

Experimental Validation of Multipoint Joint Processing of Range Measurements via Software-Defined Radio Testbed

Georgy Mashkov*, Eugene Borisov*, Grigorii Fokin*

* State University of Telecommunication, 22 Prospekt Bolshhevikov, St. Petersburg, Russia

MashkovGM@sut.ru, kafedra@sut.ru, grihafokin@gmail.com

Abstract— In this paper we present an algorithm for multipoint joint processing of range measurements with trial results accumulation, evaluate its accuracy analytically in terms of range mean square error and finally validate it through an experiment. The methods used for analytical evaluation is matrix analysis and nonlinear least squares estimation theory. The approach for experimental validation is a software-defined radio (SDR) transceiver working on National Instruments (NI) Universal Software Radio Peripheral (USRP) hardware with LabVIEW software, performing transmission, reception and processing of signals. Presented solution was validated in static scenario and achieved position errors of several meters depending on the number of range measurement trials. The resulting error was substantially affected by synchronization uncertainty due to network organization of transceiver stations and measurement processing unit. However, in the conclusion we can state that joint processing of range measurements with trial results accumulation can substantially improve positioning accuracy and it was experimentally validated with software-defined radio testbed.

Keywords— Multipoint Joint Processing, Range Measurements, Software Defined Radio, Testbed, Mean Square Error, Field Trials, USRP, LabVIEW

I. INTRODUCTION

The field of application of presented results are navigation, radar and sonar systems which consist of multiple transceiver stations gathering measurements, object of location estimation and measurement processing unit.

In estimation process of coordinates on the basis of joint (cooperative) measurement processing redundancy is often used as excess relative to the number of unknown parameters to provide more accurate results [1], [2].

Mathematically the state of redundant measurements of the system at discrete points in time is described by a system of linear or nonlinear equations relating the input and output values, and the parameters known by type conversion functions of the measuring channel. This potentially ensures

not only automatic reduction of errors of measurement results with non-linear in general and volatile transformation functions of measuring channel as a whole, but also the determination of the current values of the parameters of the conversion function [3].

However, in [1] – [3] the ways to improve the accuracy of estimation of unknown parameters during their joint measurements had not been studied yet. This problem occurs in radar, sonar and navigation systems, when the position estimation is subject to ranges, the sum and difference of ranges, as well as other independent or functionally related parameters. If all or several transceiver stations received retransmitted signals from the object of location it could improve the accuracy of range measurements which could be applied to navigation, radar and sonar systems [4] – [5].

Previous works concerning the problem of multipoint joint processing had already shown that redundancy in the number of transceiver measurement stations and measurement trials makes it possible to achieve higher accuracy. Nevertheless practical application-specific algorithms investigation and their experimental validation in realistic deployment are still necessary.

The material in the paper organized in the following order. Multipoint radio system under consideration, algorithm for joint processing and its analytical accuracy performance evaluation are presented in the second part. Developed software-defined radio testbed, including experimental scenario and its accuracy performance evaluation results are presented in the third part. Finally, we draw the conclusions in the fourth part.

II. ANALYTICAL RESULTS ON MULTIPOINT JOINT PROCESSING OF RANGE MEASUREMENTS

Multipoint joint processing radio system under consideration operates in the following way. Transceiver stations transmit unique signals based on pseudorandom binary sequence and then receive their retransmitted copies from object of location and thus gather time of arrival (TOA). Object of location estimation is an active reflector which retransmits unique signals. Finally measurement processing unit gathers TOA measurements from transceiver stations and derives range estimates between transceiver stations and object of location and after that performs joint processing of derived ranges to locate the object in a three-dimensional space based on known transceiver stations reference positions.

In general case multipoint joint processing radio system presented above is formulated as follows. We assume that it includes N transceiver stations for gathering of measurements. Thus there are N possible range measurements between transceiver stations and object of location denoted by $\hat{R}_1, \hat{R}_2, \dots, \hat{R}_N$ and $N(N-1)$ sum of range measurements denoted by $\hat{R}_{\Sigma 12}, \hat{R}_{\Sigma 21}, \dots, \hat{R}_{\Sigma N(N-1)}$. It is a well-known fact, that system is redundant when $N \geq 3$ [6] – [9], that's why approach used for independent measurements acquisition enables us to form system of N^2 linear equations which is substantially redundant:

$$(1) \quad \begin{cases} \hat{R}_1 = 1 \cdot \tilde{R}_1 + 0 \cdot \tilde{R}_2 + 0 \cdot \tilde{R}_3 + \dots + 0 \cdot \tilde{R}_N, \\ \hat{R}_2 = 0 \cdot \tilde{R}_1 + 1 \cdot \tilde{R}_2 + 0 \cdot \tilde{R}_3 + \dots + 0 \cdot \tilde{R}_N, \\ \vdots \\ \hat{R}_1 = 0 \cdot \tilde{R}_1 + 0 \cdot \tilde{R}_2 + 0 \cdot \tilde{R}_3 + \dots + 1 \cdot \tilde{R}_N, \\ \hat{R}_{\Sigma 12} = 1 \cdot \tilde{R}_1 + 1 \cdot \tilde{R}_2 + 0 \cdot \tilde{R}_3 + \dots + 0 \cdot \tilde{R}_N, \\ \hat{R}_{\Sigma 21} = 1 \cdot \tilde{R}_1 + 1 \cdot \tilde{R}_2 + 0 \cdot \tilde{R}_3 + \dots + 0 \cdot \tilde{R}_N, \\ \hat{R}_{\Sigma 13} = 1 \cdot \tilde{R}_1 + 0 \cdot \tilde{R}_2 + 1 \cdot \tilde{R}_3 + \dots + 0 \cdot \tilde{R}_N, \\ \hat{R}_{\Sigma 31} = 1 \cdot \tilde{R}_1 + 0 \cdot \tilde{R}_2 + 1 \cdot \tilde{R}_3 + \dots + 0 \cdot \tilde{R}_N, \\ \vdots \\ \hat{R}_{\Sigma N} = 1 \cdot \tilde{R}_1 + 0 \cdot \tilde{R}_2 + 0 \cdot \tilde{R}_3 + \dots + 1 \cdot \tilde{R}_N. \end{cases}$$

In matrix form system of N^2 linear equations (1) can be represented as

$$(2) \quad \hat{\mathbf{H}} = \mathbf{A} \tilde{\mathbf{S}},$$

where $\hat{\mathbf{H}}^T = [\hat{R}_1, \hat{R}_2, \dots, \hat{R}_N, \hat{R}_{\Sigma 12}, \hat{R}_{\Sigma 21}, \hat{R}_{\Sigma 13}, \dots, \hat{R}_{\Sigma N}]$ – row vector of range and sum of range measurements of size $1 \times N^2$; $\mathbf{A}$ – matrix of coefficients of size $N^2 \times N$, with elements equal to one for available in (1) range measurement and zero otherwise; $\tilde{\mathbf{S}}^T = [\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_N]$ – row vector of unknown range estimates of size $1 \times N$.

Solving (2) with nonlinear least squares estimation theory methods [6]–[8] we get the following solution:

$$(3) \quad \tilde{\mathbf{S}} = [(\mathbf{A}^T \mathbf{A} \mathbf{W}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{A} \mathbf{W}^{-1}] \hat{\mathbf{H}},$$

where $\mathbf{W}$ – square matrix of range measurements accuracy of size $N^2 \times N^2$, with range and sum of range measurements variance $\sigma_{R1}^2, \sigma_{R2}^2, \dots, \sigma_{R_{\Sigma N}}^2$ as diagonal elements, and possible correlations between them as non-diagonal elements:

$$(4) \quad \mathbf{W} = \begin{bmatrix} \sigma_{R1}^2 & \dots & \dots & \dots \\ \vdots & \sigma_{R1}^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \dots & \dots & \dots & \sigma_{R_{\Sigma N}}^2 \end{bmatrix},$$

$\mathbf{A}$ – diagonal matrix of coefficients j of size $N^2 \times N^2$ with diagonal elements equal to one for available in (1) range measurement and zero otherwise:

$$(5) \quad \mathbf{A} = \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & j & 0 & 0 \\ 0 & 0 & \ddots & \\ 0 & 0 & \dots & j \end{bmatrix},$$

Consider multipoint range measurement radio system depicted in Figure 1.

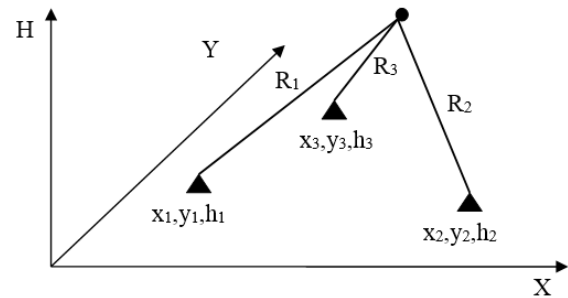


Figure 1. Multipoint range measurement radio system

Multipoint range measurement radio system depicted in Figure 1 has 3 spatially distributed transceiver stations with known reference positions $(x_1, y_1, h_1), (x_2, y_2, h_2), (x_3, y_3, h_3)$ in a three-dimensional

space for gathering of range measurements R_1, R_2, R_3 between them and object of location estimation.

For the case of 3 spatially distributed transceiver stations when $N=3$ we can get a row vector of range and sum of range measurements and matrix of coefficients respectively [6] – [9]:

$$\hat{\mathbf{H}}^T = [\hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{R}_N, \hat{R}_{\Sigma 12}, \hat{R}_{\Sigma 21}, \hat{R}_{\Sigma 13}, \hat{R}_{\Sigma 31}, \hat{R}_{\Sigma 23}, \hat{R}_{\Sigma 32}], \quad (6)$$

$$\mathbf{A}^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}. \quad (7)$$

Substituting (6) and (7) in (3) and taking into account diagonal elements equal to one in (4) and (5) and applying least squares method we can get following analytical solutions for of unknown range estimates:

$$\begin{aligned} \tilde{R}_1 &= \frac{7}{27} \hat{R}_1 - \frac{2}{27} \hat{R}_2 - \frac{2}{27} \hat{R}_3 + \frac{5}{27} \hat{R}_{\Sigma 12} + \frac{5}{27} \hat{R}_{\Sigma 21} + \dots \\ &\quad \frac{5}{27} \hat{R}_{\Sigma 13} + \frac{5}{27} \hat{R}_{\Sigma 31} - \frac{4}{27} \hat{R}_{\Sigma 23} - \frac{4}{27} \hat{R}_{\Sigma 32}, \\ \tilde{R}_2 &= -\frac{2}{27} \hat{R}_1 + \frac{7}{27} \hat{R}_2 - \frac{2}{27} \hat{R}_3 + \frac{5}{27} \hat{R}_{\Sigma 12} + \frac{5}{27} \hat{R}_{\Sigma 21} - \dots \\ &\quad - \frac{4}{27} \hat{R}_{\Sigma 13} - \frac{4}{27} \hat{R}_{\Sigma 31} + \frac{5}{27} \hat{R}_{\Sigma 23} + \frac{5}{27} \hat{R}_{\Sigma 32}, \\ \tilde{R}_3 &= -\frac{2}{27} \hat{R}_1 - \frac{2}{27} \hat{R}_2 + \frac{7}{27} \hat{R}_3 - \frac{4}{27} \hat{R}_{\Sigma 12} - \frac{4}{27} \hat{R}_{\Sigma 21} + \dots \\ &\quad + \frac{5}{27} \hat{R}_{\Sigma 13} + \frac{5}{27} \hat{R}_{\Sigma 31} + \frac{5}{27} \hat{R}_{\Sigma 23} + \frac{5}{27} \hat{R}_{\Sigma 32}. \end{aligned} \quad (8)$$

Under the condition when $\sigma_{R1}^2 = \sigma_{R2}^2 = \sigma_{R3}^2 = \sigma_R^2$ and $\sigma_{R\Sigma 12}^2 = \sigma_{R\Sigma 21}^2 = \sigma_{R\Sigma 13}^2 = \sigma_{R\Sigma 31}^2 = \sigma_{R\Sigma 23}^2 = \sigma_{R\Sigma 32}^2 = \sigma_{R\Sigma}^2$ we can get error covariance matrix for system (3) in the case of range and sum of range measurements accumulation during n trials for one and for all transceiver stations positions respectively:

$$\sigma_{RC}^2 = \text{diag} \begin{bmatrix} \frac{7}{20n+7} & -\frac{2}{7n+20} & -\frac{2}{7n+20} \\ -\frac{2}{7n+20} & \frac{5n+16}{3(7n+20)} & -\frac{2(n+2)}{3(7n+20)} \\ -\frac{2}{7n+20} & -\frac{2(n+2)}{3(7n+20)} & \frac{5n+16}{3(7n+20)} \end{bmatrix} \sigma_R^2, \quad (9)$$

$$\sigma_{RC}^2 = \text{diag} \begin{bmatrix} \frac{7}{27n} & -\frac{2}{27n} & -\frac{2}{27n} \\ -\frac{2}{27n} & \frac{7}{27n} & -\frac{2}{27n} \\ -\frac{2}{27n} & -\frac{2}{27n} & \frac{7}{27n} \end{bmatrix} \sigma_R^2. \quad (10)$$

For the case of one measurement trial, when $n=1$, from (10) we can derive the following error covariance matrix:

$$\sigma_{RC}^2 = \text{diag} \begin{bmatrix} \frac{7}{27} & -\frac{2}{27} & -\frac{2}{27} \\ -\frac{2}{27} & \frac{7}{27} & -\frac{2}{27} \\ -\frac{2}{27} & -\frac{2}{27} & \frac{7}{27} \end{bmatrix} \sigma_R^2. \quad (11)$$

Thus, varying the number of trials with results accumulation in the case of range and sum of range measurements for one and for all transceiver stations positions we can get various potential accuracy performance. Figure 2 shows the Mean Square Error (MSE) dependence of range estimates from number of trials accumulation for various ways of measurement processing organization.

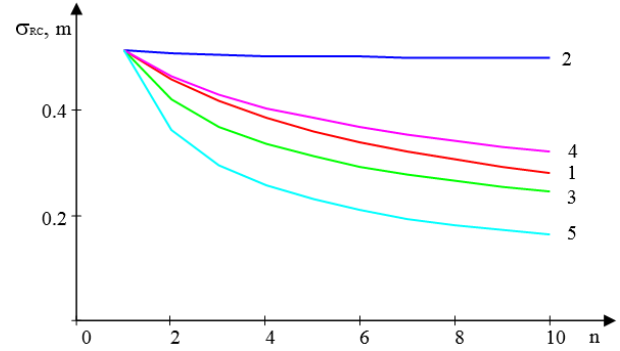


Figure 2. Analytical range MSE dependence from the number of trials

We have the following notations in Figure 2 considering analytical trial results accumulation gain in terms of MSE: 1 and 3 are the range MSE in meters for the case of range and sum of range measurements accumulation with regard to the first transceiver station positions, 2 and 4 are the range MSE in meters for the case of range and sum of range measurements accumulation with regard to the second and third transceiver station positions and finally 5 is the MSE variance in meters for the case of range and sum of range measurements accumulation with regard to all transceiver station positions. It can be seen from the Figure 2, that increasing the number of trial results accumulations and the number of transceiver stations realizing this procedure can substantially increase positioning accuracy estimation in terms of MSE.

Finally measurement processing unit gathers multiple range estimates from transceiver stations

and based on their known reference positions derives object location in a three-dimensional space in terms of rectangular coordinates solving the following system of equations

$$R = \sqrt{(x_t - x_i)^2 + (y_t - y_i)^2 + (h_t - h_i)^2}, i = 1, \dots, N, \quad (12)$$

where (x_t, y_t, h_t) – required object of location estimation coordinates and (x_i, y_i, h_i) – transceiver stations known reference position coordinates.

Note that the problem of increasing the estimation accuracy of coordinates in the LTE system is considered in the work [10] but the possibility of cooperative processing of measurements in this article are not considered.

III. EXPERIMENTAL RESULTS ON MULTIPOINT JOINT PROCESSING OF RANGE MEASUREMENTS

To realize experimental validation of proposed algorithm for multipoint joint processing of range measurements with trial results accumulation we developed a software-defined radio testbed by means of model based design via software defined radio [11], [12] on National Instruments hardware NI USRP-2932 [13] and LabVIEW software [14]. It consists of three transceiver stations gathering measurements, one transceiver station realizing functions of an active reflector, and one processing unit. Layout of developed multipoint software-defined radio testbed for joint processing of range measurements experimental validation is depicted in Figure 3.

Transceiver stations are realized on National Instruments hardware NI USRP-2932 and work under specially developed LabVIEW software applications.

Developed software-defined radio testbed operates in the following way. Transceiver stations based on NI USRP-2932 and working on Core I7 desktop PC under Windows 7 [15] with LabVIEW application “Server” transmit unique signals based on pseudorandom binary sequence and then receive their retransmitted copies from object of location and thus gather TOA. Object of location estimation is an active reflector based on NI USRP-2932 and working on Core I7 desktop PC under Windows 7 with LabVIEW application “Repeater” which retransmits unique signals, received from “Server”. Finally measurement processing unit working on

Core I7 desktop PC under Windows 7 with LabVIEW application “Active Positioning” gathers multiple TOA measurements from transceiver stations process them and visualize result.

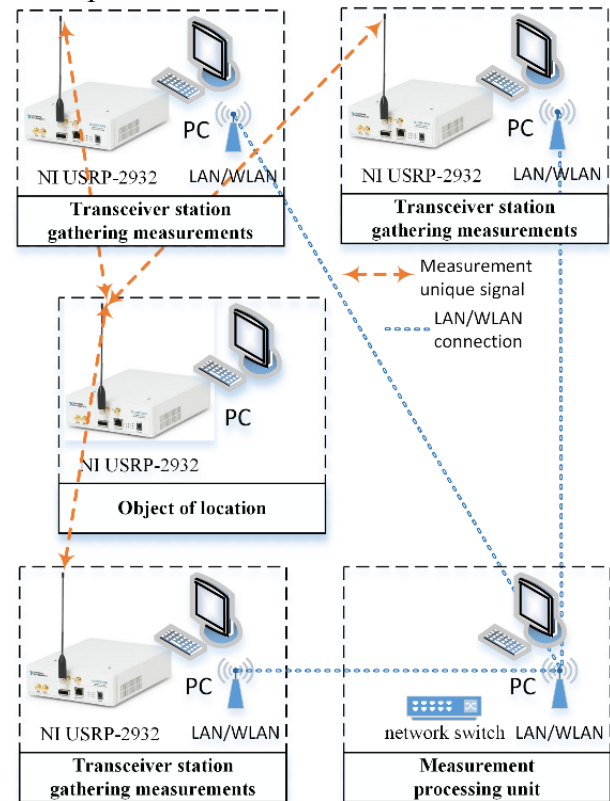


Figure 3. Multipoint Software-Defined Radio Testbed Layout for Joint Processing of Range Measurements Experimental Validation

Experiment was carried out in a static scenario when transceiver stations and object of location were placed in the room with spatial separation of several decades of meters and worked stationary under line of sight (LOS) conditions. NI USRP-2932 worked in 433 MHz Low Power Devices (LPD) band for communication with binary phase shift keying signals modulated by unique pseudorandom binary sequence with 2 MHz bandwidth and power constraint of 20 dBm.

Network connection between transceiver stations and processing unit is based on Gigabit Ethernet (GbE) LAN. To achieve fast deployment capability of such radio system it is preferably to realize network connection via WLAN or radio modem.

Hardware and software structure of developed multipoint software-defined radio testbed for joint processing of range measurements experimental validation is presented in Table 1.

TABLE 1. HARDWARE AND SOFTWARE STRUCTURE OF SOFTWARE-DEFINED RADIO TESTBED TESTBED

	Hardware	Software	Realized Functions
Measurement gathering station	NI USRP-2932, PC, AN/WLAN	Windows 7, Server (LabVIEW application)	transmit and receive unique signals and measure TOA; send TOA measurements to processing unit
Processing unit	PC, LAN/WLAN, network switch	Windows 7, Active Positioning (LabVIEW application)	set up multipoint radio system parameters, process TOA measurements, display results
Object of location	NI USRP-2932, PC	Windows 7, Repeater (LabVIEW application)	receive and retransmit unique signals

During experimental validation we got following results in terms of MSE depicted in Figure 4.

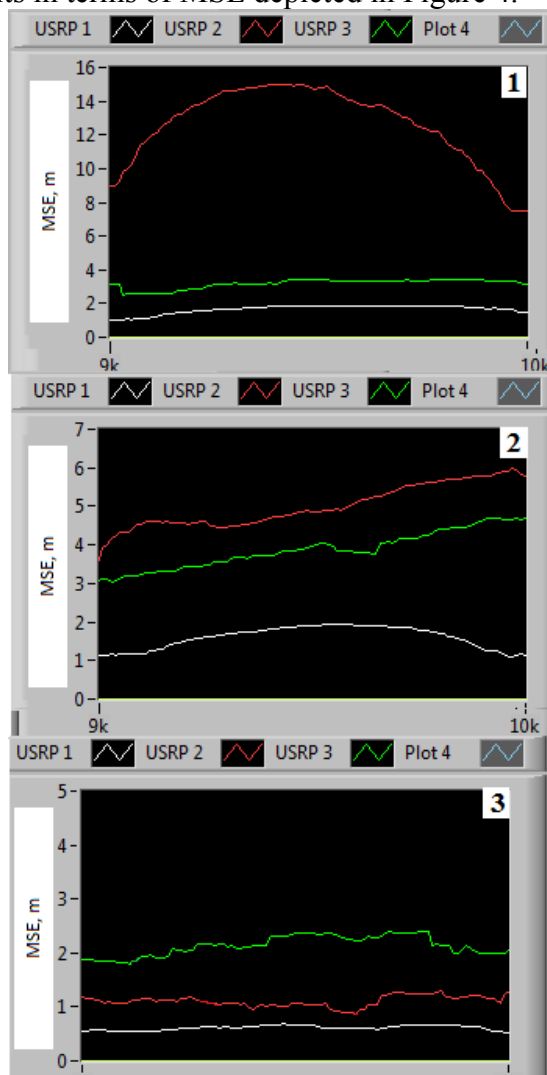


Figure 4. MSE for range measurements experimental validation

We have the following notations in Figure 4 considering experimental validation trial results accumulation gain in terms of MSE: 1 is the range MSE in meters for the case without joint processing,

2 is the range MSE in meters for the case of joint processing with regard to all transceiver stations and one trial result, 3 is the range MSE in meters for the case of joint processing with regard to all transceiver stations and ten trial results accumulation. It can be seen from the Figure 4, that increasing the number of trial results accumulation and the number of transceiver stations realizing this procedure jointly can substantially increase positioning accuracy estimation in terms of MSE.

When comparing analytical results depicted in Figure 2 with experimental results depicted in Figure 4 we can easily detect substantial increase of MSE in the latter case. This can be explained by synchronization uncertainty due to network organization via GBE LAN.

IV. CONCLUSIONS

In the conclusion we can state that joint processing of range measurements with trial results accumulation can substantially improve positioning accuracy. An algorithm for such processing was proposed, then it was evaluated analytically and finally it was experimentally validated with software-defined radio testbed. In the future investigations it is preferable to replace LAN connection with WLAN so that to achieve fast deployment capability of such radio system.

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Dr. Sc. Georgiy Mashkov working in St.Petersburg University of Telecommunication as first Vice-rector (Vice rector for Academic Affairs). He was born in 1954 in in Uzlovaya, a town and the administrative center of Uzlovsky District in Moscow Region. G. Mashkov graduated Kiev Higher engineering school in 1977, then received Ph. D and D. Sc there in 1984 and 1993 respectively. He is is author of over 150 scientific papers. G. Mashkov is Honorary Worker of Higher Professional Education of Russia.



Telecommunication.

Ph. D. Evgeny Borisov working in St.Petersburg University of Telecommunication as Professor of Department of Radio Systems and Signal Processing. He was born in 1967 in Novosibirsk. E. Borisov graduated Kiev Higher engineering school in 1990 then received Ph. D in 2000 in Smolensk Higher engineering school. He is is author of 3 monographs, more than 120 scientific papers, and 28 patents. E. Borisov is leading researcher of St.Petersburg University of



Ph. D. Grogorii Fokin working in St.Petersburg University of Telecommunication as Associate Professor of Department of Radio Communications and Signal Broadcasting. He was born in 1984 in Pskov, graduated St.Petersburg University of Telecommunication in 2005 the received Ph. D there in 2009. From 2010 till 2014 G. Fokin worked as Researcher, Senior Researcher in Radio Research and Development Institute (NIIR). He is is author of 26 scientific papers and 3 tutorials.