

Uplink Spectral Efficiency for Non-Orthogonal Multiple Access in Rayleigh Fading

Pongsatorn Sedtheetorn, Tatcha Chulajata

Department of Electrical Engineering, Faculty of Engineering, Mahidol University

25/25 Phuttamonthon 4 Road, Salaya, Nakornpathom, Thailand

pongsatorn.sed@mahidol.ac.th, tatcha.chu@mahidol.ac.th

Abstract— This paper introduces original analysis on the uplink spectral efficiency of non-orthogonal multiple access (NOMA) in Rayleigh fading environment. According to the accurate Gaussian-based evaluation, a closed-form expression of the spectral efficiency is proposed. Also, this work is extended to the practical case in which the number of active users is random. Validated by the simulation, the presented closed form benefits us to calculate the exact average of uplink NOMA spectral efficiency at different system parameters, such as signal-to-noise ratio, active probability, number of employed subcarriers.

Keywords— non orthogonal multiple access, uplink, spectral efficiency, future radio access, Rayleigh fading

I. INTRODUCTION

Owing to the massive growth in demand and traffic, there are great challenges on the research related mobile communications. Radio access technology is one of the challenges. In the fourth generation (4G) communications, orthogonal frequency division multiple access (OFDMA) is the key radio access technique. Based on OFDMA, users are allocated different subcarriers to convey their information [1]. This technique comes with the advantage on low-complexity multi-user detection (MUD). However, the issues of spectral efficiency and power utilization are still disputable [2].

To fulfil the requirement, non orthogonal multiple access (NOMA) has been introduced, e.g. [3]–[8]. With NOMA, each user is able to take the whole spectrum and multiplexed from one another in power domain by the known successive interference cancellation (SIC) method [9]. With SIC, the desired signal can be extracted by cancelling (subtracting) stronger inter-user interferences with superposition coding. This power multiplexing technique provides 30% more throughput than the traditional orthogonal multiple access (OMA) or OFDMA [6]. Therefore, NOMA is expected to replace OFDMA and used as the main radio access technology for the fifth generation (5G) mobile communications.

In the aspect of research, the challenges are still open wide both downlink and uplink transmissions. For instance, [5]–[6] focus on the spectral efficiency evaluation in which all parameters are however fixed constantly. There is some work focusing on, for example, the analysis of the outage probability [7], the rate optimization problem [8], and the maximization problem [4]. Nonetheless, some random-nature

parameters therein the work, such as channel gains, are conditioned unchanged due to the sake of simplicity.

On the contrary, in this work we concern on the exact evaluation of the uplink spectral efficiency in Rayleigh fading channel whose practical channel gains are actually random. Although, this leads us into the difficulty of computation and complexity of finding the resultant expression of spectral efficiency, we fortunately have some strong background knowledge on probability and random processes [10]–[12]. This knowledge is found to be practical for the new access technique such as NOMA as well. Thus, the exact average of NOMA uplink spectral efficiency is formulated. This makes this work outstandingly distinguished from the literature.

This paper is organized as follows. Section II illustrates the system model. Section III shows the mathematical analysis on NOMA uplink spectral efficiency in Rayleigh fading. Then the exact closed form is presented. Section IV extends our work to the general case in which the number of actual users is random. Section V demonstrates the numerical and simulation results. Section VI draws the conclusion of this work.

II. SYSTEM MODEL

In this section, the system model is explained. In Figure 1, a scenario of uplink transmissions on a single-cell mobile cellular network is presented. The base station, renamed as eNodeB [3], serves its user equipments (UEs). Every UE is able to transmit their information through multiple subcarriers, which is called multi-carrier transmissions. Based on NOMA technique, UEs are allowed to share the same resource simultaneously both in the aspects of frequency spectrum and time. Therefore, the receiver at the eNodeB is required to operate MUD in order to distinguish signals of individual UEs. One of favourite MUD techniques is SIC in which the desired signal is recovered from the subtraction (cancellation) of interferences.

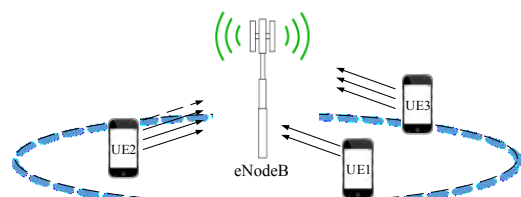


Figure 1. Uplink multicarrier NOMA

Here the channel is defined as independent and identically distributed Rayleigh fading. This means that the channel gains remain constant over a slot and become independent from one slot to another. Rayleigh channel is matched to urban environment in which there is no line of sight between transmitters and receivers. Mathematically, Rayleigh model can be formulated by a complex Gaussian random variable with zero mean [13].

To this point, the signal received at the eNodeB can be expressed as

$$S_n = \sum_{i=1}^N \sum_{l=1}^L P_{i,l} |h_{i,l}|^2 + N_0 \quad (1)$$

where N is the total number of UEs in the cell. L is the maximum number of subcarriers that each UE is allow to use.

$P_{i,l}$ and $|h_{i,l}|^2$ are the power and the gain of the signal from UE i on particular subcarrier l . Hint that $h_{i,l}$; $i = 1, 2, \dots, N$; $l = 1, 2, \dots, L$ models Rayleigh fading. N_0 is the power spectral of zero-mean additive Gaussian white noise (AWGN).

In the next section, Shannon formula is employed to evaluate the spectral efficiency of uplink NOMA in Rayleigh fading. Then, a new closed-form expression is proposed and extended to the general case of random users in Section IV.

III. UPLINK SPECTRAL EFFICIENCY

Regarding the system model on the section above, the uplink spectral efficiency of each individual mobile UE n can be shown as [4]

$$C_n = \sum_{l=1}^L \log_2 \left(1 + \frac{P_{n,l} |h_{n,l}|^2}{\sum_{i=1, i \neq n}^N P_{i,l} |h_{i,l}|^2 + N_0} \right) \text{ bps/Hz} \quad (2)$$

where $\sum_{i=1, i \neq n}^N P_{i,l} |h_{i,l}|^2$ is considered as the interferences of the desired signal $P_{n,l} |h_{n,l}|^2$ on subcarrier l .

On Rayleigh fading, the power gains, $|h_{i,l}|^2$; $i = 1, 2, \dots, N$; $l = 1, 2, \dots, L$, are assumed as exponential random variables [13]. Therefore, the spectral efficiency becomes a function of multi random variables with the average

$$C_{n,avg} = \sum_{l=1}^L E[\log_2(1 + \text{SINR}_l)] = \sum_{l=1}^L \left\{ \int_0^\infty \log_2(1+z) f_{\text{SINR}_l}(z) dz \right\} \quad (3)$$

where SINR_l (signal to interference plus noise ratio of subcarrier l) is equal to $P_{n,l} |h_{n,l}|^2 / \sum_{i=1, i \neq n}^N P_{i,l} |h_{i,l}|^2 + N_0$. Since the probability density function of SINR_l , $f_{\text{SINR}_l}(z)$, is the function of random-value power gains, exponentially distributed, it is quite difficult to find the resultant expression of (3). Fortunately, a new efficient method to tackle this problem is introduced here as below.

Change the logarithmic base in (3), then

$$C_{n,avg} = \log_2 e \sum_{l=1}^L \left\{ \int_0^\infty P(\text{SINR}_l > z) \frac{dz}{1+z} \right\} \quad (4)$$

where $f_{\text{SINR}_l}(z) dz = dP(\text{SINR}_l > z)$. To find the closed form of above equation, the property of an exponential random variable X , $P(X > \alpha) = e^{-\alpha}$, when α is a constant, is applied.

Owing to the fact that $|h_{n,l}|^2$ is also exponentially distributed, therefore

$$P(\text{SINR}_l > z) = e^{-\frac{z}{P_{n,l}} \left(\sum_{i=1, i \neq n}^N P_{i,l} |h_{i,l}|^2 + N_0 \right)} \quad (5)$$

To find the expectation value of spectral efficiency, we need to average out the cumulative function $P(\text{SINR}_l > z)$.

Fortunately, the average of the cumulative function can be determined by calculating the moment generating function (MGF) of $|h_{n,l}|^2$. Recall [13] in which the MGF of any exponential random variable X is $E[e^{-\beta X}] = 1/(1+\beta)$ for a constant β . Then,

$$E \left[e^{-\frac{z}{P_{n,l}} \left(\sum_{i=1, i \neq n}^N P_{i,l} |h_{i,l}|^2 + N_0 \right)} \right] = e^{-z N_0 / P_{n,l}} \prod_{i=1, i \neq n}^N \left(\frac{1}{1 + (P_{i,l} / P_{n,l}) z} \right) \quad (6)$$

Replace the MGF derived in (6) into (4). As a result, the closed-form expression of the spectral efficiency of non-orthogonal multiple access for the future radio resource management is

$$C_{n,avg} = \log_2 e \sum_{l=1}^L \left\{ \int_0^\infty \frac{e^{-z N_0 / P_{n,l}}}{1+z} \prod_{i=1, i \neq n}^N \left(\frac{1}{1 + (P_{i,l} / P_{n,l}) z} \right) dz \right\} \quad (7)$$

Note that this closed-form expression computes the exact average of the spectral efficiency without any loss of generality in Rayleigh fading environment. Assume all N UEs are active simultaneously, then the total spectral efficiency is simply as

$$C_{tot} = N C_n \quad (8)$$

However, the number of active UEs is practically random and distributed from 0, 1, 2, ..., N . In the next section, the total spectral efficiency is calculated when the randomness of the amount of active users is taken into account.

IV. RANDOM NUMBER OF USERS

Logically, the number of active UEs is assumed to be binomially distributed. The probability mass function, when the number of active UEs is k , can be defined as

$$P(k) = \binom{N}{k} p^k (1-p)^{N-k}; \quad k \in \{0, 1, 2, \dots, N\} \quad (9)$$

with the active probability p and $\binom{N}{k} = \frac{N!}{(N-k)!k!}$.

Hint that, when the number of interfering UEs is random, we need to recalculate (7). Recall (6) and assume all transmitted

uplink powers are identical. Without the loss of generality, (6) becomes

$$\mathbb{E} \left[e^{-\frac{z}{P_{n,j}} \left(\sum_{i=1, i \neq n}^N P_{i,j} |h_{i,j}|^2 + N_0 \right)} \right] = e^{-zN_0/P_{n,j}} \left(\frac{1}{1+z} \right)^{N-1} \quad (10)$$

Lemma 1: Let $V(\cdot)$ be any function and Y be a binomial random variable with parameters (N, p) . Then, we have

$$\mathbb{E}[V^Y(\cdot)] = [1 - p + pV(\cdot)]^N \quad (11)$$

Proof: if Y is a binomial random variable with parameters (N, p) . The moment generating function (MGF) of Y is $\mathbb{E}[e^{tY}]$ in which t is a constant. Then,

$$\begin{aligned} \mathbb{E}[e^{tY}] &= \sum_{k=0}^N e^{tk} \binom{N}{k} p^k (1-p)^{N-k} \\ &= \sum_{k=0}^N \binom{N}{k} (pe^t)^k (1-p)^{N-k} = [1 - p + pe^t]^N. \end{aligned} \quad (12)$$

Replace e^t with any function $V(\cdot)$, thus we yield the same result as in (11).

Apply (11) to (10). The exact average uplink spectral efficiency is

$$C_{n,avg} = \log_2 e \sum_{l=1}^L \left\{ \int_0^\infty \frac{e^{-zN_0/P_{n,l}}}{1+z} \left[1 - p + p \left(\frac{1}{1+z} \right) \right]^{N-1} dz \right\}. \quad (13)$$

As a result, the total spectral efficiency is

$$C_{tot} = pNC_n \quad (14)$$

where pN is the average value of a binomial random variable with parameter (N, p) .

V. NUMERICAL RESULTS

This section presents the numerical results of NOMA uplink spectral efficiency calculated from the proposed closed-form expression in (13). The simulation, so-called Monte Carlo simulation, of (2) is used to validate our proposed expression. To achieve such reliable results, we average out over 2,000,000 samples of the power gains.

Consider a single-cell environment with 3 UEs, namely UE1, UE2, and UE3, respectively. Assume every UE is active and all available bandwidth is divided into 3 subcarriers. Note that UE1, UE2 and UE3 use respectively 1, 2, and 3 subcarriers for their uplink transmissions. In Figure 2, the solid lines represent the results from our closed-form expression in (13) and the cross symbols dedicates the simulation results of (2). Hint that signal-to-noise ratio (SNR) is defined as $\text{SNR} = P_{n,j} / N_0$ in decibel (dB).

First of all, it is obvious that our numerical results is matched to the simulation. This validates the proposed expressions in this work. Secondly, the spectral efficiency seems to increase with the number of employed subcarriers. However, it is not directly proportional because added subcarriers could be shared and interfered by other UEs.

In the next experiment, the total spectral efficiency versus SNR with varying active probability in (13) is concerned. Here the total number of UEs is $N=10$. All UEs use 3 subcarriers to convey their information ($L=3$). In Figure 3, the active probability (p) varies from 0.1, 0.3, 0.5, 0.7, and 0.9. It is seen that high value of active probability activates UEs to transmit their signals and thus interfere one another. This leads to the drop of the overall spectral efficiency.

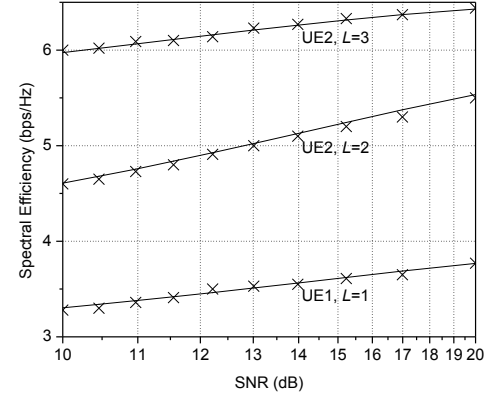


Figure 2. Spectral efficiency of individual UEs; $N = 3$ and $p = 1$.

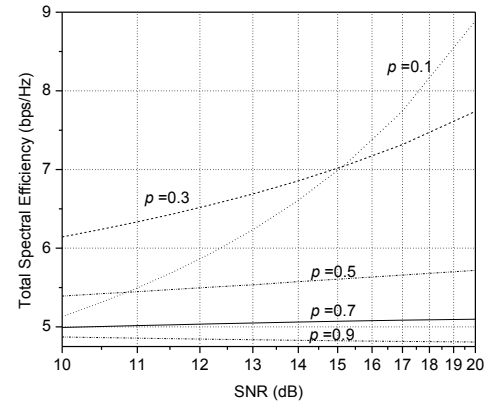


Figure 3. The total spectral efficiency with different active probability p with $N = 10$ and $L = 3$

VI. CONCLUSION

In this paper, a closed-form expression of the uplink NOMA spectral efficiency in Rayleigh fading is proposed. With the accurate analysis, the closed form yields the numerical results that matched (validated) to Monte Carlo simulation. Moreover, the scope of this work is extend to the practical case in which the number of active UEs is random. Here the distribution of the random process is assumed to be binomial which is suit to the on-off nature of mobile users. From the results, we can compute the spectral efficiency at different system parameters, for example SNR, active probability, number of employed subcarriers.

REFERENCES

- [1] Carl Wijting *et al.*, "Key technologies for IMT-advanced mobile communication systems," *IEEE Trans. on Wireless Commun.*, vol. 16, no. 3, pp. 76-85, June 2009.
- [2] Jeffrey G. Andrews *et al.*, "What will 5G be?," *IEEE Journal on Selected Areas in Commun.*, vol. 32, no. 6, pp. 1065-1082, June 2014.
- [3] Docomo 5G white paper, "5G radio access: requirements, concept and technologies," NTT Docomo Inc., 2014.
- [4] Mohammed Al-Imari *et al.*, "Uplink non-orthogonal multiple access for 5G wireless networks," in *IEEE Proceeding of ISWCS*, vol. 1, August 2014, pp. 781-785.
- [5] Yuya Saito *et al.*, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *IEEE Proceeding of VTC Spring*, vol. 1, June 2013, pp. 1-5.
- [6] Anass Benjebbour *et al.*, "Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access," in *IEEE Proceeding of ISPACS*, vol. 1, November 2013, pp. 770-774.
- [7] Zhiquo Ding *et al.*, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," *IEEE Trans. on Signal Processing Lett.*, vol. 21, no. 12, pp. 1501-1505, December 2014.
- [8] Stelios Timotheou *et al.*, "Fairness for non-orthogonal multiple access in 5G systems," *IEEE Trans. on Signal Processing Lett.*, vol. 22, no. 10, pp. 1647-1651, October 2015.
- [9] Mazen O. Hasna *et al.*, "Performance analysis of mobile cellular systems with successive co-channel interference cancellation," *IEEE Trans. on Wireless Commun.*, vol. 2, issue 1, pp. 29-40, February 2003.
- [10] Pongsatorn Sedtheetorn *et al.*, "Theoretical analysis on bit error rate of VSG CDMA in Nakagami fading," *IEEE Trans. on Information Theory*, vol. 57, issue 6, pp. 3405-3410, May 2011.
- [11] Pongsatorn Sedtheetorn *et al.*, "Accurate packet error rate analysis of variable spreading gain-code division multiaccess and multicode division multiaccess wireless communication networks," *IET Trans. on Commun.*, vol. 5, issue 16, pp. 2407-2417, November 2011.
- [12] Pongsatorn Sedtheetorn *et al.*, "A linear programming approach to cross-layer optimization in multi-class VSG-CDMA in Rayleigh fading," *IEEE Trans. on Vehicular Technol. Commun.*, vol. 57, issue 3, pp. 1864-1875, May 2008.
- [13] John G. Proakis *et al.*, *Digital Communications*, 5th edition, McGrawHill, 2008.
- [14] Sheldon M. Ross, *Introduction to probability models*, 7th edition, Harcourt Academic Press, 2000.



Pongsatorn Sedtheetorn (M'03) received the B.Eng. and M.Eng. degrees from Chulalongkorn University, Thailand, in 1998 and 2001, and the Ph.D. degree from the University of Manchester, United Kindom, in 2007. He is currently an Associate Professor with the Department of Electrical Engineering, Mahidol University, Thailand. His research interests are in the areas of wireless communications, information theory, as well as enterprise architecture.



Tatcha Chulajata (M'97) received the B.Eng. from Kasetsart University, Thailand, in 1992. He received the M.S and the Ph.D. degrees from Wichita State University, USA, in 1996 and 2003, respectively. He is currently a Senior Lecturer with the Department of Electrical Engineering, Mahidol University, Thailand. His research interests are in the areas of wireless communications, communication network, and enterprise architecture.