

On the Spectral Analysis of Discrete-Time Faster-Than-Nyquist Signaling

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Abstract—The main purpose of the FTN signaling is to increase data rate without using additional bandwidth or transmission power. To analyze system characteristics of the FTN-based system, such as system capacity, there have been some studies on the power spectral density (PSD) of the FTN signaling. The present study analyzes the PSD characteristics of a discrete-time model of the FTN signaling, and compares it to those of the existing work. From the analysis, we obtain the following meaningful observations: a certain relation between the truncation of the FTN data spectrum and the optimal choice of the FTN factor τ in terms of achieving the highest system capacity, and a possibility to reduce the down-sampling rate of the receivers to avoid aliasing when the bandwidth of the transmission pulse is less than the symbol rate.

I. INTRODUCTION

Recently, research communities related to 5G mobile communication and satellite broadcasting system have considered the Faster-Than-Nyquist (FTN) signaling motivated by J. E. Mazo [1] as a new waveform solution or an advanced multiple access technology to enhance throughput of the system [2]. FTN signaling is a novel signal transmission method that transmits symbols at faster rate than Nyquist signaling using neither additional bandwidth nor transmission power [3]. It can increase the spectral efficiency of a system, however inter-symbol-interference (ISI) is necessarily caused in the transmitted signal. The ISI leads to a significant receiving performance degradation since the minimum Euclidean distance among possible signal waveforms decreases. In [1], It has been shown that the minimum distance doesn't change as symbols are transmitted at up to rate of about 25% faster than Nyquist rate when binary modulation and sinc transmission pulses are assumed.

In an effort to analyze a theoretical performance limit of FTN-based communication system, Rusek *et al.* [4] have presented a meaningful result on the system capacity of the FTN signaling. They have shown that if a communication system has excess bandwidth, i.e., the transmission pulse of the system is not a sinc, there is an opportunity to recover the system capacity loss by using the FTN signaling. Furthermore, they also have demonstrated the optimal choice of FTN factor τ to achieve the highest possible system capacity for given transmission pulse and system bandwidth.

There also have been numerous attempts to investigate the power spectral density (PSD) characteristics of the FTN signaling [5–8]. In [6], the PSD characteristics of the pre-coded and un-pre-coded FTN signal have been analyzed as data symbols are wide-sense stationary. From the analysis, It has been observed that the FTN data spectrum defined in [6] is truncated by a band-limited channel. Hefinawy *et al.* [7], [8] have applied the spectrum sharing to FTN signaling and demonstrated its impact on the spectral efficiency of the system. Also a solution of the best spectral shape to achieve high spectral efficiency has been proposed under a transmission power constraint.

In this paper, a discrete-time model of the FTN signaling is considered to analyze its PSD characteristics. By exploiting spectral analysis of the discrete-time FTN signaling, we revealed two meaningful observations. First we demonstrate a certain relation between the optimal choice of τ presented in [4] and the truncation of the FTN data spectrum described in [6]. We also suggest a possibility to reduce the minimum down-sampling rate at the receiver of FTN-based communication system, when the FTN data spectrum is truncated. This paper is organized as follows: Section II describes the FTN signaling and the discrete-time FTN system model considered in this paper. Section III presents the spectral analysis of the system and compares it to existing works on the continuous-time FTN system. Section IV gives some observations from the spectral analysis, and the conclusion is discussed in Section V.

II. SYSTEM MODEL

A. Continuous-Time Model of FTN Signaling

The continuous-time FTN signal can be written as follows:

$$s(t) = \sum_k a_k h(t - k\tau T), \quad (1)$$

where $\{a_k\}$ are independent and identically distributed (i.i.d.) data symbols with zero-mean, $h(t)$ is a baseband transmission pulse whose baseband bandwidth is W , and τ is the FTN factor ($0 < \tau \leq 1$) [3]. The data symbols are transmitted $1/\tau$ times faster than those in Nyquist signal, hence the transmission symbol rate is $1/(\tau T)$. Let us assume that $h(t)$ has unit energy,

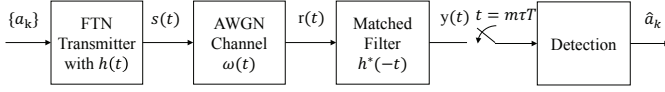


Fig. 1. Structure of the continuous-time model of FTM signaling

$$\int_{-\infty}^{\infty} |h(t)|^2 dt = 1, \quad (2)$$

and the PSD of $h(t)$ is $|H(f)|^2$. It is obvious that $|H(f)|^2$ is independent of τ and only depends on $h(t)$.

The additive white Gaussian noise (AWGN) with $N_0/2$ PSD is added to the transmitted signal, and a conventional matched filter $h^*(-t)$ is applied to the received signal. Sampled at the rate of $1/\tau T$ after the matched filter, the received FTM symbol sequence can be described as follows:

$$y(m\tau T) = \sum_k a_k g((m-k)\tau T) + \tilde{\omega}(m\tau T), \quad (3)$$

where $g(t) = h(t) * h^*(-t)$ and $\tilde{\omega}(t)$ is a colored noise obtained by filtering the AWGN. Fig. 1 shows a schematic of the considered continuous-time model of FTM signaling.

B. Discrete-Time Model of the FTM Signaling

In this section, we present a discrete-time model of FTM signaling and compare that to the continuous-time model in the previous subsection.

In discrete-time domain, the FTM transmitter in Fig.1 can be translated into two successive discrete-time signal processes, up-sampling and discrete-time pulse shaping [2]. Up-sampling process inserts zero-valued samples between adjacent data symbols, and the number of zero-valued samples is determined by τ . An up-sampled data symbol sequence can be written as follows:

$$p[n] = \sum_k a_k \delta[n - k\tau N], \quad (4)$$

where $\delta[n]$ is an unit impulse function and N is the Nyquist signaling period in discrete-time domain. After the up-sampling, pulse shaping filter $h[n]$ which is a sampled version of $h(t)$ is applied to the up-sampled sequence. Since N is the Nyquist signaling period, $h[n]$ crosses with zero at $n = N, 2N, 3N, \dots$. The consecutive processes gives a discrete-time FTM signal which includes inter-symbol-interferences.

To form a desired discrete-time FTM signal, the up-sampling rate of data symbols and the interpolation rate of the pulse shaping filter should be determined properly. Let us assume that τ can be expressed by a fraction representation as follows:

$$\tau = \frac{\tau_n}{\tau_d}, \quad (5)$$

where τ_n is the numerator and τ_d is the denominator of the fraction. From the representation, the up-sampling rate of data symbols and the interpolation rate of the pulse shaping filter can be determined to τ_n and τ_d , respectively, and a desired FTM signal can be generated by this way.

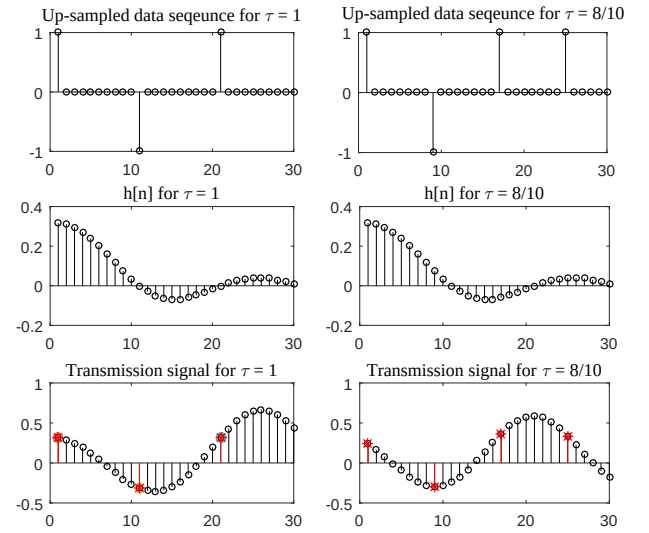


Fig. 2. Instances of the discrete-time FTM signal generation

Simple instances of generating discrete-time FTM signals are presented in Fig. 2. In $\tau = 1$ case, the up-sampling rate and the interpolation rate are the same, and a Nyquist transmission signal is generated. However, in $\tau = 8/10$ case, the up-sampling rate is less than the interpolation rate. The pulse shaping filter $h[n]$ is the same regardless of τ , but the number of zeros-valued samples inserted between adjacent data symbols is varied with τ , and the FTM signal including inter-symbol-interferences can be generated.

A discrete-time FTM signal $x[n]$ is formed by a linear convolution of $p[n]$ and $h[n]$:

$$x[n] = p[n] * h[n] = \sum_k a_k h[n - k\tau N]. \quad (6)$$

The discrete-time AWGN channel $\omega[n]$ presents the received signal $r[n] = x[n] + \omega[n]$. Using the conventional matched filter, the received FTM symbol sequence can be written as follows:

$$\begin{aligned} y[n] &= (x[n] + \omega[n]) * h^*[-n] \\ &= \sum_k a_k g[n - k\tau N] + \tilde{\omega}[n] \end{aligned} \quad (7)$$

$$y[m\tau N] = \sum_k a_k g[(m-k)\tau N] + \tilde{\omega}[m\tau N], \quad (8)$$

where $g[n] = h[n] * h^*[-n]$ and $\tilde{\omega}[n] = \omega[n] * h^*[-n]$ which is a discrete-time colored noise. Since the Nyquist signaling period N is equal to the interpolation rate of the pulse shaping filter τ_d , (8) is equivalent to the following:

$$y[m\tau_n] = \sum_k a_k g[(m-k)\tau_n] + \tilde{\omega}[m\tau_n]. \quad (9)$$

Fig. 3 illustrates a block diagram of the considered discrete-time FTM signaling.

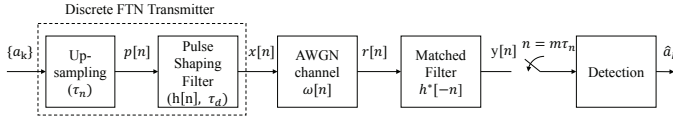


Fig. 3. Structure of the considered discrete-time FTN-based system

III. SPECTRAL ANALYSIS OF FTN SIGNALING

A. Spectral Analysis of the Continuous-Time FTN Signaling

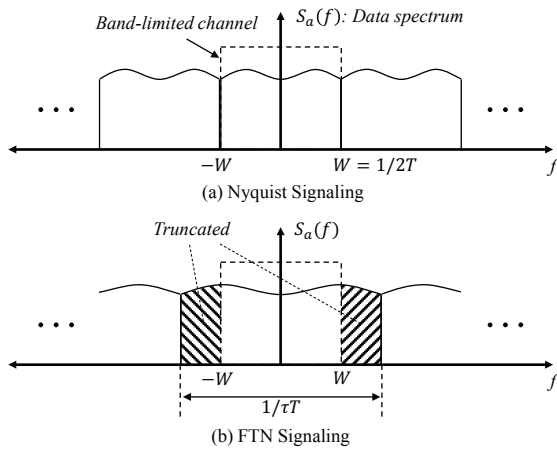
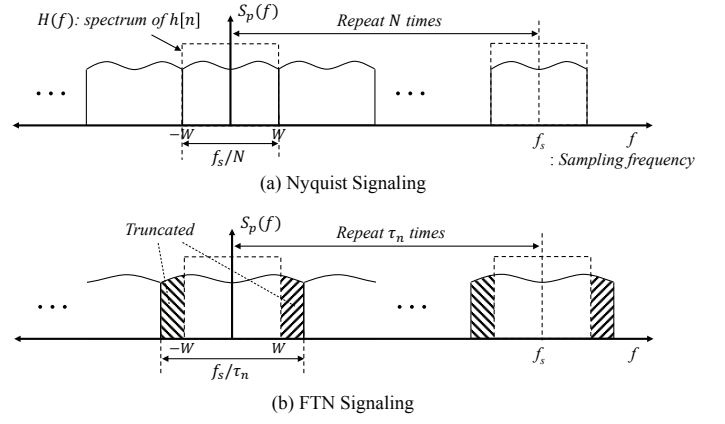
This subsection reviews the previous studies on the PSD characteristics of the continuous-time FTN signaling presented in [5], [6]. The PSD of FTN signals assuming the data symbols are wide-sense stationary is derived as follows [6]:

$$S(f) = \frac{1}{\tau T} S_a(f) |H(f)|^2, \quad (10)$$

where $H(f)$ is the Fourier transform of the transmission pulse, and the data spectrum $S_a(f)$ is defined by

$$S_a(f) \triangleq \sum_{k=-\infty}^{\infty} R_a(k) e^{j2\pi f k \tau T}. \quad (11)$$

In (11), $R_a(k)$ is the autocorrelation function of the data symbols, and the equation is similar with the discrete-time Fourier transform of $R_a(k)$ [6]. From the derived PSD of the FTN signal, it can be shown that the FTN data spectrum $S_a(f)$ is truncated by the band-limited channel. Therefore, a part of the data spectrum is lost in the process of FTN transmission by band-limited pulse $h(t)$. Fig. 4 shows the PSD characteristics of Nyquist and FTN signaling. As shown in the figure, the data spectrum of FTN signal is truncated by the band-limited channel, because the bandwidth of the data spectrum is wider than that of the channel.

Fig. 4. The data spectrum $S_a(f)$ of the continuous-time a) Nyquist and b) FTN signalingFig. 5. The data spectrum $S_p(f)$ of the discrete-time a) Nyquist and b) FTN signaling

B. Spectral Analysis of the Discrete-Time FTN Signaling

This subsection demonstrates the PSD characteristics of the discrete-time FTN signaling. It is well known that the discrete-time FTN signaling can be interpreted as a multi-rate digital signal processing (DSP) system [2], since a discrete-time FTN signal waveform is determined by the up-sampling rate and the interpolation rate of the pulse. Hence, the spectrum of discrete-time FTN signal also can be analyzed by the multi-rate DSP system interpretation.

Let us assume that the data symbol sequence are uncorrelated, i.e., the spectrum of the data symbol sequence is white. When the up-sampling process with rate of τ_n is applied to the data symbol sequence, the spectrum of the sequence is repeated τ_n times in the range of the sampling frequency. Note that the sampling frequency denoted by f_s does not vary with τ and remains constant in the system. The spectrum of the up-sampled data symbol sequence consists of τ_n replicas of the white data spectrum, and a single replica has f_s/τ_n bandwidth. Fig. 5 illustrates the spectrum of the up-sampled data symbol sequence denoted by $S_p(f)$ in the Nyquist and FTN signaling case. Note that N , the Nyquist signaling period in discrete-time domain, is equal to τ_n in the Nyquist signaling case.

The pulse shaping process, a convolution operation with the interpolated pulse $h[n]$, is then conducted to the up-sampled data symbol sequence. Since the process is a multiplying operation with the spectrum of $h[n]$ in the frequency domain, a truncation of the spectrum also occurs when the spectrum of $h[n]$ has narrower bandwidth than that of the up-sampled data symbol sequence. As shown in Fig. 5, the spectrum of the up-sampled sequence is truncated by the spectrum of the pulse shaping filter $H(f)$ where f_s/τ_n is less than the bandwidth of the spectrum of $h[n]$.

After the matched filtering and the down-sampling at rate of τ_n , the fundamental sampling frequency is changed from f_s to f_s/τ_n . Therefore, $S_y(f)$ which denotes the spectrum of the down-sampled data sequence $y[m\tau_n]$ can be illustrated by Fig. 6. In Nyquist signaling, the down-sampling rate is equal to the bandwidth of the pulse-shaped data symbol sequence, and an

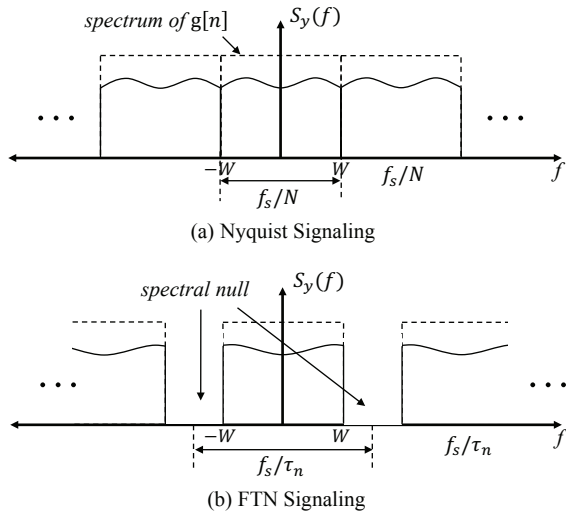


Fig. 6. The spectrum of the down-sampled sequence in the discrete-time a) Nyquist and b) FTN signaling

un-truncated and continuous spectrum can be obtained after the down-sampling. However, in FTN signaling, the down-sampling rate is greater than the width of the pulse-shaped data symbol sequence because of the spectral truncation. Hence, the spectrum of the down-sampled data sequence in FTN signaling can have discontinuities. In Fig. 6, spectral nulls are shown in FTN signaling case.

IV. OBSERVATIONS ON FTN SIGNALING FROM THE SPECTRAL ANALYSIS

A. Spectral Truncation and the Optimal Choice of τ

Rusek *et al.* have shown that there is an opportunity to regain the extra system capacity by using the FTN signaling when the transmission pulse $h(t)$ is not a sinc [4]. Furthermore, They have derived the optimal choice of τ in terms of achieving the highest possible system capacity. For a given non-sinc pulse $h(t)$ which is bandlimited to W ,

$$\tau = \frac{1}{2WT} \quad (12)$$

is the optimal value of τ to achieve the highest FTN system capacity [4], where T is the Nyquist signaling period in (1).

In the considered discrete-time model of the FTN signaling, the Nyquist signaling rate $1/T$ corresponds to f_s/N . Hence, the optimal choice of τ derived in [4] can be translated as follows:

$$\tau = \frac{\tau_n}{\tau_d} = \frac{f_s}{2NW} \iff \frac{f_s}{\tau_n} = 2W. \quad (13)$$

From the spectral analysis of discrete-time FTN signaling in the previous section, f_s/τ_n is the spectrum width of the up-sampled data sequence. Therefore, (13) implies that the spectrum width of the up-sampled data sequence is exactly equal to that of the transmission pulse.

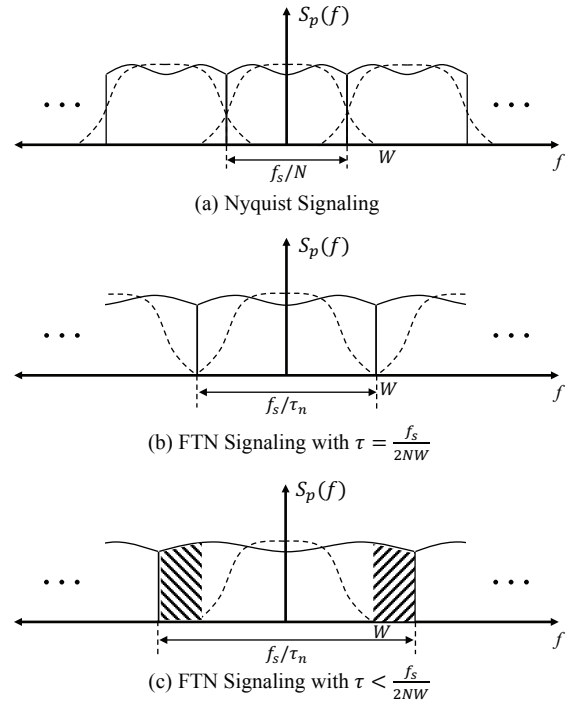


Fig. 7. The data spectrum $S_p(f)$ of the discrete-time FTN signaling according to τ

Fig. 7 illustrates the spectrum of the down-sampled data sequence under following conditions of τ : $\tau = 1$, $\tau = f_s/2NW$ and $\tau < f_s/2NW$. As shown in the figure, the data spectrum $S_p(f)$ becomes wider as τ decreases, but the spectrum of the transmission pulse remains the same. It can be seen that the spectrum is necessarily truncated under the condition of $\tau < f_s/2NW$, and $\tau = f_s/2NW$ is the minimum value of τ dodging the spectral truncation. Therefore, the maximum transmission rate without spectral truncation can be achieved when $\tau = f_s/2NW$. Although the truncation of the data spectrum does not necessarily imply the data loss [6], the highest system capacity cannot be achieved when the spectral truncation occurs. So it can be seen that there is a relation between the spectral truncation and the optimal choice of τ , and the optimal τ is the minimum value under the constraint of avoiding the spectral truncation.

B. Receiver Down-Sampling Rate to Avoid Aliasing

In the conventional receiver of the communication system, the receiver sampling rate is equal to the signaling rate. When the receiver sampling rate is greater than or equal to the signaling rate, the spectrum of the transmitted data can be recovered without aliasing. In the discrete-time system model, the receiver down-sampling rate should be greater than or equal to the up-sampling rate of the transmitter, assuming the fundamental sampling frequency f_s is constant. However in the FTN-based system, the condition can be relaxed.

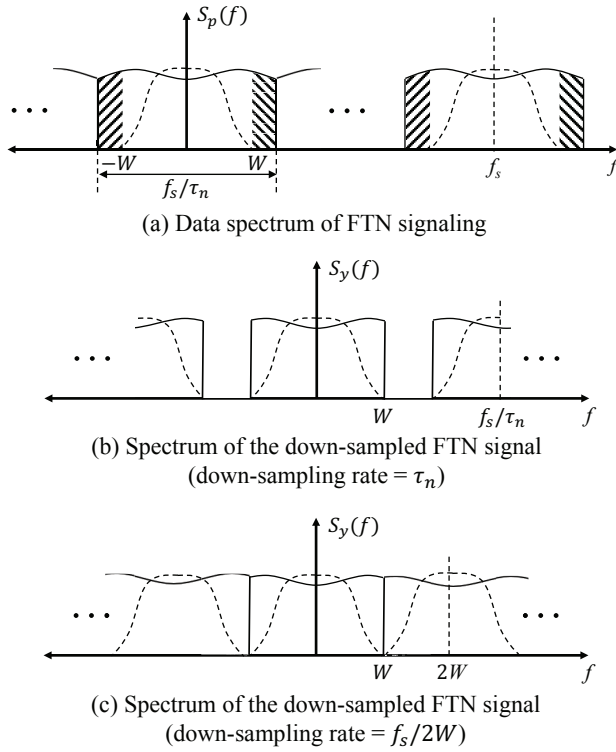


Fig. 8. The spectrums of a) the up-sampled data sequence and b), c) the down-sampled data sequence with 2 different down sampling rates: τ_n , $f_s/2W$

Note that the signaling rate of the discrete-time FTN signaling is f_s/τ_n , and the condition of the spectral truncations is

$$\tau < \frac{f_s}{2NW}. \quad (14)$$

If the condition (14) is satisfied, the spectrum of the up-sampled data sequence can be illustrated by Fig.8 (a). When the receiver down-sampling rate is equal to the up-sampling rate, i.e., the receiver sampling rate is equal to the signaling rate f_s/τ_n as in the conventional receiver, the spectrum of the down-sampled data sequence can be shown as Fig.8 (b). In this case, the bandwidth of the data spectrum becomes narrower due to the spectral truncation, and spectral nulls occur. So there is a chance to reduce the minimum down-sampling rate to avoid aliasing. In Fig.8 (c), the minimum down-sampling rate for the truncated spectrum is presented. As shown in the figure, although we reduce the receiver down-sampling rate to $f_s/2W$ not τ_n , the aliasing does not occur because the receiver sampling rate $2W$ satisfies the Nyquist sampling criterion. Therefore we can conclude that the receiver down-sampling rate can be reduced from the up-sampling rate of the transmitter, i.e., the receiver sampling rate could be smaller than the signaling rate in the FTN-based system when the spectral truncation occurs.

V. CONCLUSION

In this paper, we analyzed the PSD characteristics of the discrete-time FTN signaling and compared it with that of the continuous-time case. We interpreted the process of the discrete-time FTN signaling as a special case of multi-rate DSP, and found two interesting observations from the analysis. First we demonstrated a relation between the spectral truncation and the optimal choice of τ to achieve the highest system capacity. The optimal τ is the possible minimum value without causing the spectral truncation. Also we presented that FTN signaling can give opportunity to reduce the receiver down-sampling rate to avoid aliasing, when the spectral truncation occurs.

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