

An Improved Spectrum Sensing Algorithm Based on Random Matrix Theory

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Abstract—Spectrum sensing is an essential problem in cognitive radio. Blind detection techniques such as the algorithm based on random matrix theory which is shown to outperform energy detection especially in case of noise uncertainty, sense the presence of a primary user's signal without prior knowledge of the signal characteristics, channel and noise power. In this paper, we improve the maximum and minimum eigenvalue algorithm from two aspects. Using some recent random matrix theory results, a new threshold based on the distribution of minimum eigenvalue is introduced first. Then the signals received by each cognitive user are decomposed into I and Q components to ensure maximum exploitation of signal correlation among the temporal, spatial and phase correlation (between I and Q components) present in the received signals. Numerical simulations show that the proposed detection rule perform better than the traditional eigenvalue-based algorithm while also proving to be more robust.

Keywords—Cognitive radio; spectrum sensing; random matrix theory; distribution of minimum eigenvalue; I and Q components.

I. INTRODUCTION

Nowadays, due to the exponential growth of new wireless networks and services, the disadvantage of fixed spectrum allocation policy becomes more obvious. Besides different multiplexing modes, Cognitive Radio (CR) has emerged as a promising technology to solve the problem [1-2]. The first step of CR is spectrum sensing. Classical spectrum sensing algorithms are developed from the traditional signal detection methods, including Energy Detection(ED), Cyclostationary Feature Detection (CFD), and Matched Filtering (MF) [3]. ED is simple, but it relies on the accurate estimation of noise power, which means that its performance is affected by noise power uncertainty. The calculation of CFD is complexed and it is hard to be real-time. MF requires the prior knowledge of received signal, however, the prior knowledge of signal is unknown in practice.

Recently, with the situation of Multi-user and Multi-antenna, Random Matrix Theory (RMT) is applied on the areas of communication widely and one of the fields is spectrum sensing in cognitive radio. The main advantage of blind algorithms including ED and CFD is that they do not require any prior information on the primary signal or on the noise power, which means that they are more practical. Among blind algorithms, the spectrum sensing algorithm based on RMT was shown to outperform classical methods, especially in case of noise uncertainty [7]. Most of the algorithms based on RMT utilize the distribution of maximum and minimum eigenvalue of sample covariance matrix and construct the test statistic through their ratio or difference, including Difference between the Maximum and the Minimum Eigenvalue (DMM) [4], New Difference between the Maximum and the Minimum Eigenvalue (NDMM) [5], Leonardo S.Cardoso (LSC) [6], Maximum and Minimum Eigenvalue (MME) [7].

Recent research indicates that the description of the distribution of minimum eigenvalue is more accurate than that of maximum eigenvalue, especially when the number of Cognitive Users (CUs) or the number of samples is limited. And the paper [9] shows that the decomposition of signal acquires more information of correlation. Therefore this paper will improve the performance of MME algorithm from two aspects. Using some recent RMT results, the paper first introduces a new semi-asymptotic threshold based on the distribution of minimum eigenvalue. Then, decomposing the signal received into I (in-phase) and Q (quadrature) components is also introduced into the new algorithm. And then we obtain a novel spectrum sensing algorithm that outperforms the traditional eigenvalue-based detector.

The rest of the paper is organized as follows: Sec. II reviews the sensing model and the traditional thresholds. Sec. III deals with the threshold optimization problem and the decomposition of signal. Simulation results are presented and discussed in Sec. IV. Sec. V contains the conclusions.

II. SENSING MODEL

A. System Model

Denote with K the number of CUs and with N the number of samples sampled by each CU in one sensing period. There are two hypotheses: under H_0 , signal does not exist; under H_1 , signal exists. The received signal samples under two hypotheses show therefore respectively as follows:

$$x_i(n) = \begin{cases} w_i(n) & H_0 \\ s_i(n) + w_i(n) & H_1 \end{cases} \quad i = 1, 2, \dots, K \quad (1)$$

where $s_i(n)$ is the Primary User (PU) signal and $w_i(n)$ is the white Gaussian noise with zero mean and variance σ^2 . Let $X_i = [x_i(1) \ x_i(2) \ \dots \ x_i(N)]$ be a $1 \times N$ vector containing N samples collected by the i^{th} CU and the sensing matrix constructed by the samples received in one sensing period is written as:

$$X = \begin{bmatrix} x_1(1) & x_1(2) & \dots & x_1(N) \\ x_2(1) & x_2(2) & \dots & x_2(N) \\ \dots & \dots & \ddots & \dots \\ x_K(1) & x_K(2) & \dots & x_K(N) \end{bmatrix} \quad (2)$$

The sample covariance matrix is $R(N) = \frac{1}{N} X X^T$, where

$\lambda_{\max}$ and $\lambda_{\min}$ are the maximum eigenvalue and minimum eigenvalue of $R(N)$ respectively.

B. Previous Work

The normalized sample covariance matrix is $R'(N) = \frac{N}{\sigma_n^2} R(N)$ whose maximum and minimum eigenvalue are $l_{\max}$ and $l_{\min}$. Under H_0 , $R'(N)$ will be a Wishart matrix [6] and its eigenvalues converge to a limited value according to the Marcenko-Pastur law. However, $R'(N)$ under H_1 is not a Wishart matrix anymore. The test statistic $T = l_{\max} / l_{\min} = \lambda_{\max} / \lambda_{\min}$ can be constructed by utilizing this difference and denote as γ the decision threshold. The detector decides for H_0 if $T < \gamma$, for H_1 otherwise. There are three approaches to set γ .

1) Asymptotic Approach[6]

Under H_0 , due to the asymptotical properties of Wishart matrices, it is known that in the limit of $\lim_{N \rightarrow \infty} \frac{K}{N} = c \in (0, 1)$ maximum eigenvalue and minimum eigenvalue of $R'(N)$ converge to:

$$\begin{aligned} l_{\min} &\rightarrow a = (N^{1/2} - K^{1/2})^2 \\ l_{\max} &\rightarrow b = (N^{1/2} + K^{1/2})^2 \end{aligned} \quad (3)$$

However, $l_{\max} \rightarrow b' > b$ under H_1 and the asymptotic threshold is $\gamma_{as} = \frac{b}{a}$.

2) Semi-asymptotic Approach[7]

Because the asymptotic threshold is a fixed value and it cannot achieve Constant False Alarm Rate (CFAR) detection. Although the distribution of $l_{\max}$ is unknown under H_0 , its linear transformation $L_{\max} = \frac{l_{\max} - b}{v}$ converges in distribution to the Tracy-Widom law with $v = (N^{1/2} + K^{1/2})(N^{1/2} - K^{1/2})^{1/3}$.

And the false alarm probability can be written as:

$$\begin{aligned} P_{fa} &= P\left(\frac{l_{\max}}{l_{\min}} > \gamma | H_0\right) \\ &= P(l_{\max} > \gamma l_{\min} | H_0) \\ &= P(l_{\max} > \gamma (N^{1/2} - K^{1/2})^2) \\ &= P\left(\frac{l_{\max} - b}{v} > \frac{\gamma (N^{1/2} - K^{1/2})^2 - b}{v}\right) \\ &= 1 - F_{TW}\left(\frac{\gamma (N^{1/2} - K^{1/2})^2 - b}{v}\right) \end{aligned} \quad (4)$$

Then semi-asymptotic threshold is:

$$\gamma_{sa} = \left(\frac{v F_{TW}^{-1}(1 - P_{fa}) + b}{(N^{1/2} - K^{1/2})^2} \right) \quad (5)$$

where F_{TW}^{-1} is the inverse Tracy-Widom Cumulative Distribution Function (CDF) which can be stored in a look-up table since it does not have a closed-form expression.

3) Non-asymptotic Approach[10]

The semi-asymptotic approach uses the asymptotical limit for the smallest eigenvalue and the Tracy-Widom CDF for the largest one. Recent research shows that $L_{\min} = \frac{l_{\min} - a}{\mu}$ converges in distribution to the Tracy-Widom law with $\mu = (K^{1/2} - N^{1/2})(K^{1/2} + N^{1/2})^{1/3}$ under H_0 . Paper [10] uses the Tracy-Widom CDF both for the smallest and the largest eigenvalue and the non-asymptotic threshold is written as:

$$\gamma_{rd} = F_T^{-1}(1 - P_{fa}) \quad (6)$$

where F_T^{-1} is the CDF of

$$f_T = \left[\int_0^{+\infty} x f_{l_{\max}}(tx) f_{l_{\min}}(x) dx \right] \cdot I_{\{t>1\}} \quad (7)$$

with

$$\begin{aligned} f_{l_{\max}}(x) &= \frac{1}{v} f_{TW}\left(\frac{x-b}{v}\right), f_{l_{\min}}(x) \\ &= \frac{1}{|\mu|} f_{TW}\left(\frac{a-x}{\mu}\right) = \frac{1}{\mu} f_{TW}\left(\frac{x-a}{\mu}\right) \end{aligned} \quad (8)$$

However it is quite difficult to calculate the integration of Tracy-Widom Probability Density Function (PDF).

III. NEW THRESHOLD AND DECOMPOSITION OF SIGNAL

Among the thresholds existed, the asymptotic threshold cannot achieve CFAR detection. It is hard to calculate the non-asymptotic threshold because f_{TW} is unknown and the semi-asymptotic threshold can achieve CFAR detection. Recent research [8] shows that the description of the distribution of minimum eigenvalue is more accurate than that of maximum one, especially in low dimensions. This paper uses the asymptotical limit for the largest eigenvalue and the Tracy-Widom CDF for the smallest one. Recalling that $\mu < 0$ and the false alarm probability can be shown as:

$$\begin{aligned}
 P_{fa} &= P\left(\frac{l_{\max}}{l_{\min}} > \gamma | H_0\right) \\
 &= P\left(\frac{l_{\max}}{\gamma} > l_{\min} | H_0\right) \\
 &= P\left(l_{\min} < \frac{(N^{\frac{1}{2}} + K^{\frac{1}{2}})^2}{\gamma}\right) \\
 &= P\left(\frac{l_{\min} - a}{\mu} > \frac{\frac{(N^{\frac{1}{2}} + K^{\frac{1}{2}})^2}{\gamma} - a}{\mu}\right) \\
 &= 1 - F_{TW}\left(\frac{\frac{(N^{\frac{1}{2}} + K^{\frac{1}{2}})^2}{\gamma} - a}{\mu}\right)
 \end{aligned} \quad (9)$$

And the new semi-asymptotic threshold is:

$$\gamma_{nsa} = \frac{(N^{\frac{1}{2}} + K^{\frac{1}{2}})^2}{\mu F_{TW}^{-1}(1 - P_{fa}) + a} \quad (10)$$

The literature [9] points out that the decomposition of signal can acquire more information of correlation and increase the number of logical signal. Let $X_i = [x_i(1) \ x_i(2) \ \dots \ x_i(N)]$ be a $1 \times N$ vector containing N samples collected by the i^{th} CU. X_i is decomposed into I and Q components and they are expressed as:

$$\begin{aligned}
 X_i^I &= X_i * \sin\left(\frac{2\pi f_c n}{f_s}\right) \quad n = 1, 2, \dots, N \\
 X_i^Q &= X_i * \cos\left(\frac{2\pi f_c n}{f_s}\right) \quad n = 1, 2, \dots, N
 \end{aligned} \quad (11)$$

where f_c is the carrier frequency and f_s is the sampling frequency. The new $M \times N$ dimension sensing matrix can be constructed by recombining $M=2K$ signal vectors:

$$Y = \begin{bmatrix} X_1^I \\ X_1^Q \\ \vdots \\ X_K^I \\ X_K^Q \end{bmatrix} = \begin{bmatrix} x_1^I(1) & x_1^I(2) & \dots & x_1^I(N) \\ x_1^Q(1) & x_1^Q(2) & \dots & x_1^Q(N) \\ \vdots & \vdots & \ddots & \vdots \\ x_K^I(1) & x_K^I(2) & \dots & x_K^I(N) \\ x_K^Q(1) & x_K^Q(2) & \dots & x_K^Q(N) \end{bmatrix} \quad (12)$$

The new sample covariance matrix is $R_y(N) = \frac{1}{N} Y Y^T$.

Compared with the classical sample covariance matrix $R(N)$, the new sample covariance matrix $R_y(N)$ ensures maximum exploitation of signal correlation among the temporal, spatial or phase correlation (between I and Q components) present in the received signal. And then we have the improved MME detector:

1) Decompose the signal received into I and Q components and compute $R_y(N)$;

2) Compute maximum and minimum eigenvalues $\lambda_{y_{\max}}$ and $\lambda_{y_{\min}}$;

3) Compute the test statistic $T = \frac{\lambda_{y_{\max}}}{\lambda_{y_{\min}}}$ and H_1 is true if $T > \gamma$, where γ is the new semi-asymptotic threshold determined by desired P_f .

It should be noted that in the calculation of maximum and minimum eigenvalues, the time complexity $O(L^3)$ is related to L , and L is the number of dimension of the covariance matrix. Therefore, compared with the traditional MME algorithm, the new method is unavoidable to introduce higher computational overhead, almost eight times.

IV. SIMULATION RESULT

In this section, simulation results will be shown to evaluate the performance of the proposed algorithm. The PU's signal is Amplitude Modulation (AM) signal whose carrier frequency is 702 KHz and the sample ratio is 4MHz. There are 10 CUs in the cognitive network and 100 samples for each CU. The signal-to-noise ratio (SNR) is -10dB.

A. Comparison of ROC with Different Thresholds

Fig.1 provides a performance comparison of eigenvalue-based detectors with asymptotic threshold, semi-asymptotic threshold and new semi-asymptotic threshold respectively. Except asymptotic threshold, the detection probability of both semi-asymptotic threshold and new semi-asymptotic threshold varies with the false alarm probability, which shows they can achieve CFAR detection. And because the new semi-asymptotic threshold uses the distribution of minimum eigenvalue instead of the distribution of maximum eigenvalue, the new semi-asymptotic threshold is more accurate than the traditional one and that is why its detection probability is slightly higher.

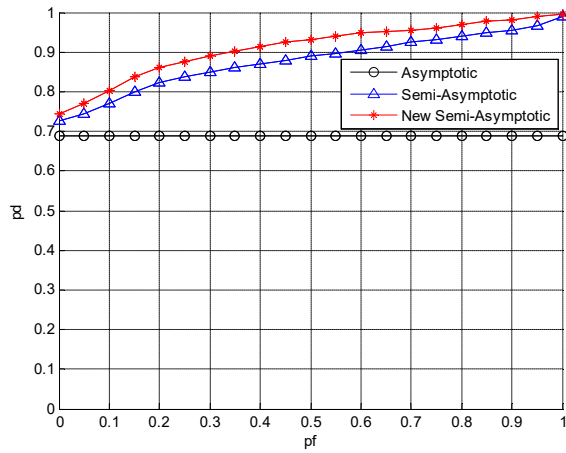
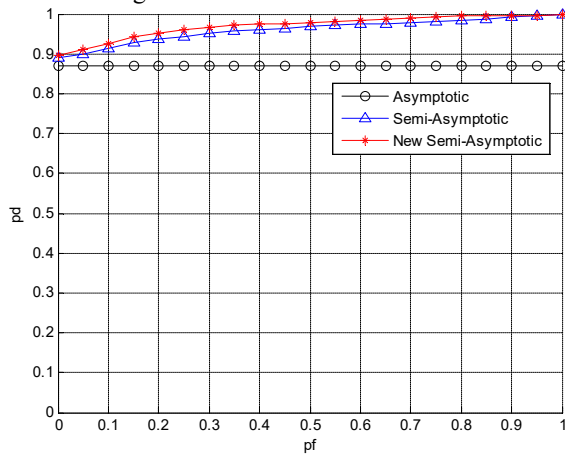
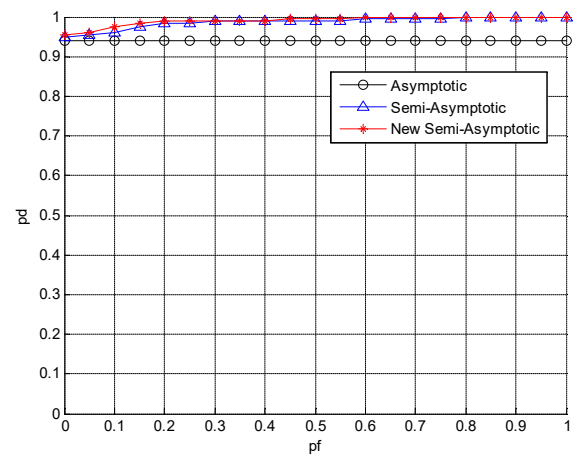
Figure 1. Comparison of ROC with different thresholds when $N = 100$.

Fig.2 and Fig.3 represent a performance comparison of eigenvalue-based detectors with different thresholds while sampling number changes. When N increases to 140, the performances are all improved. Due to the increase of N , the matrix starts to satisfy the asymptotical condition

$$\lim_{N \rightarrow \infty} \frac{K}{N} = c \in (0,1) .$$

And the detection probability of traditional semi-asymptotic threshold approximates to that of new semi-asymptotic threshold. However, the performance of new semi-asymptotic threshold is still slightly better than that of traditional one. The ROC curves will eventually coincide until N grows to some extent.

Figure 2. Comparison of ROC with different thresholds when $N = 140$ Figure 3. Comparison of ROC with different thresholds when $N = 200$

B. Comparison of ROC of Decomposing Signal

Fig.4 shows the performance of eigenvalue-based detectors without decomposing signal and compares it to the method with decomposing signal received, when the number of CUs in the cognitive network is 10. It is shown that the performance with decomposing signal improves significantly. It is because that the new sample covariance matrix constructed by decomposing signal into I and Q components contains more information of correlation, including the temporal, spatial and phase correlation (between I and Q components). In fact, the number of CUs is always limited and it is necessary to increase the number of logical signals, which means this measure conforms to the real situation.

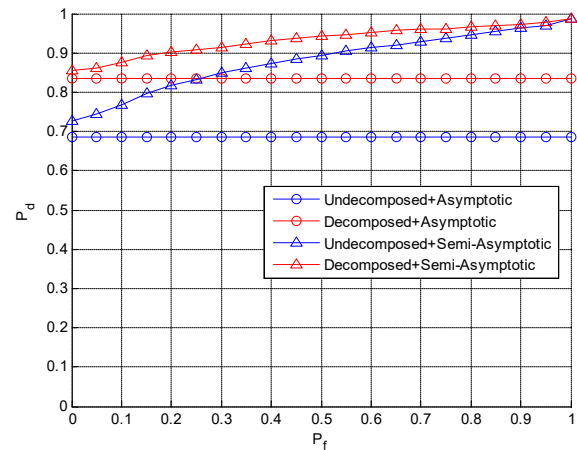
Figure 4. Comparison of ROC of decomposing signal when $K=10$

Fig.5 shows a performance comparison when $K=14$. Obviously, the larger K is, the better performance is. With the same number of sample, the probability of detection increases when K is larger. In addition, as K increases, the speed of increment in detection probability gets lower.

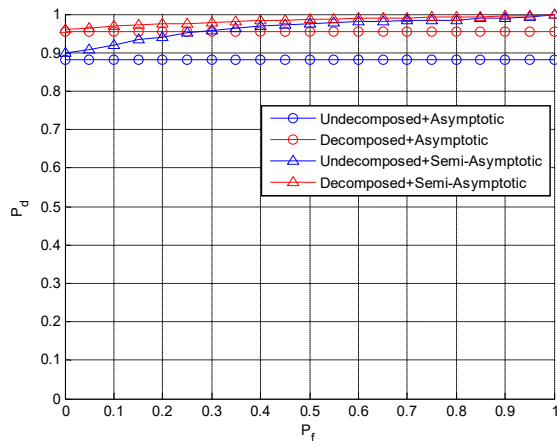


Figure 5. Comparison of ROC of decomposing signal when K=14

C. Comparison of ROC of Traditional Method and New One

The combination of the new semi-asymptotic threshold and the decomposition of signal contributes to the new spectrum sensing algorithm. Compared with the traditional MME detector with the traditional semi-asymptotic threshold without decomposing signals, Fig.6 represents that the new method has a better performance. It is not only because the decomposition of signal bring more information of correlation, but also because the new semi-asymptotic threshold is more accurate.

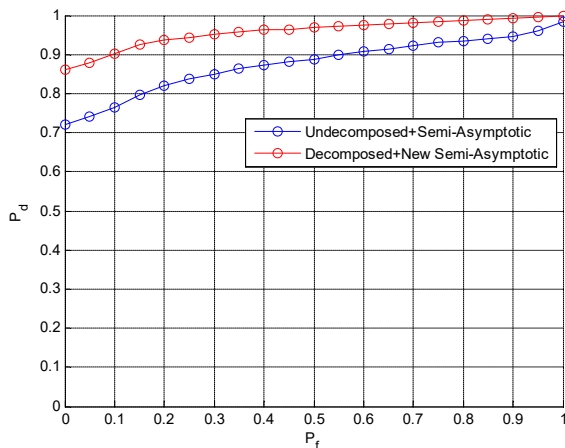


Figure 6. Comparison of ROC of traditional method and new one

In Fig.7, we plot the probability of detection against the SNR based on the application of MME algorithm and the improved one. The performance of improved MME is better than that of the traditional one.

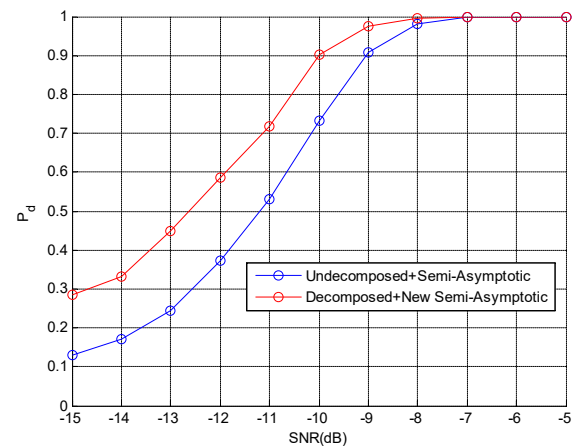


Figure 7. Probability of detection vs signal-to-noise ratio.

V. CONCLUSION

The paper improves the spectrum sensing method based on RMT. On the one hand, the paper introduces a new semi-asymptotic threshold based on the distribution of minimum eigenvalue. Compared with the traditional semi-asymptotic threshold, it is more accurate. On the other hand, the decomposition of signal increase the number of logic signal, which can acquire more information of correlation. Simulation results show that it improves the performance of algorithm obviously. The combination of the new semi-asymptotic threshold and the decomposition of signals contributes to the new spectrum sensing algorithm which has a better performance than the traditional MME algorithm. However, the proposed method increases the dimension of the covariance matrix and its computational complexity is also higher. Besides the eigenvalues, the eigenvectors also can be utilized to construct the spectrum sensing algorithms in the future.

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