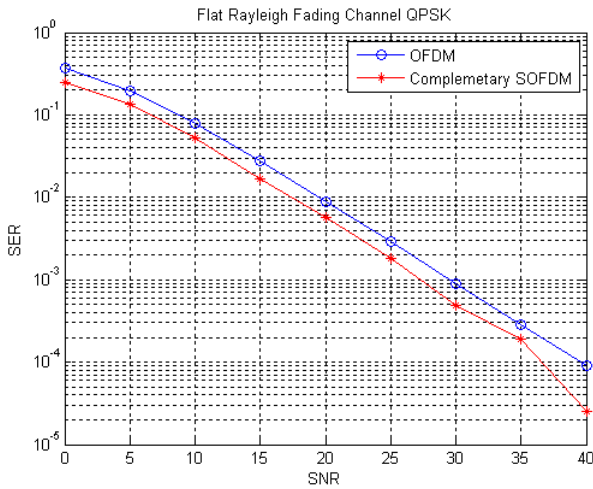


Fig. 8. Symbol error rate (SER) performance of SOFDM and complementary SOFDM of total 4 subcarriers with QPSK modulation in independent flat rayleigh fading channel

In Fig. 7, the OFDM and complementary SOFDM have identical performance in AWGN channel because they are both orthogonal MC system. However, in flat Rayleigh fading channel, time diversity gain is observed for complementary SOFDM in Fig. 8.

C. Flexible complementary SOFDM

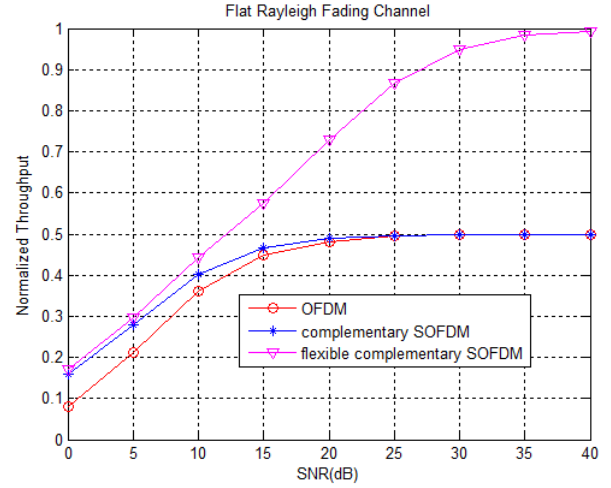


Fig. 9. Normalized throughput performance of OFDM, complementary SOFDM and flexible complementary SOFDM of total 4 subcarriers with QPSK modulation in independent flat rayleigh fading channel

In Fig. 9, complementary SOFDM throughput is higher than OFDM because of time diversity gain. And throughput of flexible complementary SOFDM is beyond that of complementary SOFDM because the complementary part is not always transmitted in the former case. If SNR is high enough, the complementary part transmission becomes unnecessary so that the throughput of the flexible complementary SOFDM can be doubled than complementary SOFDM.

V. CONCLUSION

In this paper, we introduce the concept of SOFDM which has a halved subcarrier than OFDM. The SOFDM is only orthogonal in real field but non-orthogonal in complex field. Then we provide two linear equalization algorithms base on inter carrier cross-correlation matrix to cancel the ICI. However, SOFDM still suffers a performance loss. So we design a group-wise complementary SOFDM scheme which can perfectly cancel the ICI without performance loss by transmitting two special designed MC symbols in each group. As a result, it won't have spectral efficiency gain but will has a diversity gain over OFDM. And then a HARQ-style flexible complementary SOFDM is proposed in which spectral efficiency gain and performance gain can be obtained simultaneously.

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