

A Cooperative Spectrum Sensing Algorithm Based on Leading Eigenvector Matching

Yuhui SONG *, Yigang ZHOU **

*School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, China

** School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, China

syhhit@yeah.net, zhoyug@yeah.net

Abstract—Spectrum sensing is an essential problem in cognitive radio. Blind detection techniques such as the algorithm based on random matrix theory which is shown to outperform energy detection especially in case of noise uncertainty, sense the presence of a primary user's signal without prior knowledge of the signal characteristics, channel and noise power. In this paper, we propose a cooperative spectrum sensing method based on the Leading Eigenvector Matching (LEM). LEM detector uses the feature blindly learned from Feature Learning Algorithm (FLA) as prior knowledge. The correlation coefficient between feature learned and leading eigenvector of sample covariance matrix serves as the test statistic for signal detection. The closed-form expression of the threshold is also derived in this paper. Numerical simulations show that the proposed detection algorithm performs better than the MME algorithm while also proving to be more robust and it does not suffer from a noise power uncertainty problem.

Keywords— Cognitive Radio; spectrum sensing; covariance matrix; leading eigenvector; feature learning.

I. INTRODUCTION

Cognitive Radio (CR) is different from traditional radio systems because Cognitive User (CU) could sense the environment and utilize the vacant spectrum accordingly [1]. Spectrum sensing is an essential problem in CR which can detect the Primary User's (PU) signal presence [2]. It is simple to detect signal when the signal to noise (SNR) is high, but in practice sensing the presence of PU's signal becomes demanding because of the low SNR and shadow fading. Spectrum sensing algorithms existed can be divided into non-blind detector and blind detector according to whether it requires prior knowledge about the signal and the channel characteristics. Non-blind techniques (such as matched filter detection and cyclostationary feature detection [3]) that rely on prior knowledge give a better performance, but it is difficult to acquire prior knowledge in practice. On the other hand, blind sensing (e.g. energy detection) that do not require prior knowledge is flexible in their application. Compared with energy detection, the spectrum sensing algorithms based on Random Matrix Theory (RMT) were shown to outperform classical methods as a blind detector, especially in case of noise uncertainty. Most of the algorithms based on RMT utilize the differences between the distributions of eigenvalues

of sample covariance matrix under H_0 and H_1 , including Maximum and Minimum Eigenvalue (MME) [4], Energy with Minimum Eigenvalue (EME) [5], Maximum Eigenvalue Detection (MED) [6]. Besides eigenvalue, eigenvector is another characteristics of the covariance matrix. Feature Template Matching (FTM) [7] has been proposed as a single-node spectrum sensing technique which is based on the leading eigenvector and shown that it performs better than MME and the CAV (Covariance Absolute Value) algorithms. Multiple Feature Matching (MFM) [8] algorithm applied the FTM algorithm in MIMO system. But the methods above did not derive the closed-form expression of the threshold and were limited in single-node spectrum sensing.

In this paper, we propose a cooperative spectrum sensing method based on the Leading Eigenvector Matching (LEM). LEM detector uses the feature blindly learned from Feature Learning Algorithm (FLA) as prior knowledge. The correlation coefficient between feature learned and leading eigenvector of sample covariance matrix serves as the test statistic for signal detection. The closed-form expression of the threshold is also derived. Simulation shows that its performance is better than MME and it does not suffer from a noise power uncertainty problem.

The rest of the paper is organized as follows: Sec. II reviews the sensing model and basic theory of the proposed algorithm. Sec. III deals with the description of LEM and the threshold derivation problem. Simulation results are presented and discussed in Sec. IV. Sec. V contains the conclusions.

II. SYSTEM MODEL AND BASIC THEORY

A. System Model

Denote with N the number of CUs. There are two hypotheses: under H_0 , signal does not exist; under H_1 , signal exists. The received signal samples under two hypotheses show therefore respectively as follows:

$$x_i(n) = \begin{cases} w_i(n) & H_0 \\ s_i(n) + w_i(n) & H_1 \end{cases} \quad i = 1, 2, \dots, N \quad (1)$$

where $s_i(n)$ is the PU's signal and $w_i(n)$ is the white Gaussian noise with zero mean and variance σ^2 . Let $X_i(n) = [x_i(n) \ x_i(n+1) \ \dots \ x_i(n+S-1)]$ be a $1 \times S$ vector

containing S consecutive samples collected by the i^{th} CU and the j^{th} sensing segment constructed by the samples received is written as:

$$X_j = \begin{bmatrix} X_1(j) \\ X_2(j) \\ \vdots \\ X_N(j) \end{bmatrix} = \begin{bmatrix} x_1(j) & x_1(j+1) & \dots & x_1(j+S-1) \\ x_2(j) & x_2(j+1) & \dots & x_2(j+S-1) \\ \vdots & \vdots & \ddots & \vdots \\ x_N(j) & x_N(j+1) & \dots & x_N(j+S-1) \end{bmatrix} \quad (2)$$

The sample covariance matrix is $R_j = \frac{1}{S} X_j X_j^T$ and its leading eigenvector is φ_j .

B. Basic Theory

Denote with x_s a 2×1 vector containing AM signal and x_n a 2×1 vector containing White Gaussian Noise (WGN). Let x_{s+n} be 2×1 vector containing AM signal with WGN, which means $x_{s+n} = x_s + x_n$. The elements of the vector are x_1 and x_2 respectively. Fig.1 shows that the distribution of WGN is random, while x_s has the same characteristic angle with x_{s+n} , which means that their leading eigenvector is similar. The algorithm proposed uses the different characteristics to distinguish signal and noise. The similar situation in three-dimension is shown in Fig.2.

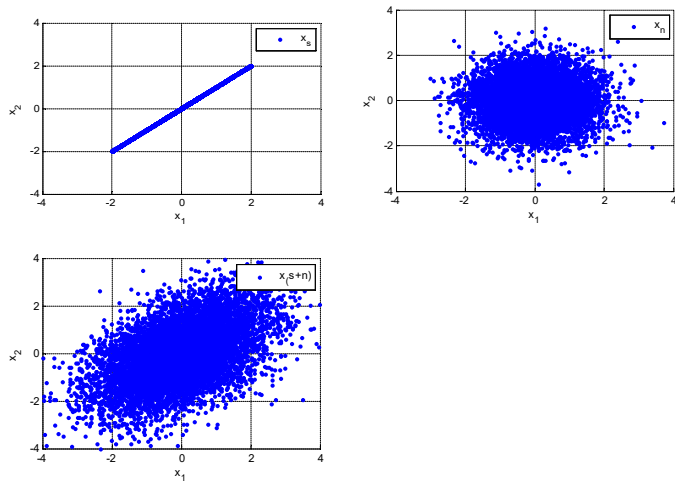


Figure 1. The distribution under two-dimension

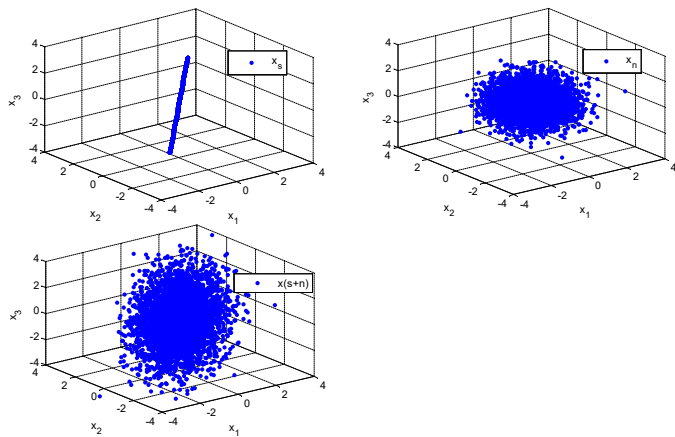


Figure 2. The distribution under three-dimension

According to the definition of the eigenvalues and eigenvectors, the relation among sample covariance matrix R , its maximum eigenvalue λ and leading eigenvector I can be written as the equation

$$RI = \lambda I \quad (3)$$

While noise exists, it can be expressed as

$$(R + \sigma^2 E)I = (\lambda + \sigma^2)I \quad (4)$$

It is shown that the leading eigenvector I of matrix R is also the leading eigenvector of $R + \sigma^2 E$. Therefore, I remains stable regardless of the change of noise variance, which is most robust against noise.

III. DETECTION ALGORITHM

A. Leading Eigenvector Matching Algorithm

Leading eigenvector is called signal feature in pattern recognition and it has the greatest mutual information with original signal. Compared with the randomness of leading eigenvector of WGN, the leading eigenvector of WGN is more stable. If PU's signal exists, highly correlated leading eigenvector can be detected in consecutive sensing segments X_j and X_{j+1} . Due to the robustness of signal feature, it can be learned by blind FLA.

The feature φ_s can be learned blindly from J sensing segments by following steps:

- 1) Extract feature φ_j and φ_{j+1} from X_j and X_{j+1} ;
- 2) Compute correlation coefficient via cosine similarity formula:

$$\rho_{j,j+1} = \frac{\langle \varphi_j, \varphi_{j+1} \rangle}{\|\varphi_j\| \|\varphi_{j+1}\|} = \left| \varphi_j^T \varphi_{j+1} \right| \quad (5)$$

- 3) $J-1$ correlation coefficients can be calculated from J sensing segments. If $\rho_{m,m+1} = \max_{j=1,2,\dots,J-1} \{\rho_{j,j+1}\}$, signal feature φ_s is learned as φ_{m+1} .

With the prior knowledge φ_s , we have LEM detector:

- 1) Compute the received signal sample covariance matrix $X_{current}$ and corresponding leading eigenvector $\varphi_{current}$;
- 2) Compute correlation coefficient $\rho_{s,current}$ between φ_s and $\varphi_{current}$;
- 3) H_1 is true if $\rho_{s,current} > \varepsilon$, where ε is the threshold determined by desired P_f .

Compared with MME detector, both of the algorithms need to solve eigenvector and eigenvalue problem and their time complexity is almost same. But the LEM detector need to learn the feature φ_s by FLA which requires extra computation and time. In practical applications, the feature can be learned ahead and stored in local CU's memory.

B. Threshold

It is necessary to derive the expression of the false alarm probability in CR. On the one hand, it is an important parameter to evaluate the detector's performance. On the other hand, the decision threshold is set according to it. The probability of false alarm is defined as:

$$P_f = p(|\varphi_s^T \varphi_{current}| > \varepsilon | H_0) \quad (6)$$

Under H_0 , we have the following result:

$$R_s, R_{current} \sim \frac{1}{S} \text{wishart}(S, \sigma^2 I) \quad (7)$$

where $\text{wishart}(\bullet)$ is Wishart distribution. Let φ_s and $\varphi_{current}$ be the leading eigenvectors of R_s and $R_{current}$, and the normalized covariance matrices are defined as:

$$C_s = \frac{1}{\sigma^2} R_s, C_{current} = \frac{1}{\sigma^2} R_{current} \quad (8)$$

$$C_s, C_{current} \sim \frac{1}{S} \text{wishart}(S, I)$$

Let A and B be the matrices containing the normalized eigenvectors of C_s and $C_{current}$ respectively. We have $A = [a_1, a_2 \dots a_N]$, $B = [b_1, b_2 \dots b_N]$ where the eigenvectors are arranged in descending order. And the matrices A^T and B^T converge in distribution to Haar [9]. It is known that A , B and B^T are unitary matrices and we have following result:

$$f(A^T B) = f(A^T (B^T)^T) = f(A^T) \quad (9)$$

where $f(\bullet)$ is the Probability Density Function(PDF). Because $A^T B$ and A^T converge in same distribution, the elements of the matrices also converge in same distribution and we have following result:

$$f(|a_1^T b_1|) = f(|a_{11}|) \quad (10)$$

where a_{11} is the element at the first row and the first column in matrix A .

According to the property of unitary matrix, it can be known that a_{11}^2 converges in distribution to Beta with parameters $\alpha = \frac{1}{2}$ and $\beta = \frac{N-1}{2}$. The PDF of

$T(n) = |a_1^T b_1| = |\varphi_s^T \varphi_{current}|$ can be written as:

$$f(T|N) = \frac{\Gamma(\frac{N}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{N-1}{2})} \frac{(1-T^2)^{(N-3)/2}}{T} \quad (11)$$

And we have the expression of false-alarm probability as follows:

$$P_f = p(|\varphi_s^T \varphi_{current}| > \varepsilon | H_0) \quad (12)$$

$$= \frac{\Gamma(\frac{N}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{N-1}{2})} \int_{\varepsilon}^1 \frac{(1-T^2)^{(N-3)/2}}{T} dT$$

Hence, we derive the decision threshold as a function of the false-alarm probability:

$$\varepsilon = \sqrt{F_{Beta}^{-1}(1-P_f, \frac{1}{2}, \frac{N-1}{2})} \quad (13)$$

where $F_{Beta}^{-1}(\cdot)$ is the inverse cumulative distribution function of Beta distribution. It is shown that the threshold is a function of the target P_f and the dimension of the covariance matrix N . So the algorithm proposed in this paper is a blind sensing algorithm that do not require prior knowledge about the signal and the channel characteristics.

IV. SIMULATION

In this section, simulation results will be shown to evaluate the performance of the proposed algorithm. The PU's signal is Amplitude Modulation (AM) signal whose carrier frequency is 702 KHz and the sample ratio is 4MHz. The numbers of CUs and samples are 32 and 1000 respectively, and the signal-to-noise ratio (SNR) is -20dB.

A. Comparison of ROC under noise uncertainty

Compared with MME algorithm, Fig.3 shows that LEM has a better performance. Generally speaking, the signal detection techniques need to achieve Constant False Alarm Rate (CFAR) detection and the required false-alarm probability of spectrum sensing in CR is under 10%. As seen in the figure, when the false-alarm probability is 5%, the detection probability of MME and LEM is 65% and 95% respectively. It also shows that the impact of noise uncertainty on LEM is slight, which means LEM does not suffer from a noise uncertainty problem. However, the detection probability of MME under noise uncertainty improves a little and it will be explained later.

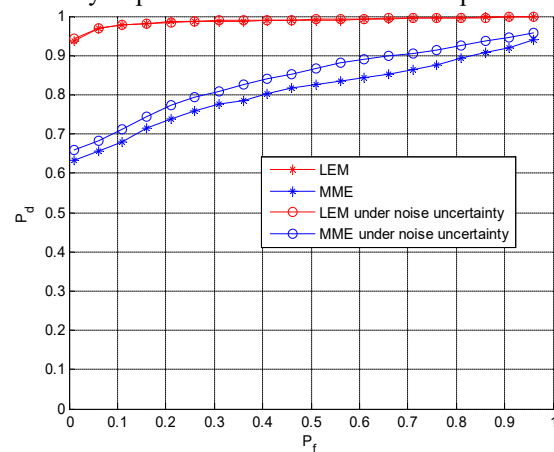


Figure 3. ROC under noise uncertainty for MME and LEM

The abscissa coordinate of Fig.4 is the theoretical false-alarm probability and the vertical coordinate is the empirical false-alarm probability. It shows that the impact of noise uncertainty on LEM is slight and the false-alarm probability of MME increases a little. That is why the detection probability of MME under noise uncertainty improves without practical significance. Fig.4 also shows that the false-alarm probability curve of LEM is roughly diagonal, that is, the theoretical false-alarm probability is approximately equal to the theoretical false-alarm probability, which proves the correctness of the decision threshold derivation.

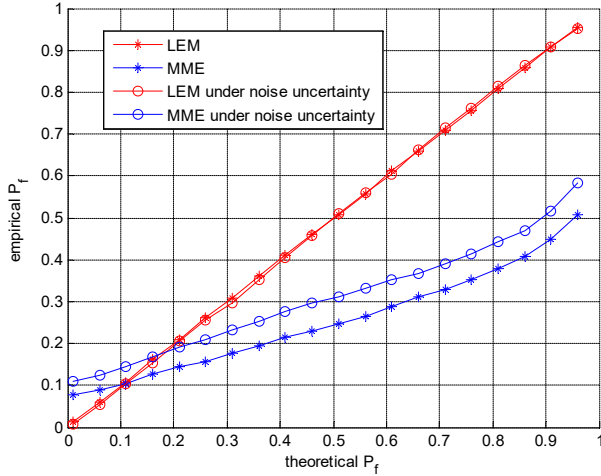


Figure 4. Empirical P_f vs. theoretical P_f for MME and LEM

B. Comparison of performance under different SNR

Fig.5 represents that the detection probability of MME decreases sharply with the decrease of SNR and the detection probability of LEM decreases slowly. When the SNR is -20dB, the detection probability of MME and LEM is 65% and 95% respectively. In Fig.6, we investigate the probability of false alarm versus SNR. The false-alarm probability of LEM reaches 5% and fluctuates slightly around it. The false-alarm probability of MME is slightly higher than LEM algorithm, this is because the threshold derivation of LEM is more accurate.

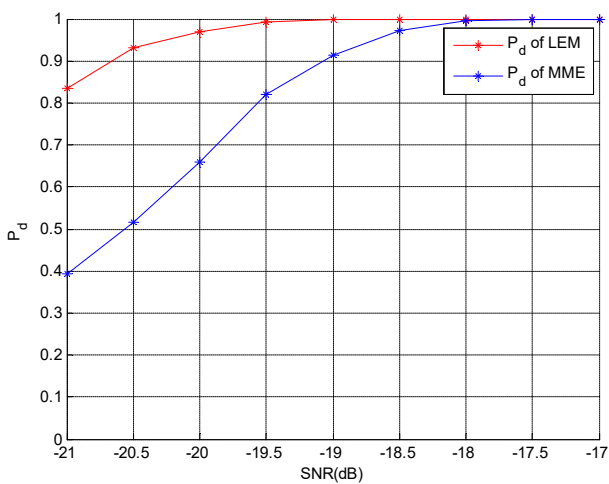


Figure 5. P_d vs. SNR at $P_f = 5\%$ for MME and LEM

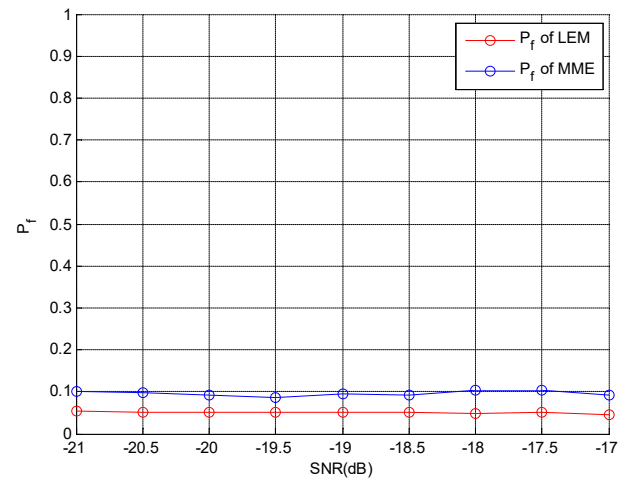


Figure 6. P_f vs. SNR at $P_f = 5\%$ for MME and LEM

C. The impact of the parameter N and S

Fig.7 and Fig.8 show the ROC curves of LEM detector with various numbers of CUs N and different sample sizes S . It is obvious that the performance rises when N and S grow. The results show that the sample size S and the number of cooperative CUs N play the same role in detection performance. In addition, as S increases, the speed of increment in detection probability gets lower.

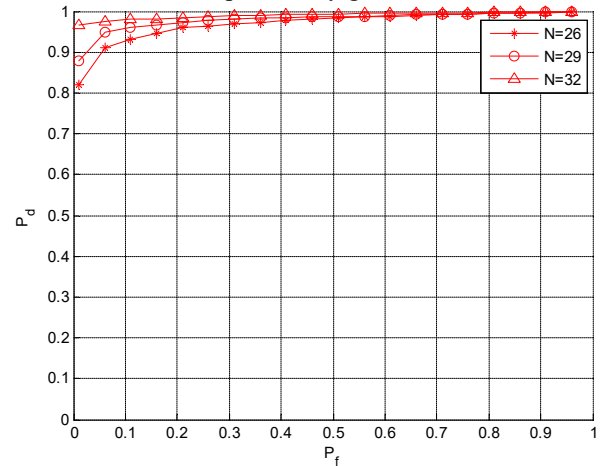


Figure 7. ROC for various numbers of CUs N

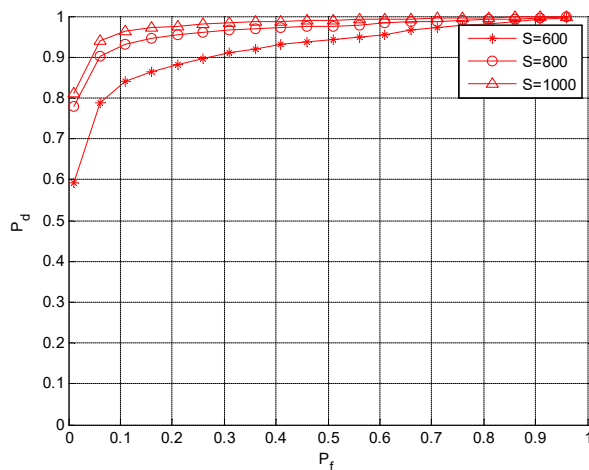


Figure 8. ROC for different sample sizes S

V. CONCLUSIONS

In this paper, we propose a cooperative spectrum sensing method based on the leading eigenvector matching. LEM detector uses the feature blindly learned from FLA as prior knowledge. The correlation coefficient between feature learned and the leading eigenvector of sample covariance matrix serves as the test statistic for signal detection. The closed-form expression of the threshold is also derived in this paper. Simulation shows that the performance of LEM is better than that of MME and it does not suffer from a noise power uncertainty problem. However there are some inherent flaws in this approach. A feature can only be learned in the presence of the desired PU signal, it cannot be learned in the presence of noise or in the presence of any other signal.

REFERENCES

- [1] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," in *IEEE Personal Communications*, vol. 6, no. 4, pp. 13-18, Aug 1999.
- [2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," in *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201-220, Feb. 2005.
- [3] D. Cabric, S. M. Mishra and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," *Signals, Systems and Computers*, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on, 2004, pp. 772-776 Vol.1.
- [4] F. Penna, R. Garello and M. A. Spirito, "Cooperative spectrum sensing based on the limiting eigenvalue ratio distribution in wishart matrices," in *IEEE Communications Letters*, vol. 13, no. 7, pp. 507-509, July 2009.
- [5] Y. Zeng and Y. C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE Transactions on Communications*, vol. 57, no. 6, pp. 1784-1793, 2009.
- [6] Y. C. Liang, G. Pan, and Y. Zeng, "On the performance of spectrum sensing algorithms using multiple antennas," in *IEEE Global Telecommunications Conference (GLOBECOM)*, 2010, pp. 1-5.
- [7] P. Zhang, R. Qiu and N. Guo, "Demonstration of Spectrum Sensing with Blindly Learned Features," in *IEEE Communications Letters*, vol. 15, no. 5, pp. 548-550, May 2011.
- [8] F. A. Bhatti, G. B. Rowe and K. W. Sowerby, "Spectrum sensing using feature vectors," *Communication Systems (ICCS)*, 2012 IEEE International Conference on, Singapore, 2012, pp. 448-452.

- [9] Li Yingxue, Lei Jing, Zhong Shiyuan, Huangchunming, Huangchao. Covariance Blind Detection Method Based on Eigenvector in Cognitive Radio Networks [J]. *Telecommunications Science*, 2015, 31(11): 2015306.



Yuhui SONG received the B.S. degree in electronic and information engineering from Harbin Engineering University, Harbin, China, in 2011. He is currently pursuing the M.S. degree in Harbin Institute of Technology, Harbin, China. His research interests include spectrum sensing in cognitive radio



Yigang ZHOU received the B.S. degree in electronic engineering from Tianjin University, Tianjin, China, in 1985 and the M.S. degree in Harbin Institute of Technology, Harbin, China, in 1988. He is currently an Associate Professor with the Department of Information Engineering, Harbin Institute of Technology. His current research interests include acquisition and storage of the data in high speed and information security.