

Bit Error Rate Expression of MC-CDMA System in α - μ Fading Channel

Chirawat Kotchasarn

Telecommunication Program, Faculty of Engineering, Rajamangala University of Technology Thanyaburi
39 Moo 1 Rangsit-Nakornnayok Rd. Klong 6, Thanyaburi, Pathumthani, 12110, THAILAND

Chirawat.k@en.rmutt.ac.th

Abstract—Multicarrier code division multiple access (MC-CDMA) is a technique that combines orthogonal frequency division multiplexing (OFDM) and CDMA. The performance analysis of MC-CDMA system with correlated fading channels with α - μ distribution is investigated in this paper. We employ the binary phase shift keying (BPSK) modulation scheme and the maximal ratio combining (MRC) technique at the receiver. By using the definition of Q function expression together with the moment generating function (MGF) method, a simple form of the bit error rate (BER) performance is derived. The results show that the more subcarriers the better BER. The influence of MAI on the system performance is significant and BER saturates at high SNR.

Keywords α - μ distribution, multicarrier CDMA, bit error rate, maximal ratio combining

I. INTRODUCTION

Multicarrier code division multiple access (MC-CDMA), which efficiently combines CDMA with orthogonal frequency division multiplexing (OFDM), has gained considerable attention as a promising multiple access technique for mobile communications [1,2]. MC-CDMA is a spread spectrum technique where the signal is spread in the frequency domain. Since the MC-CDMA technique possesses the advantages of both OFDM and CDMA, it has the properties desirable for future systems such as insensitivity to frequency-selective fading channels, frequency diversity, and the capability of supporting multirate service by applying either multicode or variable spreading factor techniques [1].

The performance of MC-CDMA has been studied both for the uplink and the downlink of a mobile communication system in [3] which perfect time synchronization among users was assumed. The BER performance of MC-CDMA systems over Rayleigh fading channel has been investigated in [4] and [5]. The authors in [4] analyzed the BER performance of MC-CDMA systems with a frequency offset, where the central limit theorem (CLT) approximation was used in the analysis. A performance analysis using the law of large number (LLN) approximation of an MC-CDMA system employing an antenna array at the base station has been presented in [5]. The bit error probability in multipath channels was analyzed based on the CLT approximation.

Nakagami fading channels have received considerable attention in the study of various aspects of wireless systems [6, 7]. The Nakagami distribution provides a more general and versatile way to model wireless channels [8]. The authors investigated the BER performance of MC-CDMA with an equal ratio combining (EGC) receiver and Nakagami- m fading channels [1] with the same fading parameter m on different subcarriers. In [9], the authors proposed a technique that combines OFDM and CDMA by using the characteristic function and Gaussian approximation method. The BER expressions of coherent demodulation with maximal ratio combining (MRC) and EGC are derived.

Although, in general, it has been found that the fading statistics of the mobile radio channel may well be characterized by Nakagami- m , situations are easily found for which other distributions such as Hoyt, Rice, and Weibull. More importantly, situations are encountered for which no distributions seem to adequately fit experimental data, although one or another may yield a moderate fitting.

Some researches [10] even question the use of the Nakagami distribution because its tail does not seem to yield a good fitting to experimental data, better fitting being found around the mean or median. The well-known fading distributions have been derived assuming a homogeneous diffuse scattering field, resulting from randomly distributed point scatters. The assumption of a homogeneous diffuse scattering field is certainly an approximation, because the surfaces are spatially correlated characterizing a nonlinear environment [11]. With the aim at exploring the nonlinearity of the propagation medium, a general fading distribution—the α - μ distribution was proposed [12]. Hence, its parameters are directly associated with the physical properties of the propagation medium. The α - μ distribution is general, flexible, and has easy mathematical tractability. It includes important distributions such as Gamma, Erlang, central Chi-squared, Nakagami, exponential, Weibull, one-sided Gaussian, and Rayleigh. Its density, cumulative frequency, and moments appear in simple closed form expressions. All these features combined make the α - μ distribution very attractive.

In this work, we propose a performance analysis of MC-CDMA communications system over the α - μ distribution. We adopt the BPSK modulation scheme with an alternative

expression for Q function to evaluate the bit error rate (BER) performance of the MC-CDMA system. The analysis is done by using the maximal ratio combining (MRC) technique. The close form expression of BER is derived.

The rest of this paper is organized as follow, section II gives a description of the MC-CDMA system model including fading characteristics. In section III, the BER expression with MRC combining of α - μ fading channel is derived. The numerical results and discussions are shown in section IV, then section V summarizes our contribution.

II. SYSTEM MODEL

In this section, the model of an MC-CDMA system is described. We assume that there are K simultaneous users, each having N subcarriers. The block diagrams of the considered MC-CDMA transmitter and receiver with one tap frequency domain equalizers in the uplink transmission are depicted in Figure 1.

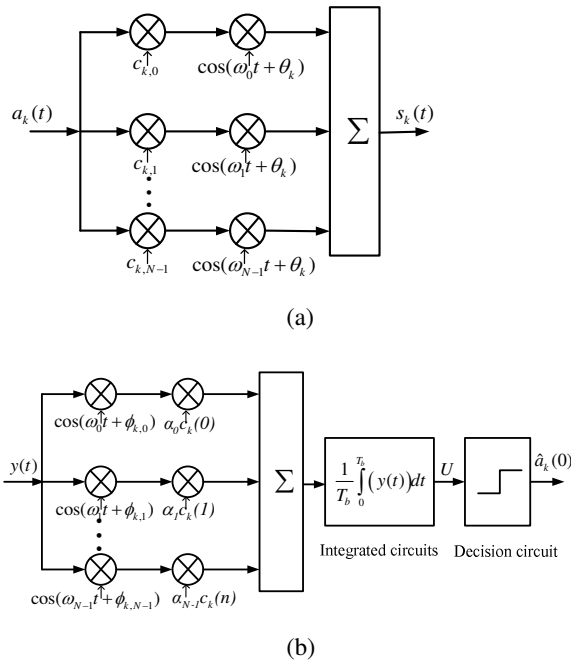


Figure 1. Block diagram of MC-CDMA system (a) transmitter and (b) receiver.

The transmitted signal of user k -th is given by

$$s_k(t) = \sqrt{\frac{2P}{N}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} c_k(n) a_k(m) p(t - mT_b) \cos(\omega_n t + \theta_k), \quad (1)$$

where $\omega_n = \omega_c + 2\pi(n-1)\frac{F}{T_b}$, which is the angular frequency of the n -th carrier. In this equation, P is the transmitted power. $c_k(n)$ represents the signature sequence of the k -th user,

$p(t - mT_b)$ is a rectangular pulse with duration of T_b and $a_k(m)$ represents the m -th input data bit for user k -th which belong to 1 or -1 with equal probability.

Signal is transmitted in a slow fading channel, the channel amplitude for the n -th carrier of the k -th user is denoted as $r_{k,n} e^{j\phi_{k,n}}$, with phase $\phi_{k,n}$. We assumed that phase of the signal is uniformly distributed over $[-\pi, \pi]$. Let $r_{k,n}$ denoted the envelope of the k -th user with carrier n and α is an arbitrary constant with $\alpha > 0$. The root mean square value is defined as $\hat{r}_{k,n} = \sqrt{E[r_{k,n}^2]}$. The α - μ probability density function of $r_{k,n}$ is denoted by $p(r_{k,n})$ and can be show as [13]

$$p(r_{k,n}) = \frac{\alpha \mu^\mu r_{k,n}^{\alpha\mu-1}}{\hat{r}_{k,n}^{\alpha\mu} \Gamma(\mu)} \exp(-\mu \frac{r_{k,n}^\alpha}{\hat{r}_{k,n}^\alpha}). \quad (2)$$

Let us define the instantaneous signal to noise ratio ($\gamma_{k,n}$) of the n -th carrier for user k as

$$\gamma_{k,n} = \left(\frac{r_{k,n}}{\hat{r}_{k,n}} \right)^2 \hat{\gamma}_{k,n}, \quad (3)$$

where $\hat{\gamma}_{k,n} = E[\hat{r}_{k,n}^2] \frac{E_b}{N_0}$, and $\frac{E_b}{N_0}$ is the signal energy per bit.

Then, the pdf of $\gamma_{k,n}$ can be found by performing random variable transformation based on (2) and using (3), which results in

$$p(\gamma_{k,n}) = \frac{\alpha \mu^\mu (\gamma_{k,n})^{\frac{\alpha\mu}{2}-1}}{2 \Gamma(\mu) (\hat{\gamma}_{k,n})^{\frac{\alpha\mu}{2}}} \exp \left(-\mu \left(\frac{\gamma_{k,n}}{\hat{\gamma}_{k,n}} \right)^{\frac{\alpha}{2}} \right). \quad (4)$$

This distribution is a general fading distribution that can reduce to the other distributions such as Rayleigh ($\alpha = 2, \mu = 1$), Nakagami-m (μ as fading parameter by setting $\alpha = 2$), Weibull ($\alpha = 0.5$ as fading parameter by setting $\mu = 1$). This general property of the α - μ distribution makes this distribution be a general fading characteristic and attractive by many researchers.

At the receiver, with assumption perfect synchronization, the received signal is given by

$$y(t) = \sqrt{\frac{2P}{N}} \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} r_{k,n} c_k(n) a_k(m) \cos(\omega_n t + \phi_{k,n}) p(t - mT_b - \tau(k)) + z(t) \quad (5)$$

with $\phi_{k,n} = \theta_k + \phi_{k,n}$ and $z(t)$ is an additive white Gaussian noise with zero mean and two sided power spectrum density

of $\frac{N}{2}$ W/Hz. Using Matched Filter (MF) and maximal ratio combining (MRC) technique, the decision variable (U) can be expressed as

$$U = \frac{1}{T_b} \int_0^{T_b} y(t) \sum_{n=0}^{N-1} \alpha_{0,n}(m) c_0(n) \cos(\omega_n t + \phi_{k,n}) dt. \quad (6)$$

We assume that user 0 is a reference one, and using MRC, therefore $\alpha_{0,n} = r_{0,n}$. In summary, after some manipulation, the decision variables can be written as

$$U = \sum_{n=0}^{N-1} S_n + \sum_{n=0}^{N-1} \sum_{k=1}^{K-1} I_{k,n} + \sum_{n=0}^{N-1} Z_n = S + I + Z, \quad (7)$$

with

$$S = \sqrt{\frac{P}{2N}} \sum_{n=0}^{N-1} a_0 r_{0,n}^2, \quad (8)$$

$$I = \sqrt{\frac{P}{2N}} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_k(n) c_k(n) r_{k,n} r_{0,n} \cos(\tilde{\phi}_{k,n}), \quad (9)$$

$$\text{and } Z = \sum_{n=0}^{U-1} z_n, \quad Z_n = \frac{1}{T_b} \int_0^{T_b} z(t) \sum_{n=0}^{N-1} c_0(n) r_{0,n} \cos(\omega_n t + \phi_{k,n}) dt. \quad (10)$$

Finally, the signal to noise (SNR) ratio can be calculated as

$$SNR = \frac{\frac{P}{2N} \sum_{n=0}^{N-1} r_{0,n}^2}{\frac{P}{4N} (K-1) E[r_{k,n}^2] \sum_{n=0}^{N-1} r_{0,n}^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} r_{0,n}^2}, \quad (11)$$

We notice that the SNR depends on the fading characteristic. Using user 0 as the reference signal, and $E[r_{k,n}^2] = 1$,

$P = \frac{E_b}{T_b}$ and $r_{0,n}^2 \frac{E_b}{N_0} = \gamma_{0,n}$. Therefore, the SNR (γ) can be written in a compact form as

$$\gamma = \left[\frac{1}{\frac{E_b}{N_0} \frac{(K-1)}{2} + \frac{N}{2}} \right] \sum_{n=0}^{N-1} \gamma_{0,n}. \quad (12)$$

III. PERFORMANCE ANALYSIS

The conditional BER of BPSK modulation can be shown as [14]

$$P_{e|0,n} = \mathcal{Q}(\sqrt{2\gamma}), \quad (13)$$

where $\mathcal{Q}(\cdot)$ denotes Q-Function and can be represented as in the form [15]

$$\mathcal{Q}(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta. \quad (14)$$

Hence, equation (13) can be alternatively expressed as

$$P_{e|0,n} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma}{\sin^2\theta}\right) d\theta. \quad (15)$$

To compute the average BER, we must statistically average (15) over the joint probability density function of the instantaneous SNRs. Using the alternative Q-function and the assumption of independent fading channels at different subcarriers, the average BER is given by

$$\bar{P}_e = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp\left(-\frac{\gamma}{\sin^2\theta}\right) p_{\gamma_{0,0}}(\gamma_{0,0}) \dots p_{\gamma_{0,N-1}}(\gamma_{0,N-1}) d\gamma_{0,0} \dots d\gamma_{0,N-1} d\theta. \quad (16)$$

Consider the innermost integrate of (16), the moment generating function (MGF) is calculated as

$$\begin{aligned} \mathcal{M}_\gamma(s) &= \int_0^{\infty} \exp(-s\gamma) p_\gamma(\gamma) d\gamma \\ &= \frac{\alpha \mu^\mu}{2\Gamma(\mu) \left(\hat{\gamma}\right)^{\frac{\alpha\mu}{2}}} \frac{k^{1/2} l^{\frac{\alpha\mu-1}{2}}}{(2\pi)^{\frac{l+k-2}{2}} s^{\frac{\alpha\mu}{2}}} G_{l,k}^{k,l} \left(\left(\frac{\mu}{\hat{\gamma}^{1/2}} \right)^k \frac{l^l}{s^{l,k}} \middle| I(l, 1-\alpha\mu/2) \right), \end{aligned} \quad (17)$$

with $I(n, m) = \frac{m}{n}, \frac{m+1}{n}, \dots, \frac{m+n-1}{n}$. Let $\frac{\alpha}{2} = \frac{l}{k}$, then the greatest common division, $\gcd(l, k) = 1$, and the value of μ can be both integer and non-integer. After some calculations, the average BER can be expressed as

$$\bar{P}_e = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{n=0}^{N-1} J_{0,n}(\text{SINR}_{0,n} \theta) d\theta, \quad (18)$$

$$\text{where } J_{0,n} = \int_0^{\infty} \exp\left(\frac{-\gamma_{0,n}}{\sin^2\theta} \frac{N}{2} + \frac{E_b}{N_0} \left(\frac{K-1}{2} \right) \right) p_{\gamma}(\gamma_{0,n}) d\gamma_{0,n}$$

$$= \frac{\alpha \mu^\mu}{2\Gamma(\mu) \left(\hat{\gamma}_{0,n}\right)^{\frac{\alpha\mu}{2}}} \frac{k^{1/2} l^{\frac{\alpha\mu-1}{2}}}{(2\pi)^{\frac{l+k-2}{2}} \text{SINR}_{0,n}^{\frac{\alpha\mu}{2}}} G_{l,k}^{k,l} \left(\left(\frac{\mu}{\hat{\gamma}_{0,n}^{1/2}} \right)^k \frac{l^l}{\text{SINR}_{0,n}^{\frac{l}{k}}} \middle| I(l, 1-\alpha\mu/2) \right) I(k, 0) \quad (19)$$

We observe that the average signal to interference plus noise ratio (SINR) for the n -th carrier of the first user is written as

$$\text{SINR}_{0,n} = \frac{(P/N) E[\beta_{0,n}^2]}{N_0 + \sum_{k=1}^{K-1} (P/N) E[\beta_{0,n}^2]}. \quad (20)$$

Finally, the average BER per user is calculated as

$$\bar{P}_e = \frac{1}{K} \sum_{k=0}^{K-1} P_e(k) \quad (21)$$

IV. NUMERICAL RESULTS

In this section, we show the numerical result of the system by using MATLAB software. In figure 3, we show BER of the system over the different values of α and μ over correlated fading channel. The number of user (K) equals to 25 and 1024 subcarriers. For $\alpha = 2$ and $\mu = 0.5$ the system is known as one-sided Gaussian channel. While $\alpha = 2, \mu = 1$ the system is known as Rayleigh fading channel, $\alpha = 2, \mu = 4/7$, this system is Nakagami fading channel.

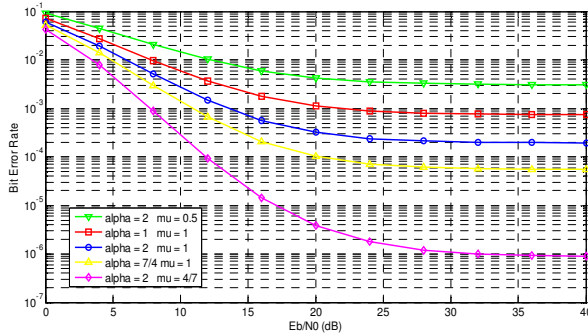


Figure 2. BER for MC-CDMA over different channels.

In figure 3, we show the system performance in term of BER for $\alpha = 2$ and $\mu = 1$ by letting number of user equals to 25 for the different number of subcarriers $N = 64, 128, 256, 512$ and 1024. We notice that the much more subcarrier the better BER performance.

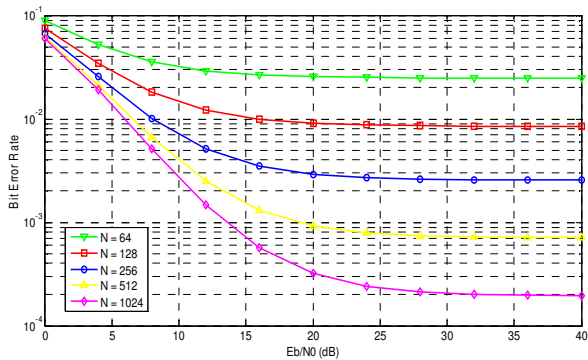


Figure 3. BER for MC-CDMA over $\alpha = 2, \mu = 1$ channel.

While in Figure 4 shows the BER of the system for the different number of user (K) for correlated fading channel for different channel. By letting the transmitted signal power over noise power (E_b/N_0) equal to 10 dB and 128 subcarriers. The significant impacts of multiple access interference can be observed. We can see that the more user, the more BER.

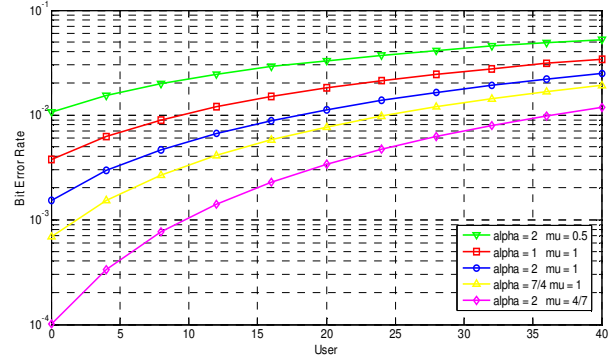


Figure 4. BER for MC-CDMA over different channels at $E_b/N_0 = 128$.

V. CONCLUSIONS

In this paper, we analyze the BER expression of the correlated MC-CDMA system over $\alpha - \mu$ fading channel. We perform the SNR of the system and the expression of BER was calculated by using moment generating function (MGF) approach. The system performances over different $\alpha - \mu$ are observed. We notice that the more subcarrier the better BER performance. In addition, it was also seen that the influence of multiple access interference (MAI) on the system performance is significant and that the BER saturates at high SNR region. We observe that increasing the user degrades the system BER.

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Chirawat Kotchasaran received the B.Eng. in electronics engineering from KMITL in 1996 and the M. Eng. in Electrical Engineering from KMUTT in 1999. He also received the Ph.D. in telecommunications from Asian Institute of Technology, Thailand in 2008. Currently, he is an assistant Professor at department of electronics and telecommunication engineering, RMUTT, Thailand. His main researches are digital communications, mobile communication and signal processing in communications..