

Probabilistic Risk Assessment of Multi-State Systems Based on Bayesian Networks

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Abstract—Probabilistic approaches are common in the risk assessment of complex engineering systems. Although conventional methods such as fault tree (FT) have been used effectively in probabilistic risk assessment (PRA), they are only suitable to binary-state systems. As an extension of FT, multi-state fault tree (MSFT) is a good way in the modelling of multi-state systems, but it suffers severely limitation of efficient analysis and assess for systems risk, which is of great significance in PRA. Due to the difficulty of risk assessment in the multi-state systems, a new method based on Bayesian networks (BNs) is proposed. The BN model is constructed by converting MSFT and logic operators of the system through a mapping algorithm. Then the calculations of consequence probability and importance degree for each component are proposed based on Bayesian inference. Also, diagnose of failure system states can be achieved by posterior inference of BN. Finally, an example is illustrated to verify the effectiveness and feasibility of the proposed method.

Keywords—risk assessment, multi-state systems, MSFT, BNs, consequence probability, importance degree

I. INTRODUCTION

Engineering decisions often involve probabilistic risk assessment of the system's working status under evolving and uncertain information. PRA is widely used as it can not only estimates the likelihood and consequences of accidents, but also categorizes their potential scenarios [1]. Traditional PRA analysis methods, such as Fault Tree Analysis (FTA) and Event Tree Analysis (ETA), are based on the assumption that the events involved are binary events (Normal or Fault). However, in many real situations, both the system and its components are capable of being in different failure modes [2]. Furthermore, any system and its component with one failure mode can be divided into lots of failure states according to the degree of the failure, and binary-state system can be treated as a special case of the multi-state system.

Recently, significant amounts of research efforts have been devoted to deal with the PRA analysis of the multi-state systems. In [3], a multi-state fault tree analysis (MFTA) model is proposed to assess the reliability of the complex system, and the MFTA model is constructed based on multi-state block diagrams using binary variables. Xue [4] introduced the discrete function theory into multi-state system analysis, and adapted two methods (inclusion-exclusion and enumeration) to calculate the probability of each state. However, these

above methods are only suitable within the scope of the coherent systems. Huang [5] proposed a generic method to construct the multi-state fault tree through reliability block diagrams and decision tables. However, with the growth of the system scale and types of states, analysing the MSFT based risk assessment becomes a rather complex problem. Therefore, an efficient general method is need to efficiently assess the risk of the multi-state systems.

Currently, Bayesian networks have been proposed in many researches as an ideal method for risk assessment and failure analysis in complex systems [6]. The BN is a graphical model consisting of nodes and directed links, which respectively represent random variables and their probabilistic dependencies. The variables may represent the states of the components of a system, or their capacities and demands. Lee et al. [7] presented a large engineering project risk management procedure by using BN, and then identified the major risk items which affected system performance. The main feature of BN is that, by entering evidence on one or more variables, e.g., the observed states, capacities or demands of a subset of the components, the information propagates throughout the network and updates distributions of other random variables, e.g., the states of other components or the system state, in accordance with the Bayes' rule, which provides an effective way of risk analysis in PRA. Several research have aimed at exploring the capabilities of BN in the risk assessment of binary-state systems [8, 9], which considered mapping fault tree into BN to assess the binary-state systems. However, these methods have not considered the situation of multi-state systems.

In this paper, we try to use the Bayesian network for probability risk assessment of the multi-state systems. We first introduce a brief description of the model of BN. Then, according to the MSFT structure and logical operators of the multi-state system, we proposed a mapping algorithm to convert MSFT to BN model. We use the BN inference technique to calculate the probability distribution of the systems state, and the performance of each component's contribution to a failure system statue is presented by the importance degree. Also, with the posterior inference of BN, diagnostic analysis is proposed to find out the suspected cause for a failure system. At last, through an application of the risk

assessment for water supply working system, we validate the effectiveness of the proposed method.

II. BAYESIAN NETWORKS

Bayesian networks (also known as belief nets) are graphical models to describe the dependencies among random variables, which provides a natural way of representing causal information. A BN consists of two parts.

A. Network Structure

S is a directed acyclic graph (DAG), which can be expressed by $S = \langle X, E \rangle$. $X = \{X_1, \dots, X_n\}$ is the nodes in the BN, each X_i represent a random variable, which could be events or substance. Directed acyclic E represents dependencies (or as a cause-effect relationship) between pairs of nodes, which usually indicates the cause node (parent node) points to the result node (children node). $Pa(X_i)$ is the set of parent nodes of X_i . In a BN, the nodes without incoming arcs, i.e. without parent nodes, are called root nodes, and the nodes without outgoing arcs, i.e. without children nodes, are called leaf nodes.

B. Network Parameterization

We denoted Θ as a set of conditional probability tables (CPTs), one for each variable in X , called the network parameterization. $\Theta_{X_i|pa(X_i)}$ denotes the CPT for variable X_i given its parents $pa(X_i)$ in S .

In BN analysis, what we concern is the joint probability distribution (JPD) $P(X_1, \dots, X_n)$ instead of the CPTs. Using the chain rule, the JPD can be written as

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1}^n P(X_i | X_1, X_2, \dots, X_{i-1}) \quad (1)$$

For any node X_i , we can find the minimal set of nodes in X which is not conditional independent with X_i . Assuming the minimal set is $\pi_i \subseteq \{X_1, X_2, \dots, X_{i-1}\}$, then

$$P(X_i | X_1, X_2, \dots, X_{i-1}) = P(X_i | \pi_i) \quad (2)$$

Where the variables in π_i is the parent nodes of X_i in BN. Then, JPD of $\{X_1, \dots, X_n\}$ can be written as

$$P(X) = \prod_{i=1}^n P(X_i | Pa(X_i)) = \prod_{i=1}^n \Theta_{X_i|pa(X_i)} \quad (3)$$

$\langle S, \Theta \rangle$ is used to determine a BN model. It can both forward reasoning and backward reasoning. Through marginalizing the JPD, we can achieve the probability distribution of a particular random variable (evaluate the risk of the system). Also, given the evidence (i.e., state of one or more random variables is known with certainty), we can get the marginal posterior probability for each variable (diagnostic assessment of high risk system).

III. RISK ASSESSMENT MODELLING OF MULTI-STATE SYSTEMS BASED ON BAYESIAN NETWORKS

The BN-based risk assessment of multi-state systems consists of two parts: BN model construction and BN-based risk analysis. In this section, we will briefly talk about them.

A. BN Model Construction

As the presented method, the construction of BN is to determine S and Θ . Several researchers considered using expertise knowledge to establish the BN nodes, determine the nodes prior distribution and CPT [10]. Due to the limitation of uncertainty of human interventions in knowledge acquisition, the outcome of the BN model designed by expertise knowledge may have discrepancy with real status.

FTA graphically depicts failure propagation and the logical relationship between root causes and fault paths. In a FTA, system risk is represented by top events, the basic events represent root causes which will danger the system. However, basic events may not directly affect the system risk, instead, intermediate events in FTA is the middle connection of top and basic events. The three types of events are connected by logical gate (AND, OR et al.). MSFT is a FT-based method which considers the states of the three types of events to be multi instead of binary. Zeng et al. [11] proposed a method of MSFT construction based on reliability block diagrams and decision tables. Unlike logic gates in binary-state FT, this method use a logical operator to link the conditional independent events. As BN and FT are similar in structure, it is straightforward to convert MSFT to BN.

Consider MSFT has basic events $X_1, X_2, \dots, X_M$, and $Q_i = \{q_i^1, q_i^2, \dots, q_i^{m_i}\}$ is a range of m_i states for X_i . $G_1, G_2, \dots, G_N$ represents the intermediate events, and $P_i = \{p_i^1, p_i^2, \dots, p_i^{n_i}\}$ is a range of n_i states for G_i . The top event T values in state space $\Omega = \{t_1, t_2, \dots, t_r\}$. The conversion algorithm from MSFT to BN proceeds along the following steps:

- (1) For each basic events X_i of MSFT, create a root node of same name in the BN and assign its state space the same as in MSFT;
- (2) For each intermediate events G_i of MSFT, create a same state space and name node in BN;
- (3) For the top event T in MSFT, assign to leaf node of BN, and give them the same state space and name;
- (4) Connect nodes in the BN as corresponding relations in MSFT;
- (5) Assign the prior probability to the root nodes in BN according to the relevant basic events X_i of MSFT;
- (6) The logic operator in MSFT determines the conditional probability distribution of intermediate and leaf nodes in BN.

By the mapping algorithm, it is easy to build BN model for multi-state systems. Based on the constructed BN model, we can conduct various types of analysis.

B. BN-Based Risk Analysis

Ren et al. [12] indicate that the most important use of BN is in revising probabilities in light of actual observations of events. Therefore, it is possible to calculate the probability distribution of potential safety risks and identify the most likely potential causes in occurrence of accidents.

In this paper, we mainly discuss the consequences analysis for the top event, importance degree of each basic event and diagnostic analysis for failure state of top event by using the Bayesian inference technique.

1) Consequence analysis

Consequence analysis aims to capture the probability distribution of the leaf node (top event) under a combination of root nodes (basic events). The states of each root node and intermediate node can be treated as evidence input into the BN. Unlike FTA/ETA, the Bayesian inference does not need to get minimal cut sets, which greatly increases the computational efficiency. The probability distribution of T , represented by $p(T=t)$, can be calculated as

$$p(T=t) = \sum_{x_1^o, \dots, x_M^o, g_1^o, \dots, g_N^o} p(X_1=x_1^o, \dots, X_M=x_M^o, G_1=g_1^o, \dots, G_N=g_N^o) \times p(T=t | X_1=x_1^o, \dots, X_M=x_M^o, G_1=g_1^o, \dots, G_N=g_N^o) \quad (4)$$

By Eq. (3), we can transform the above equation as

$$p(T=t) = \sum_{x_1^o, \dots, x_M^o, g_1^o, \dots, g_N^o} p(T=t | pa(T)) \times \prod_{k=1}^N p(G_k=g_k^o | pa(G_k)) \times \prod_{i=1}^M p(X_i=x_i^o) \quad (5)$$

Where $t \in \Omega$, $g_i^o \in P_i$, $x_i^o \in Q_i$ stands for the state of the three type nodes respectively.

2) Importance degree

Importance degree plays an important role in PRA, aiming at illustrating the performance of each root cause's contribution to the occurrence of an accident. Risk achievement worth (RAW) is proposed to measure the influence of each root node X_i to leaf node T . Under prior evidence of T at failure state t , the RAW of X_i , which is represented by $RAW_{X_i}(t)$, can be calculated as

$$RAW_{X_i}(t) = \frac{1}{m_i} \sum_{x_i^o \in Q_i} \frac{p(T=t | X_i=x_i^o)}{p(T=t)} \quad (6)$$

$RAW_{X_i}(t)$ can be used as an indicator to measure the degree of importance of the root node X_i , factors that are very important to the accident occurrence should be given more attention during the system running phase, in order to reduce the risk limit.

3) Diagnostic analysis

Comparing with FTA and ETA, the feature of the backward reasoning technique is unique and matchless in BN inference. Diagnostic analysis is used to obtain the posterior probability distribution of each basic event using the BN backward

reasoning technique when an accident or failure of top events happens. The posterior probability distribution of X_i , represented by $p(X_i=x_i^o | T=t)$, can be obtained by

$$p(X_i=x_i^o | T=t) = \frac{p(T=t | X_i=x_i^o) \times p(X_i=x_i^o)}{p(T=t)} \quad (7)$$

The distribution of posterior probabilities can provide reliable references for fault diagnosis. Suppose x_i^o is risk state, X_i is more likely to become the direct cause of a risk state t of T when $p(X_i=x_i^o | T=t)$ is close to 1.

IV. CASE STUDY

Take the water supply working system in [4] as an example. All states of the components in the system are mutually statistically independent. The MSFT of the working system is shown in Figure 1. The top event, intermediate events and basic events may be in any finite states. Four states are chosen to represent the performance levels of the system and its components: 0 represents failed state; 1 represents worse state; 2 represents not good state; 3 represents good state. Some components may only have two states, e.g., water level can be good or failed. The logical operator α_1 , α_2 and α_3 of the MSFT is shown in Figure 2., in which i and j represent the left and right input events of the logical operator respectively.

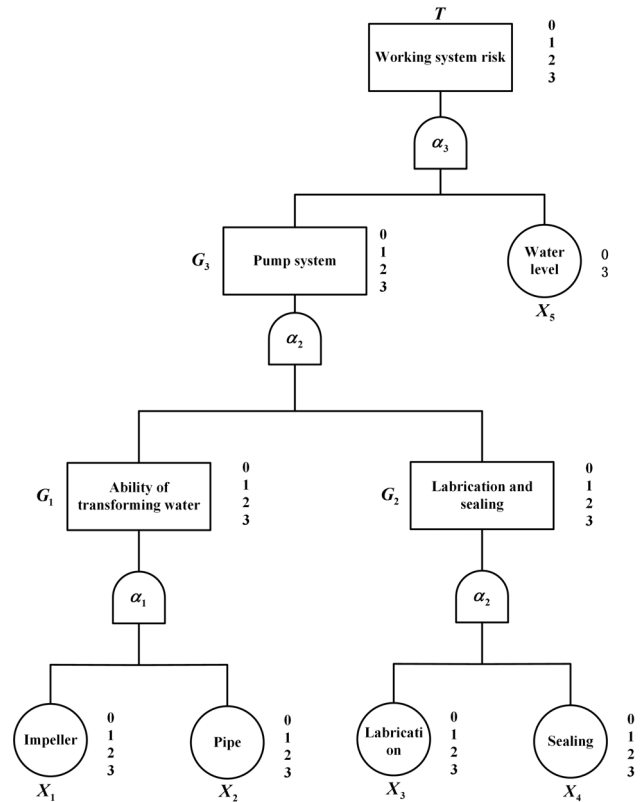


Figure 1. MSFT of the water supply working system

Operator α_1					Operator α_2					Operator α_3			
$i \backslash j$	0	1	2	3	$i \backslash j$	0	1	2	3	$i \backslash j$	0	3	
0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0	0	1	1	1	0	0	1	1	1	0	1	
2	0	1	1	2	2	0	1	1	2	2	0	2	
3	0	1	2	3	3	0	1	2	3	3	0	3	

Figure 2. Logical operators of the MSFT

Based on the mapping algorithm proposed in section 3, we can transform the MSFT of the water supply's working system into BN. The topology of the BN is shown in Figure 3.

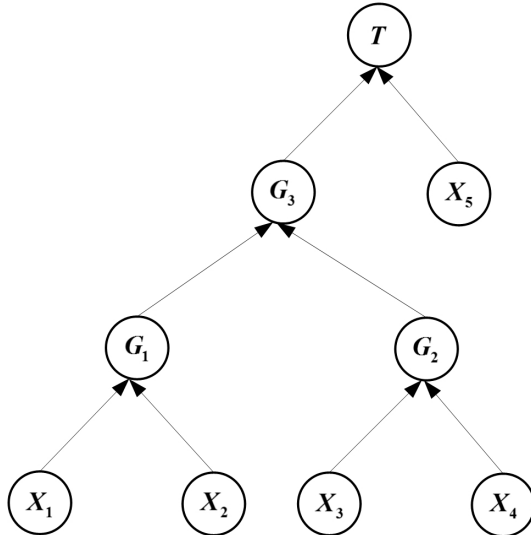


Figure 3. BN model of the water supply working system

The conditional probability distribution of the intermediate events G_i and top event T can be achieved by converting the logic operator α_1, α_2 and α_3 . We take G_1 as an example, as shown in Figure 4.

Logical operator α_1					CPT of G_1				
$i \backslash j$	0	1	2	3					
0	0	0	0	0	$P(G_1=0 X_1=0, X_2=1)=1$				
1	0	0	1	1	$P(G_1=0 X_1=1, X_2=0)=1$				
2	0	1	1	2	$P(G_1=0 X_1=1, X_2=2 \text{ or } 3)=1$				
3	0	1	2	3	$P(G_1=1 X_1=2, X_2=2)=1$				
					$P(G_1=1 X_1=2 \text{ or } 3, X_2=1)=1$				
					$P(G_1=2 X_1=2, X_2=3)=1$				
					$P(G_1=2 X_1=3, X_2=2)=1$				
					$P(G_1=3 X_1=3, X_2=3)=1$				

Figure 4. Conversion of Logic operator α_1 to CPT of G_1

The prior probability distribution of root nodes X_i is listed in Table 1.

TABLE 1. PRIOR PROBABILITY DISTRIBUTION OF X_i

	X_1	X_2	X_3	X_4	X_5
0	0.005	0.035	0.064	0.004	0.061
1	0.01	0.080	0.125	0.015	\
2	0.120	0.115	0.175	0.150	\
3	0.865	0.77	0.636	0.831	0.939

Since X_5 only have two states: 0 or 3, then we could see that the prior probability of X_5 only focus on state 0 and state 3. After establishing the BN model for the working system, we can use Bayesian analysis technology to assess the systems risk.

A. Consequence Probability

The occurrence probability $P(T=t)$ of the consequences $t(t \in \{0,1,2,3\})$ of the system working statue can be calculated by Eq. (5). Different from computations performed on MSFT, BN does not require the determination of minimal cut sets. We use the variable elimination algorithm to reduce the computation complexity. The result is shown in Table 2.

TABLE 2. PROBABILITY OF EACH STATE OF T

State	0	1	2	3
Probability	0.1602	0.1989	0.3104	0.3305

From Table 2, we can observe that the working system is more likely to be at good state (state 3), since $P(T=3)$ is higher than others.

B. Importance Degree

Importance Degree aims at identify the critical and sensitive factors in occurrence of working system risk or failures (at state 0 or 1) using the importance degree technique of Bayesian analysis. The RAW of all basic events X_i under working system risk statue $T=t, t \in \{0,1\}$, which is calculate by Eq. (6), is shown in Figure 5.

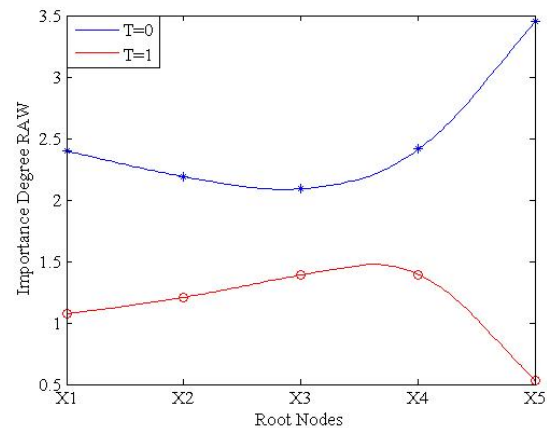


Figure 5. Importance degree of each root nodes for system at state 0 and state 1

As illustrated in Fig.5, X_5 has the top influence to cause the water supply's working system at failure state 0. Meanwhile, when working system is at state 1, X_3 turns out to be the top influence factor. Therefore, these two components are the weak part in the whole system, and they should be paid with more attention to ensure the system works at a good state.

C. Failure Diagnosis

When an accident or failure of the working system occurs while the system is working, we need to find out the suspected cause for the failure, so that we can prevent more serious losses. Using the diagnostic analysis of the BN analysis, we can construct a failure diagnosis for the working system.

In Figure 6, we present the posterior probability distribution of each root nodes of the BN under the evidence that the leaf node T , or called systems risk statue, to be at state 0 or state 1.

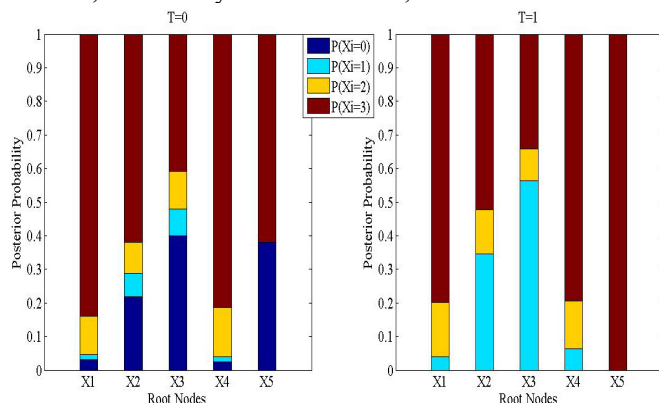


Figure 6. Failure diagnosis of system at state 0 and state 1

With regard to $T = 0$, as illustrated in the left of Fig. 6, indicate that $X_3 = 0$ (with a 40% chance) and $X_5 = 0$ (with a 38% chance) are more likely to happen. With regard to $T = 1$, as illustrated in the right of Fig. 6, indicate that $X_3 = 1$ (with a 56% chance) and $X_2 = 1$ (with a 35% chance) are more likely to happen. Therefore, X_3 is more likely to be the reason for system at a poor state. Thus, we should improve the reliability of X_3 to improve the performance level of the whole system.

V. CONCLUSIONS

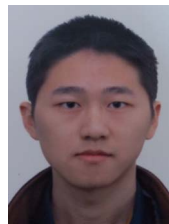
With advantages of uncertainty reasoning and figurative expression, Bayesian networks provide a probabilistic method for risk assessment of safety critical systems. In this paper, we proposed a BN model for risk assessment of multi-state systems. In order to construct the BN model, we proposed a mapping algorithm to convert MSFT to BN. Then we use the Bayesian inference technical to analysis the probability distribution of the system risk, the importance degree for basic events and failure diagnose can be obtained as well. At last, an application of risk assessment for water supply working system is shown to verify the effectiveness and feasibility of the proposed method.

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