

Adaptive Polynomials Method for FBMC Nonlinear Power Amplifier Complex Gain

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Abstract— In this paper, an adaptive polynomials for nonlinear power amplifier complex gain of a filter bank multicarrier modulation (FBMC) is modelled. One of the main problems in FBMC systems is the high peak-to-average power ratio (PAPR) of transmitting signals which leads to a significant distortion when passed through a nonlinear device such as a high power amplifier (HPA) of transmitter. Therefore, analytical expression of HPA characteristic based on an orthogonal polynomial method is investigated. From the simulation result, the HPA modelling error of the orthogonal polynomial method is better than that of the conventional polynomial method at a high input value. As a result, a combination of the orthogonal polynomial method and the conventional polynomial method, called the adaptive polynomial method, is proposed. With this method, the orthogonal polynomial method is applied for a high input value and adopts the conventional polynomial method for a low input value. A comparison with the simulation results is carried out to verify the accuracy of this method.

Keywords— power amplifier, orthogonal polynomial, nonlinear system, memoryless HPA, FBMC

I. INTRODUCTION

A system with multicarrier modulation (MCM) can achieve much higher spectral efficiency in frequency selective channels compared to that using single carrier modulation. Nowadays, the FBMC, which is a family of MCM modulation technique, is one of the most promising solutions for achieving high data rate 5G communications [1]. However, one of the most challenging issues for the FBMC signal is the high Peak to Average Power Ratio (PAPR) of transmitting signals due to the superposition of many subcarriers, in which leads to a significant in-band distortion and a spectral spreading when passed through a nonlinear device such as a high power amplifier (HPA) of transmitter. Basically, a high PAPR forces the transmit power amplifier to have a large *backoff* in order to ensure linear amplification of the signal, which cause degradation of an efficiency of the RF power amplifier. A number of approaches have been proposed to reduce the PAPR problem such as an amplitude clipping method, a block coding method, a selected mapping method (SLM), a partial transmit sequence method (PTS) and a tone reservation method (TR) [2]. Moreover, a various linearization

methods based on the concept of digital pre-distortion (DPD) have been proposed, aiming for an increased power efficiency of linearly operated HPA such as an inverse Volterra series, the rational function, Wiener-Hammerstein systems, memory polynomials, look-up table (LUT), and neural networks [3]. However, there is a lack of in-depth study on the analytical computation of the HPA characteristic. A theoretical characterization of nonlinear device effects on OFDM systems has been proposed in [4]. Referring to [4], the authors focused on the impact of the nonlinear amplitude distortions induced by three HPA models: the soft envelope limiter (SEL), the solid state power amplifier (SSPA), and the travelling wave tube amplifier (TWT). Nevertheless, except for the SEL HPA model, no closed-form HPA characteristic expression for SSPA and TWT HPA model. In [5], an approximation of the HPA conversion characteristics for multicarrier modulation schemes has been investigated for applying to any HPA model by using a polynomial fitting method of the amplitude-amplitude modulation (AM/AM) and amplitude-phase modulation (AM/PM) conversion characteristics. However, using the basic polynomial approximation could not achieve the most accuracy result [6].

This fact motivates us to investigate an analytical computation of HPA characteristic by using sophisticated mathematic method such as orthogonal polynomial method. The performance comparison in the sense of accuracy between the conventional polynomial method and the proposed method is presented. Moreover, the adaptive polynomial method by combining the orthogonal and the conventional polynomial method is presented. The contributions of this paper include an approximation expression of HPA characteristic based on the orthogonal polynomial method and the adaptive polynomial method. The rest of this paper is organized as follow. Section II introduces the system model including HPA models characteristic and the nonlinear HPA conversion characteristics for multicarrier modulation scheme. The propose method is described in Section III. In Section IV, we present the simulation results of the proposed method. Finally, the conclusion is given in Section V.

II. SYSTEM MODEL

In this section, a system model used in this research is described, regarding the FBMC system with the commonly used HPA models such as SEL, SSPA and TWTA and the HPA characteristic modelling by using the conventional polynomial method.

A. HPA Models

Before transmitting to the channel, the baseband FBMC signal is passed through a nonlinear high power amplifier (HPA). The peak power of the FBMC signal could be distorted when passing through the HPA, resulting in a substantial amount of signal distortion.

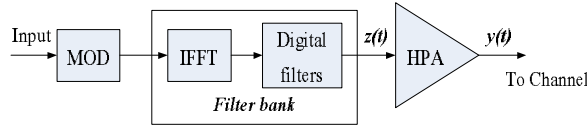


Figure1. FBMC Block Diagram

A diagram of the transmitted FBMC signal is shown in Figure 1. $z(t)$ is the baseband FBMC signal with complex envelope at the input of the HPA. The HPA model can be described by its input/output or transfer function characteristics. Basically, the amplified signal $y(t)$ can be written as

$$y(t) = S[\rho(t)] \exp[j\theta(t)] \quad (1)$$

where $\rho(t), \theta(t)$ is the signal amplitude and phase of $z(t)$, respectively. For simplicity sake, the time variable t from $\rho(t)$ and $\theta(t)$ is discarded. $S(\rho) = F_a(\rho) \exp(jF_p(\rho))$ is the complex soft envelop of the amplified signal $y(t)$. $F_a(\rho)$ is the AM/AM characteristic of the HPA. $F_p(\rho)$ is the AM/PM characteristic of the HPA. The memoryless HPA models that are commonly used are SEL, SSPA and TWTA described bellows.

1. Soft envelope limiter (SEL)

The AM/AM and AM/PM functions can be defined as [7]

$$\begin{aligned} F_a(\rho) &= \begin{cases} \rho, & \rho \leq A_{sat} \\ A_{sat}, & \rho > A_{sat} \end{cases} \\ F_p(\rho) &= 0 \end{aligned} \quad (2)$$

where A_{sat} is the HPA saturation input level.

2. Solid state power amplifier (SSPA)

This model, also known as the Rapp model [8]. They can be expressed as

$$\begin{aligned} F_a(\rho) &= \frac{\rho}{\left(1 + \left(\frac{\rho}{A_{sat}}\right)^{2\nu}\right)^{\frac{1}{2\nu}}}, \\ F_p(\rho) &= 0 \end{aligned} \quad (3)$$

where ν is a smoothness factor that controls the transition from the linear region to the saturation region, ($\nu > 0$).

3. Traveling wave tube amplifier (TWTA)

Saleh's model is generally used for modelling TWT amplifiers [9], which is the better accuracy than SEL and SSPA model. The AM/AM and AM/PM conversion characteristic of the TWTA model can be expressed as

$$\begin{aligned} F_a(\rho) &= A_{sat}^2 \frac{\rho}{\rho^2 + A_{sat}^2}, \\ F_p(\rho) &= \theta_0 \frac{\rho^2}{\rho^2 + A_{sat}^2} \end{aligned} \quad (4)$$

where θ_0 controls the maximum phase distortion introduced by this HPA model. In general, the HPA is operated at a given input back-off (IBO) from a given level for reducing the effects of nonlinearities. The IBO is defined as

$$IBO = 10 \log_{10} \left(\frac{A_{sat}^2}{\sigma^2} \right) \quad (5)$$

where σ is the variance of the input signal (mean input signal power).

B. The Nonlinear HPA Characteristic Modelling

According to the Bussgang theorem [10], the NL HPA output $y(t)$ is related to the input $z(t)$ by the following equation.

$$y(t) = K(t)z(t) + n(t) \quad (6)$$

where $n(t)$ is a zero mean noise, which is uncorrelated from $z(t)$. $K(t)$ is a complex gain with modulus $|K|$ and phase $\varphi_K(t)$. However, $K(t)$ can be determined analytically [4] by

$$K(t) = \frac{1}{2} \mathbb{E} \left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho} \right] \quad (7)$$

where $\mathbb{E}[\cdot]$ is the expectation operator.

In order to simplify the computation and obtain analytical expressions for K for any HPA model, a polynomial approximation of $S(\rho)$ have been proposed [5]. By using the polynomial approximation, the new HPA output is given by

$$y(t) = e^{j\theta} \sum_{k=1}^L \alpha_k \rho^k, \quad (8)$$

$$= e^{j\theta} S(\rho)$$

where L is the polynomial order, α_k is the complex coefficients of the polynomial approximation. The new expression of $S(\rho)$ is equal to

$$S(\rho) = \sum_{k=1}^L \alpha_k \rho^k. \quad (9)$$

The complex valued polynomial coefficients $\alpha_k, k=1,2,\dots, L$ can be easily obtained by using a classical least square (LS) method [5]. The computation of K in equation (7) can be achieved by the investigation of the expectation of a power of Rayleigh random variable, $\mathbb{E}[\rho^k]$, expressed as

$$K(t) = \frac{1}{2} \sum_{k=1}^L (k+1) \alpha_k \mathbb{E}[\rho^{k-1}] \quad (10)$$

According to [5], the computation of K can be achieved by

$$K(t) = \sqrt{\frac{\pi}{8}} \sum_{k=1, K \text{ odd}}^L (k+1) \alpha_k \sigma^k \prod_{i=0}^{\frac{k-1}{2}} (2i+1) + \frac{1}{2} \sum_{k=2, K \text{ even}}^L (k+1) \alpha_k (\sqrt{2}\sigma)^k \left(\frac{k}{2}\right)!. \quad (11)$$

III. THE PROPOSED METHOD

As mentioned in the literature, the peak power of the FBMC signal could be distorted when passing through the HPA, resulting in a substantial amount of signal distortion. Therefore, a theoretical HPA characteristic expression is an important issue. Referring to the existing research, an analytical computation of HPA characteristic using the basic polynomial approximation could not achieve the best accuracy result. In this paper, a novel method based on an orthogonal polynomial fitting of the HPA characteristic is proposed.

Based on a (quasi-) memoryless HPA, the HPA output $y(t)$ can be determined by [11]

$$y(t) = \sum_{K=0}^L \alpha_{2K+1} |z(t)|^{2K} z(t),$$

$$= \sum_{K=0}^L \alpha_{2K+1} \phi_{2K+1}(z(t)) \quad (12)$$

where $z(t)$ is the HPA input, α is the conventional polynomial coefficient at order K .

$\phi_{2K+1}(z(t)) = |z(t)|^{2K} z(t)$ is the conventional polynomial basis function. To obtain the closed-form expressions for the orthogonal polynomials, an upper triangular matrix U could be applied. The new polynomial basis function can be examined by

$$\phi_{2k+1}(z) = \sum_{K=0}^L U_{KL} \phi_{2K+1}(z(t)) \quad (13)$$

$$U_{kL} = \sum_{K=0}^L (-1)^{L-K} \frac{\sqrt{L+1}}{(k+1)!} \binom{L}{k}, k \leq L. \quad (14)$$

Table 1 shows the first nine such orthogonal polynomials.

$\phi_1(z) = z$
$\phi_3(z) = \sqrt{2}(-1 + \frac{1}{2} z ^2)z$
$\phi_5(z) = \sqrt{3}(1 - z ^2 + \frac{1}{6} z ^4)z$
$\phi_7(z) = \sqrt{4}(-1 + \frac{3}{2} z ^2 - \frac{1}{2} z ^4 + \frac{1}{24} z ^6)z$
$\phi_9(z) = \sqrt{5}(1 - 2 z ^2 + z ^4 - \frac{1}{6} z ^6 + \frac{1}{120} z ^8)z$

Therefore, (13) can be expressed as

$$\phi_{2k+1}(z(t)) = \sum_{K=0}^L (-1)^{L-K} \frac{\sqrt{L+1}}{(k+1)!} \binom{L}{k} \phi_{2K+1}(z(t)). \quad (15)$$

According to (15), the conventional polynomial HPA output in (12) can be modified as

$$y(t) = \sum_{K=0}^L \alpha_{2K+1} \phi_{2K+1}(z(t)) \quad (16)$$

$$y(t) = e^{j\theta} \sum_{K=0}^L \alpha_{2K+1} \rho^{2K} \quad (17)$$

where

$$\rho^{2K} = \varphi_{2K+1}(z(t)).$$

Then

$$S(\rho) = \sum_{K=1}^L \alpha_{2K+1} \rho^{2K}$$

and the HPA gain from equation (7) is modified based on the orthogonal polynomial as

$$\begin{aligned} K &= \frac{1}{2} \sum_{k=0}^L \mathbb{E} \left[\frac{\partial S(\alpha_{2k+1}) \rho^{2k}}{\partial \rho} + \frac{(\alpha_{2k+1}) \rho^{2k}}{\rho^{2k}} \right], \\ &= \frac{1}{2} \sum_{k=0}^L \alpha_{2k+1} \mathbb{E} \left[\frac{\partial \rho^{2k}}{\partial \rho} + \frac{\rho^{2k}}{\rho^{2k}} \right], \\ &= \frac{1}{2} \sum_{k=0}^L \alpha_{2k+1} \mathbb{E} \left[\frac{\partial \rho^{2k}}{\partial \rho} + 1 \right], \\ &= \frac{1}{2} \sum_{k=0}^L \alpha_{2k+1} (2k+1) \mathbb{E} [\rho^{2k+1}] \end{aligned} \quad (18)$$

where

$$\rho^{2k} = \varphi_{2k+1}(z(t)).$$

Therefore, the HPA gain approximation based on the orthogonal polynomial method is expressed as

$$K(t) = \frac{1}{2} \sum_{K=0}^L \alpha_{2K+1} (2K+1) \mathbb{E} [\varphi_{2K+1}(z(t))]. \quad (19)$$

Basically, a normalized mean squared error (NMSE) is defined to verify a quantitative measure of the approximation accuracy, expressed as [11]

$$nmse(dB) = 10 \log_{10} \left[\frac{\sum_{n=1}^N |y(t_n) - \hat{y}(t_n)|^2}{\sum_{n=1}^N |y(t_n)|^2} \right] \quad (20)$$

where $y(t_n)$ is the measured HPA output and $\hat{y}(t_n)$ is the HPA modelled output. N is the number of input samples.

IV. SIMULATION RESULTS

This section presents computer simulation results of the HPA characteristic modelling without noise consideration.

A normalized mean squared error (NMSE) is applied to verify the accuracy of each HPA characteristic expression methods. In our simulations, the HPA characteristic for all methods are derived assuming that the input amplitude $|\rho|$ is uniformly distributed in $[0,1]$. For the basis function coefficients of the conventional polynomial is of the order of $|\rho|^k$ [12]. To determine the NMSE, the HPA output $y(t_n)$ in equation (20) is generated according to the SSPA model (3) with parameters A_{sat} equal to 1, v equal to 1. The HPA modelling error, verified by the NMSE value, comparison of the HPA output between the conventional polynomial method and the proposed orthogonal polynomial method is presented.

Figure 2 shows the HPA modelling error comparison at IBO equal to 4 with different polynomial order (k). In Figure 2, the HPA modelling error of the orthogonal polynomial method becomes progressively smaller than that of the conventional polynomial method, especially when additional higher order polynomial term (k) is applied.

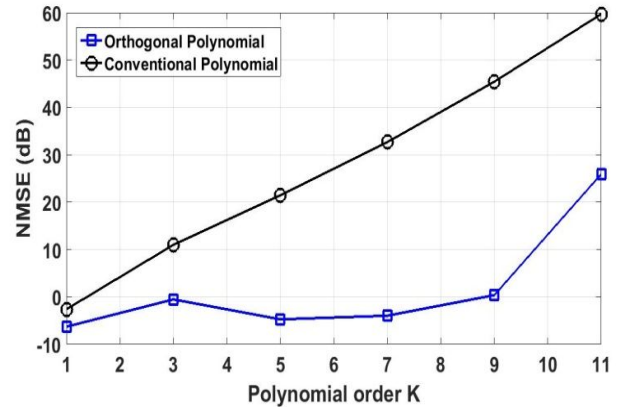


Figure 2. The HPA modelling error comparison

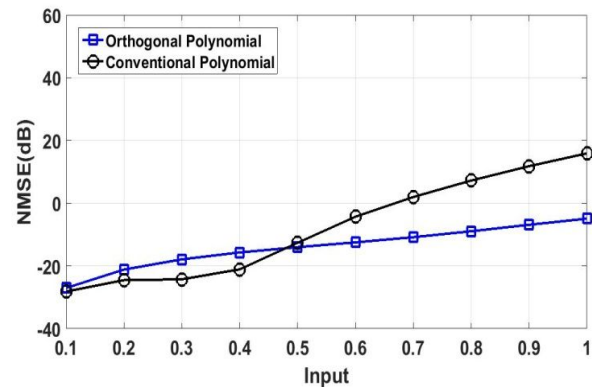


Figure 3. The HPA modelling error comparison (IBO = 2, order k = 2)

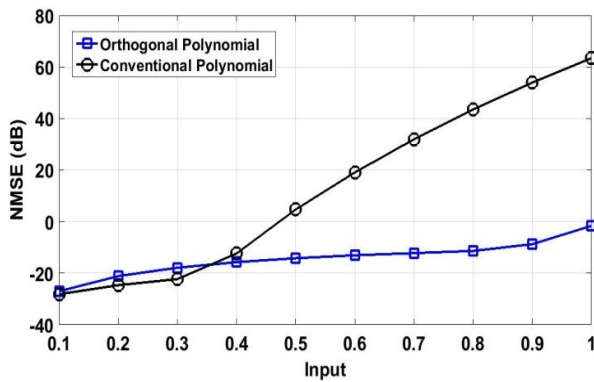


Figure 4. The HPA modelling error comparison (IBO = 2, order $k = 10$)

Figure 3 and 4 show the HPA modelling error versus different input values. From the Figure 3 and 4, the conventional polynomial method introduces more error when a high input value, in contrast with the proposed orthogonal polynomial method.

The HPA modelling error comparison at polynomial order k equal to 10 with different IBO are presented in Figure 5 and 6. From Figure 5, the HPA modelling error of the conventional polynomial method at IBO equal to 6 is higher than that of the proposed orthogonal polynomial method at the input value from 0.6. At IBO equal to 8, the HPA modelling error in Figure 6 of the conventional polynomial method is higher than that of the proposed orthogonal polynomial method at the input value from 0.7. Moreover, it could be found that the maximum HPA modelling error of the conventional polynomial method becomes larger when IBO value decline. From the figures, they could be implied that the conventional polynomial method introduces more error when a low IBO, meanwhile the proposed orthogonal polynomial method is free of such a problem. As result, an adaptive polynomial method, which is combination between the goodness the proposed orthogonal polynomial method and the conventional polynomial method is proposed. With this method, the orthogonal polynomial method is applied for a high input value and adopts the conventional polynomial method for a low input value.

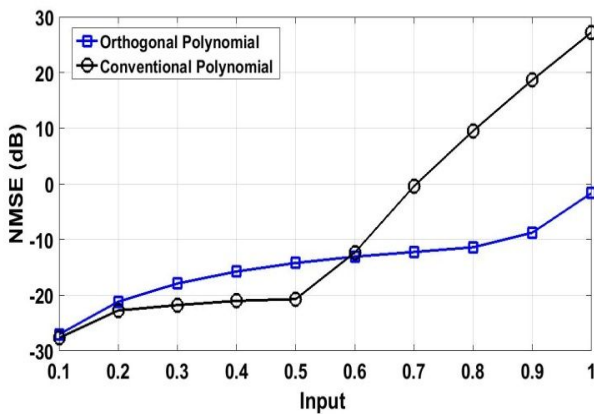


Figure 5. The HPA modelling error comparison (IBO = 6, order $k = 10$)

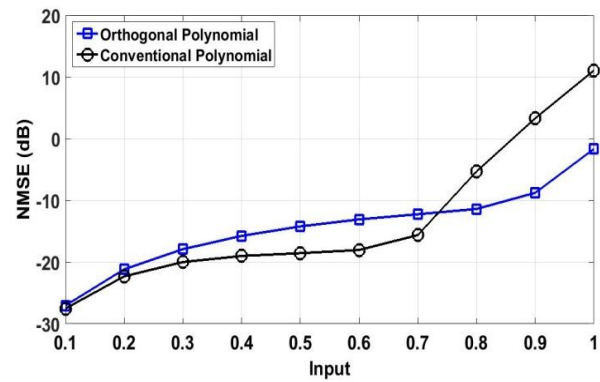


Figure 6. The HPA modelling error comparison (IBO = 8, order $k = 10$)

Figure 7 and 8 show the HPA modelling error of the adaptive polynomial method comparing with the orthogonal polynomial method and the conventional polynomial method. From the figures, the adaptive polynomial method yield better HPA modelling error than the conventional polynomial method.

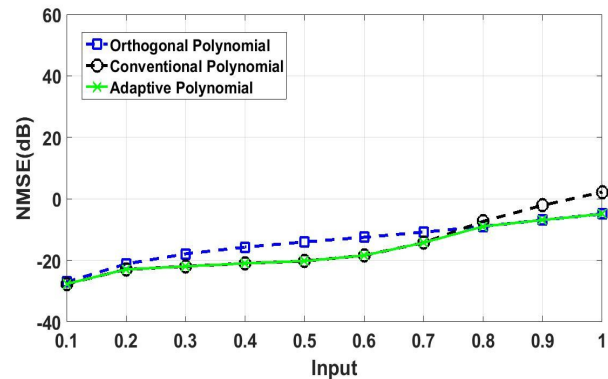


Figure 7. The HPA modelling error comparison (IBO = 6, order $k = 2$)

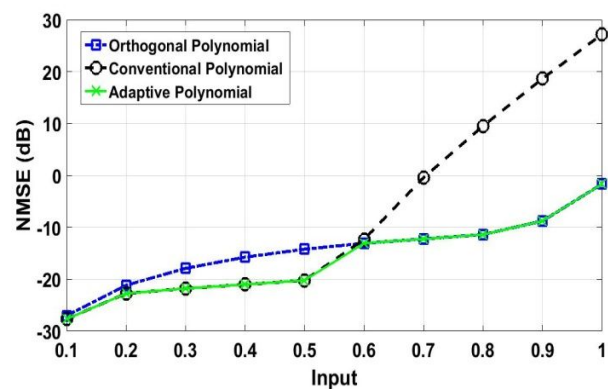


Figure 8. The HPA modelling error comparison (IBO = 6, order $k = 10$)

V. CONCLUSIONS

In this paper, the analytical computation of HPA characteristic for multicarrier modulation schemes by using the orthogonal polynomial method is investigated. The performance comparison in the sense of accuracy between the conventional polynomial method and the proposed method is presented. From the simulation result, the HPA modelling error of the orthogonal polynomial method is better than that of the conventional polynomial method when a high input value is applied, especially at a high polynomial order. Moreover, the orthogonal polynomial method is free of the different IBO, in contrast with the conventional polynomial method. Finally, the adaptive polynomial method is proposed, which is composed of the orthogonal polynomial method applied for a high input value and the conventional polynomial method applied for a low input value. Simulation results show that the adaptive polynomial method yield the better accuracy.



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REFERENCES

- [1] G. Wunder et al, "Challenging the LTE Design Paradigms of Orthogonality and Synchronicity", *IEEE VTC Spring*, 2013.
- [2] Mishmy T.S. and Sheeba V.S., "Peak to average power ratio reduction in filterbank based multicarrier system", *IEEE International Conference on Advance in Computing and Communications*, August 2013, Page(s): 377-381.
- [3] R. Zayani et al, "Adaptive predistortion techniques for non-linearly amplified FBMC-OQAM signals", *Proceeding of IEEE International Conference on Vehicular Technology (VTC Spring)*, Seoul, May 2014, pp. 1-5.
- [4] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems", *IEEE Trans on Communications*, vol. 48, no. 10, October 2000, pp. 1755-1764.
- [5] H. Bouhadda et al. "Theoretical analysis of BER performance of nonlinearly amplified FBMC/OQAM and OFDM signals", *EURASIP Journal on Advances in Signal Processing*, DOI: 10.1186/1687-6180-2014-60, May 2014.
- [6] R. Dallinger et al., "Adaptive pre-distortion techniques based on orthogonal polynomials signals", *Proceeding of IEEE International Asilomar Conference on Systems and Computers (ASILOMAR)*, Canada, November 2010, pp. 1945-1950.
- [7] He Rowe, *Memoryless Nonlinearity with Gaussian Inputs : Elementary Results*, Bell Syst. Tech. J. vol.61, no.7, 1982, pp. 1519-1526.
- [8] C. Rapp, "Effect of HPA nonlinearity on 4-DPSK-OFDM signal for digital sound broadcasting systems," *Second European Conference on Sat. Comm, Belgium*, 1997.
- [9] A. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT Amplifiers," *IEEE Trans. on Communication*, vol.29, 1981, pp.1715-1720.
- [10] A. Papoulis, He Rowe, *Propability, Random Variables, and Stochastic Process*, 3rd ed., New York Polytechnic Institute, New York, 1991.
- [11] R. Raich et al, "Orthogonal polynomial for complex gaussian process", *IEEE Trans. on Signal Processing*, vol. 52, no. 10, October 2004, pp. 2788-2796.
- [12] R. Raich et al. "Orthogonal polynomials for power Amplifier modeling and predistorter design," *IEEE Trans. on Vehicular Technology*, vol. 53, no. 5, September 2004, pp. 1468-2797.