

Spectral Efficiency Evaluation for Non-Orthogonal Multiple Access in Rayleigh Fading

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Abstract— This paper presents original analysis on the downlink spectral efficiency of non-orthogonal multiple access (NOMA) in Rayleigh fading environment. According to our accurate evaluation technique, a closed-form expression of NOMA spectral efficiency is proposed. With the closed form, the exact average of NOMA spectral efficiency can be achieved at different system parameters. Moreover, this closed form can be used to evaluate other orthogonal multiple access (OMA) techniques such as orthogonal frequency division multiple access (OFDMA).

Keywords— non orthogonal multiple access, downlink, spectral efficiency, future radio access, Rayleigh fading

I. INTRODUCTION

Radio access technology is the key factor of mobile communications. In the fourth generation (4G) era, the access technology is orthogonal frequency division multiple access (OFDMA) which multiplexes each user by different subcarriers [1]. However, due to high traffic volume, OFDMA could not fully satisfy this requirement especially in terms of spectral efficiency and power utilization [2].

Therefore, there has been numerous research work on future radio access (FRA). The aim is to achieve a novel access technique to cope with high traffic volume and to optimize both spectral efficiency and power utilization. As a result, a promising technique has been proposed, namely non-orthogonal multiple access (NOMA), e.g. [3]-[7].

According to this technique, individual user is allowed to occupy the whole spectrum and multiplexed from one another in power domain by the known successive interference cancellation (SIC) method [8]. With SIC, the weakest signal can be extracted by removing (subtracting) stronger inter-user interferences with superposition coding. Obviously, NOMA is expected to employ as the radio access technology for future mobile generations, starting with the fifth generation (5G). On the experiments in [5], NOMA offers 30% more throughput than the conventional orthogonal multiple access (OMA) or OFDMA.

To this point, the research on the new access technique is still open wide. The pioneer group of researchers (e.g. [4]-[5]) focuses on the spectral efficiency evaluation in which all parameters are set constantly. Some literature is on the analysis of the outage probability [6] or the rate optimization problem [7].

In this work, we concern on the exact calculation of the spectral efficiency in Rayleigh fading environment whose practical channel gains are naturally random. This leads to the difficulty in computation and complexity in the final expression of spectral efficiency.

Fortunately, we have some strong background knowledge on probability and random processes. This knowledge has been used in code division multiple access (CDMA) systems for both Ricean and Rayleigh fading environments e.g. [9]-[10]. Also, it is practical to apply for the new access technique such as NOMA. Thanks to our knowledge, the exact average of NOMA spectral efficiency is formulated and represented in a closed form which is outstandingly distinguished from the literature.

This paper is organized as follows. Section II illustrates the system model. Section III shows the mathematical analysis on NOMA spectral efficiency in Rayleigh fading and then the proposed exact closed form is presented. Section IV demonstrates the numerical and simulation results. Section V draws the conclusion of this research work.

II. SYSTEM MODEL

In this section, the scenario of a future mobile cellular system is explained. Here downlink communication is concerned. As in Figure 1, The base station, called eNodeB, serves multiple user equipments (UEs) [3]. The radio access technique is NOMA which multiplexes individuals in power domain. Each receiver uses the SIC technique and is able to perfectly decode the signals from the weakest ones [4].

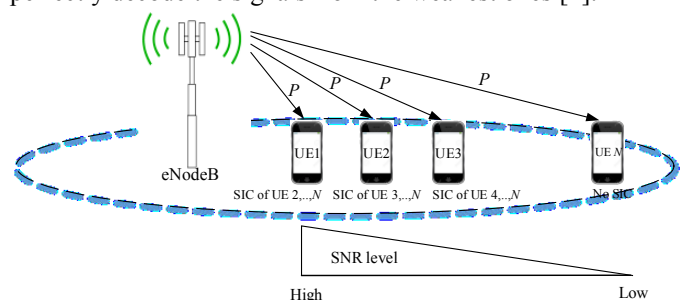


Figure 1. Downlink NOMA with SIC technique

The channel model is Rayleigh independent and identically distributed. This implies that the channel gains remain constant over a slot and become independent from one slot to

another. Rayleigh model is matched to urban environment in which there is no line of sight between transmitters and receivers. Moreover, Rayleigh channel is a complex Gaussian random variable with zero mean.

In this work, The power spectral of zero-mean additive Gaussian white noise (AWGN) is N_0 . On this channel condition, the power gains of UEs 1, 2,..., N can be defined as $|h_1|^2, |h_2|^2, \dots, |h_N|^2$. In the figure, UE 1 stays nearest to the eNodeB whereas UE 2, 3,..., N situate further respectively. Without the loss of generality, assume

$$|h_1|^2 / N_{0,1} > |h_2|^2 / N_{0,2} > \dots > |h_N|^2 / N_{0,N} \quad (1)$$

In this case, the signal power at the receiver end of UE n is

$$S_n = |h_n|^2 P + N_{0,n} \quad (2)$$

where $P = \sum_{i=1}^N P_i$ is the total signal power transmitted from the eNodeB (see Figure 1). According to NOMA technique, $P_1 < P_2 < \dots < P_N$ which is different from OMA (e.g. OFDMA) as seen in Figure 2.

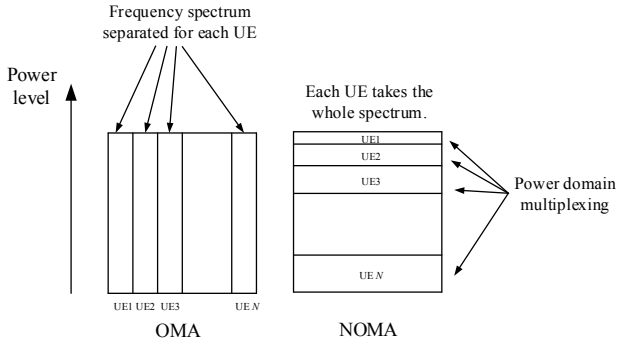


Figure 2. NOMA versus OMA (OFDMA)

Based on SIC process, UE n , $n \in \{1, 2, \dots, N\}$, can remove the inter-user interference from UE $n+1$ whose SNR (signal to noise ratio) level is smaller, $|h_{n+1}|^2 / N_{0,n+1} < |h_n|^2 / N_{0,n}$. On the assumption of band-limited waveforms in AWGN channel, the spectral efficiency of UE n can be declared as [5]-[7]

$$C_n = \log_2 \left(1 + \frac{P_n |h_n|^2}{\sum_{i=1}^{n-1} P_i |h_i|^2 + N_{0,n}} \right) \quad (3)$$

which is in bps/Hz and for $n \in \{1, 2, \dots, N\}$. As above expressions, the spectral efficiency $C_1, C_2, \dots, C_N$ can be estimated by simulating the random channel gains $|h_1|^2, |h_2|^2, \dots, |h_N|^2$ and averaging out all possible values. In some past work e.g. [4]-[5], the ratios $|h_n|^2 / N_{0,n}$, $n \in \{1, 2, \dots, N\}$ remain fixed for simplicity.

In this paper, we introduce an accurate and efficient method to compute the exact average of spectral efficiency, which is the function of complex Gaussian random variables, as shown in the following section.

III. NOMA SPECTRAL EFFICIENCY

Under the condition of Rayleigh fading, the power gains are exponentially distributed random variables [11]. Consider the system model in (3). Now the average spectral efficiency, assumed successful decoding and no error propagation, of UE n can be presented as

$$C_{n,avg} = E[\log_2(1 + \text{SINR})] = \int_0^\infty \log_2(1 + z) f_{\text{SINR}}(z) dz \quad (4)$$

where SINR (signal to interference plus noise ratio) is equal to $P_n |h_n|^2 / \sum_{i=1}^{n-1} P_i |h_i|^2 + N_{0,n}$. Now it is seen that there is some complexity on the integration of the probability density function of SINR, $f_{\text{SINR}}(z)$. To tackle such the problem, a new efficient method to calculate such (4) is introduced as below.

Rearrange (4) with the change of logarithmic base, then we have

$$C_{n,avg} = \log_2 e \int_0^\infty P(\text{SINR} > z) \frac{dz}{1 + z} \quad (5)$$

where $f_{\text{SINR}}(z) dz = dP(\text{SINR} > z)$. To find the closed form of above equation, the property of an exponential random variable X , $P(X > \mu) = e^{-\mu}$, when μ is a constant, is applied.

Due to the fact that $|h_n|^2$ is also exponentially distributed, therefore

$$P(\text{SINR} > z) = e^{-\frac{z}{P_n} \left(\sum_{i=1}^{n-1} P_i |h_i|^2 + N_{0,n} \right)} \quad (6)$$

To find the expectation value of spectral efficiency, we need to average out the cumulative function $P(\text{SINR} > z)$. Fortunately, the average of the cumulative function can be determined by calculating the moment generating function (MGF) of $|h_n|^2$. Recall [12] in which the MGF of any exponential random variable X is $E[e^{-\beta X}] = 1/(1 + \beta)$ for a constant β . Then,

$$E \left[e^{-\frac{z}{P_n} \left(\sum_{i=1}^{n-1} P_i |h_i|^2 + N_{0,n} \right)} \right] = e^{-z N_{0,n} / P_n} \prod_{i=1}^{n-1} \left(\frac{1}{1 + (P_i / P_n) z} \right) \quad (7)$$

Replace the MGF derived in (7) into (5). As a result, the closed-form expression of the spectral efficiency of non-orthogonal multiple access for the future radio resource management is

$$C_{n,avg} = \log_2 e \int_0^\infty \frac{e^{-z N_{0,n} / P_n}}{1 + z} \prod_{i=1}^{n-1} \left(\frac{1}{1 + (P_i / P_n) z} \right) dz. \quad (8)$$

Hint that this closed form presents the exact average of the spectral efficiency without any loss of generality in Rayleigh fading environment. Moreover, the closed form can be used in OMA case by simply adding the orthogonal multiplexing factor α [5].

For instance, let $\alpha_1, \alpha_2, \dots, \alpha_N$ be the orthogonal multiplexing factors of UE1, 2, ..., N and $\sum_{i=1}^N \alpha_i = 1$. Also the power

allocation for each UE is identical to one another, $P_1 = P_2 = \dots = P_N = P$, then the spectral efficiency of UE n is

$$C_{n,OMA} = \alpha_n \log_2 \int_0^\infty \frac{e^{-zN_{0,n}/P_n}}{(1+z)^n} dz. \quad (9)$$

IV. NUMERICAL RESULTS

This section presents the numerical results of NOMA spectral efficiency calculated from the proposed closed-form expression in (8). The simulation, so-called Monte Carlo simulation, of (4) is used to validate our proposed expression. To achieve such reliable results, we average out over 2,000,000 samples of the power channel gains.

Consider a single-cell environment with three UEs, namely UE1, UE2, and UE3, respectively. UE1 is the closet one to the eNodeB whereas UE2 and UE3 stay further. Then, the power allocations are assigned to individual UEs as follows; $P_1=1/6, P_2=1/3, P_3=1/2$ for UE1, UE2, and UE3, respectively. Define $P = P_1 + P_2 + P_3$ and $SNR = P / N_0$.

From Figure 3, it is found that the numerical results is positively matched to those of the simulation. This proves the accuracy of our proposed expression. In the figure, the expression is used to evaluate the NOMA spectral efficiency of each UE against the overall SNR in dB. Obviously, the spectral efficiency of UE1 is higher than others because it, staying nearest to the eNodeB, has the highest individual SINR. This result supports the principle of NOMA with SIC receivers.

Also, it is interesting to compare the spectral efficiency of NOMA with OMA (OFDMA). From Figure 4, the overall spectral efficiency of NOMA is up to 30% higher than those of OMA.

Moreover, Figure 4 shows four different power allocation plans, i.e. A, B, C, and D with various power proportions for individual UEs. Note that the spectral efficiency plotted in this figure is the total value, i.e. $C = C_1 + C_2 + C_3$. It can be seen that the power proportion of far UE should be greater to gain better overall spectral efficiency (plan A). With this power configuration, the spectral efficiency however drops when SNR is lower than 17 dB. Thus, the power allocation plan B seems the optimal solution in this scenario.

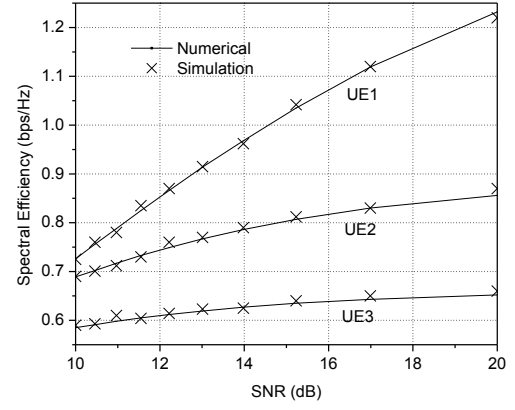


Figure 3. NOMA spectral efficiency with $P_1 = 1/6, P_2 = 1/3, P_3 = 1/2$ for UE1, UE2, and UE3, respectively

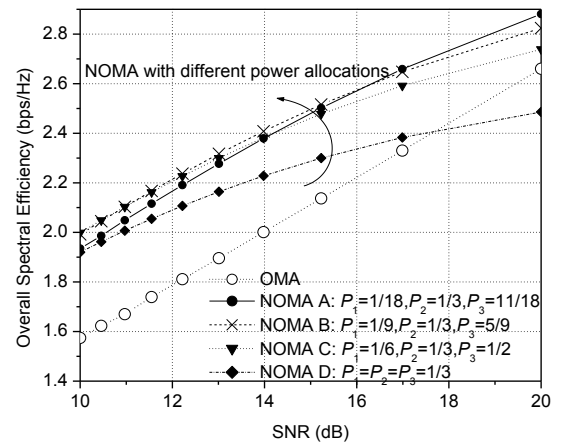


Figure 4. Overall OMA and NOMA spectral efficiency with different power allocations for UE1, UE2, and UE3

V. CONCLUSION

In this paper, a closed-form expression of downlink NOMA spectral efficiency in Rayleigh fading is introduced. Based on our accurate approximation technique, the random-distributed system model is firstly analysed. Then, the closed form is formulated. Validated with the simulation, the closed form benefits us finding the exact average of NOMA spectral efficiency at different system parameters including SNRs and user power allocations. This can be used to gain the optimal power allocation. Furthermore, we can extend the closed form utilization to OMA cases.

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