

Duality of I/Q Imbalance in RF Mixer: Application to the L_1 -norm Based Estimation and Generalization to the SVD Based Model

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Abstract— This paper addresses the duality between the gain/phase mismatch in the I/Q imbalance model of RF transceiver IC. The newly proposed L_1 -norm based estimator exploiting this duality can serve as a sound alternative to the existing L_2 -norm based estimator significantly lowering the implementation cost. Generalization of this duality empowers the single skew matrix representation of the I/Q imbalance model, mappable to an always existing unique SVD based model, to extend its compensation range limited too conservative so far with much higher level of confidence than ever before.

Keywords— I/Q imbalance, duality, L_1 -norm, cross correlation, singular value decomposition

I. INTRODUCTION

I/Q imbalance of the mixer has been one of the critical impairment in RF transceiver circuit along with the phase noise of the RF-PLL as a form of inter-carrier interference(ICI). Thereby, it bogged down the adoption of high-order modulation and MIMO techniques based on OFDM in the past because mixers were indispensable to wireless communication systems unless full software defined radio(SDR) architecture is used. Along with the development of 4G communication back to WiMAX, digital processing technology for the I/Q imbalance calibration of the mixer has greatly improved both in theory and implementation, lowering the barriers and costs for both higher-order modulation and multi-input-multi-output(MIMO).

Besides, advances in digital I/Q imbalance calibration also solved the image rejection problem of low-IF receiver which is the root cause of the failure in the test item of adjacent channel selection (ACS) in the narrow-band communication systems such as GSM/EDGE. This allowed the transceivers to take full advantage of the state-of-the-art digital VLSI technology.

Initially, the calibration algorithm relied on a priori knowledge of the received signal. Soon, methods based on blind adaptive deconvolution [1] or the higher order statistics of the signal such as properness and circularity [2] have become wide-spread.

Among various kind of I/Q imbalance models, the frequency-independent I/Q imbalance model, with a single skew matrix accounting for the mixer, is still used to provide the design specification to the RF transceiver design and test limit in their language namely image rejection ratio(IRR) for a tone signal. This simplification provides a clear understanding

of the duality between the gain and phase mismatch in the I/Q imbalance. The finding of the duality itself also helps simplify the calibration process and reduce its complexity.

Conjugate signal representation shown in Figure 1(a) is now widely used even for frequency-independent I/Q imbalance model [3,4]. However, real-numbered matrix representation as shown in Figure 1(b) is frequently used throughout this paper to discern the imaginary part of the input vector rotated by $+\pi/4$, $x_i + x_q$, from the complex number, $x_i + j \cdot x_q$, more clearly. Matrix notation also makes the singular value decomposition(SVD) based model more compact and helps to simplify the equation in L_0, L_1 -norm case [5].

The remaining of this paper is organized as follows: The duality between gain and phase mismatch is derived and its implication is explained in section 2. Then two applications are presented. In section 3, L_1 -norm based estimate of gain/phase mismatch is induced from the duality. Section 4 addresses that the generalization of the duality leads to the SVD based I/Q compensator structure using a rotator in front of the I/Q imbalance compensator. The simulation results for both applications appended at the end of each section confirm the validity of all the arguments about both L_1 -norm estimate and generalization to the SVD based model. Finally, concluding remarks are provided in section 5.

II. DUALITY OF GAIN/PHASE MISMATCH

The duality of the gain/phase mismatch, although not explicitly stated, has been first exploited in [6] to the problem I/Q imbalance calibration at first. In their works, however, only the signs of both $(y_i + y_q)$ and $(y_i - y_q)$ have been used to obtain the approximate estimate of gain mismatch, i.e.

$$\Delta \hat{\epsilon}_{g,0} \triangleq \text{sgn}(y_i^2 - y_q^2) = \text{sgn}(y_i + y_q) \cdot \text{sgn}(y_i - y_q) \quad (1a)$$

, while the approximate of the phase mismatch was straightforward as

$$\Delta \hat{\epsilon}_{p,0} \triangleq \text{sgn}(y_i \cdot y_q) = \text{sgn}(y_i) \cdot \text{sgn}(y_q). \quad (1b)$$

Then, the estimation is done iteratively as

$$\begin{aligned} \hat{\epsilon}_{g,0}[n] &= \hat{\epsilon}_{g,0}[n-1] + \mu \cdot \Delta \hat{\epsilon}_{g,0} \\ \hat{\epsilon}_{p,0}[n] &= \hat{\epsilon}_{p,0}[n-1] + \mu \cdot \Delta \hat{\epsilon}_{p,0} \end{aligned} \quad (2)$$

to obtain the ϵ_g, ϵ_p recursively from the compensator output.

Doing a simple arithmetic of rotating the given input by $+\pi/4$ based on a clue from [6], we proved that the gain imbalance of $(x_i - x_q, x_q + x_i)$ is equivalent to the phase

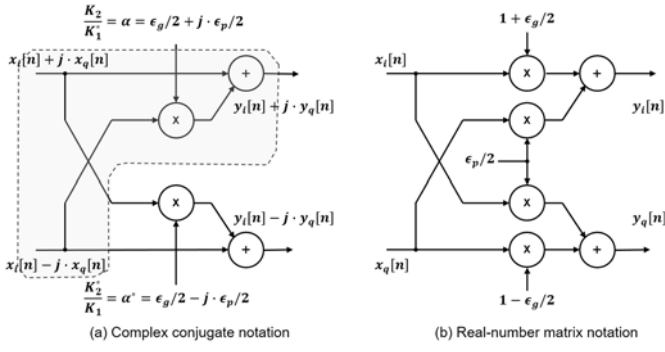


Figure 1. I/Q imbalance model which can also be used for compensator.

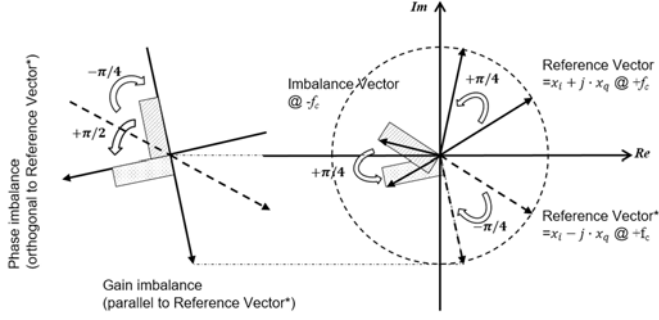


Figure 2. Duality between ϵ_g and ϵ_p in the imbalance model.

imbalance of (x_i, x_q) , $+\epsilon_p$, and the phase imbalance of $(x_i - x_q, x_q + x_i)$ is $-\epsilon_g$, the negative gain imbalance of (x_i, x_q) .

$$\begin{bmatrix} y_i \\ y_q \end{bmatrix} = \begin{bmatrix} 1 + \epsilon_g/2 & +\epsilon_p/2 \\ +\epsilon_p/2 & 1 - \epsilon_g/2 \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x_q \end{bmatrix} \quad (3a)$$

$$\begin{bmatrix} y_i - y_q \\ y_q + y_i \end{bmatrix} = \begin{bmatrix} 1 + \epsilon_p/2 & -\epsilon_g/2 \\ -\epsilon_g/2 & 1 - \epsilon_p/2 \end{bmatrix} \cdot \begin{bmatrix} x_i - x_q \\ x_q + x_i \end{bmatrix} \quad (3b)$$

This duality can also be derived using the conjugate model from [7, 8] as shown in Figure 2. From their argument, it can be said that if all the signals including input and output are rotated by $+\pi/4$ then its complex conjugate is rotated by $-\pi/4$. Thereby, the impairment, being the cross product of α and the conjugate of the desired signal, is rotated by $+\pi/2 = 2 \cdot (\pi/4)$ with respect to the conjugate of the given desired signal, i.e.

$$\begin{aligned} y &= x + \alpha \cdot x^* \\ y \cdot e^{+j\pi/4} &= x \cdot e^{+j\pi/4} + \alpha \cdot x^* \cdot e^{+j\pi/4} \\ &= x \cdot e^{+j\pi/4} + \alpha \cdot e^{+j\pi/2} \cdot (x \cdot e^{+j\pi/4})^* \end{aligned} \quad (4)$$

, where $\alpha = \epsilon_g + j \cdot \epsilon_p$. It should be noted that this duality endows us a commanding insight on L_1 -norm as in L_0 -norm and also the generalization to the SVD based model replacing $+\pi/4$ with an arbitrary angle of φ although this duality itself is trivial to L_2 -norm based estimator with the totality of

$$\Delta \hat{\epsilon}_{g,2} \triangleq y_i^2 - y_q^2 = (y_i - y_q) \cdot (y_q + y_i) \quad (5a)$$

$$\Delta \hat{\epsilon}_{p,2} \triangleq y_i \cdot y_q = \{(y_q + y_i)^2 - (y_i - y_q)^2\} / 4. \quad (5b)$$

III. L_1 -NORM BASED ESTIMATION

The main purpose of trying L_1 -norm in place of L_2 -norm is to reduce the implementation cost. As is often the case, simple squaring and correlation such as y_i^2, y_q^2 or $y_i \cdot y_q$ doubles the word-length of the internal data path. Typically, doubling of the

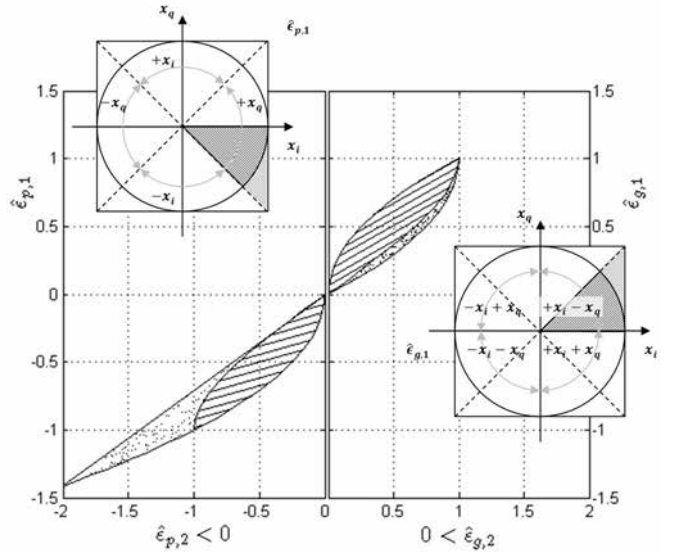


Figure 3. Plot of the proposed L_1 -norm versus L_2 -norm.

word-length caused by the multiplication makes the critical path longer and makes the timing closure of the digital logic implementation difficult. Moreover, the dynamic range after the operation gets bloated and unnecessary word-length optimization problems are introduced.

On the other hand, the L_1 -norm estimate defined in (6) does not double the word-length of the output. It is also expected to have better performance at least over the sign detection only algorithm, i.e. L_0 -norm losing the magnitude information [6]. As in Figure 3 of mapping between L_2 and L_1 , locality is kept at all points especially around the zero unlike L_0 .

$$\begin{aligned} \Delta \hat{\epsilon}_{g,1} &= |y_i| - |y_q| \\ &= \text{sgn}(\Delta \hat{\epsilon}_{g,2}) \cdot \min(|y_q + y_i|, |y_i - y_q|) \\ &= \Delta \hat{\epsilon}_{g,0} \cdot \min(|y_q + y_i|, |y_i - y_q|) \end{aligned} \quad (6a)$$

, while

$$\begin{aligned} \Delta \hat{\epsilon}_{p,1} &= \{|y_q + y_i| - |y_i - y_q|\} / 2\sqrt{2} \\ &= \text{sgn}(\Delta \hat{\epsilon}_{p,2}) \cdot \min(|y_i|, |y_q|) / \sqrt{2} \\ &= \Delta \hat{\epsilon}_{p,0} \cdot \min(|y_i|, |y_q|) / \sqrt{2} \end{aligned} \quad (6b)$$

Because both estimates of $\Delta \hat{\epsilon}_{g,1}, \Delta \hat{\epsilon}_{p,1}$ are anti-symmetric, we plotted the mapping range curves between L_2 and L_1 -norm based estimate only in half plane, respectively. Right half-plane is for $\epsilon_{g,2} > 0$ and left half-plane is for $\epsilon_{p,2} < 0$. The symmetry between the shaded region for the inside of the unit circle as opposed to the asymmetric region for the outside marked with speckles confines the valid range of duality inside the unit circle.

The second form for phase estimate in (6b) first appeared in [9], whose use is argued only with the simulation results but without any theoretical ground for using the $\min(\cdot)$ operator. Meanwhile, the duality of the gain/phase mismatch introduced and proved in Section 2 explains a more insightful form of the well working L_1 -norm based cross correlation of y_i and y_q . It is the gain mismatch of $y_i - y_q$ and $y_q + y_i$. The equivalence between the two forms can be easily proved with a simple arithmetic if the answer is given but the starting point cannot be easily unpicked without the concept of the duality.

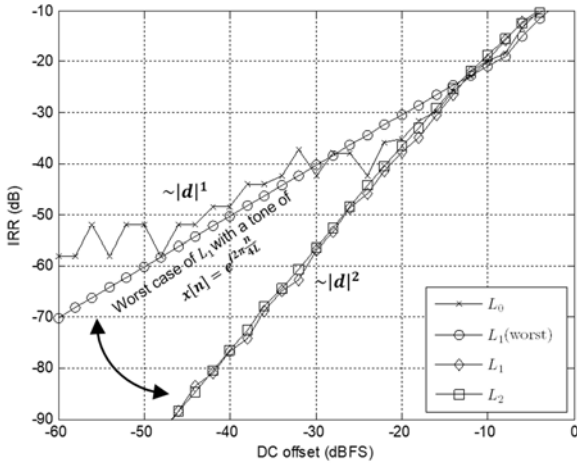


Figure 4. Comparison of the IRR as a function of DC offset, $|d|$, for L_p -norm based estimates with $p \in \{0, 1, 2\}$.

Historically, a similar form of duality although not in wireless communication traces back to the analog phase comparator in relay circuit which has long been used in power electronics [10]. Detecting a significant difference between two AC power lines, not between I/Q of a single signal, is crucial. For that purpose, the difference either in amplitude or phase is chosen but the duality between the amplitude and phase comparator is already well established.

Before proceeding further, it is noted that the use of L_1 -norm is limited only to the recursive adaptation schemes exploiting the circularity of the compensator output. For example, few algorithms proposed in [11-13] use the input samples of the compensator, (x_i, x_q) , for the estimation of the parameters where the unbiasedness of the estimator matters.

A. Comparison with L_2 -norm estimate

In digital-domain image rejection, either the word-length of the data path including ADCs or the DC offset limits the accuracy of the estimator. Figure 4 shows the image rejection ratio (IRR) of the I/Q imbalance estimator as a function of the DC offset. The conservative evaluation of [9] about the signal level dependency of L_1 -norm is partly a result from the DC offset relative to the signal level, d .

Clearly shown in Figure 4, L_1 -norm is much sensitive to the residual DC offset for some input signals in worst cases. However, the dependency of L_1 -norm estimate is close to that of L_2 -norm for most input signal distributions such as high-order modulation signal or thermal noise with AWGN characteristics. The IRR of L_1 is a function of $|d|$ with the order of 1 in worst cases while that of L_2 for any input, which is also a lower bound of L_1 , is $|d|^2$.

B. Implementation cost

The implementation cost of L_1 naturally lies between sign-only, L_0 , and L_2 . If the word-length of the input is W , then the word-length for each estimate would be 1, W and $2W$, respectively. Besides, L_2 requires multipliers or square logics.

It is remarkable that the cross-correlation in the form of “difference between the L_1 -norm” can be interpreted as the

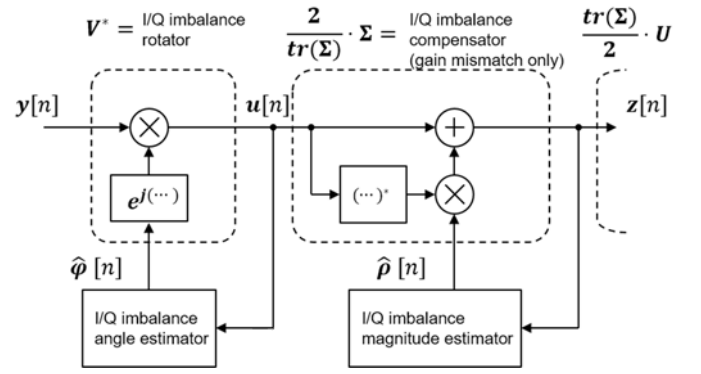


Figure 5. SVD based I/Q imbalance compensator.

ROM-less approximation of the digital quarter squaring algorithm for multiplication. The ROM for squaring operation is indispensable to L_2 -norm. A more detailed number about the implementation cost can be found in [14].

IV. GENERALIZATION TO SVD BASED MODEL

The duality between gain/phase mismatch discovered with the rotation of the signal by $+\pi/4$ can be generalized for the arbitrary rotation angle of φ . We know that the rank of the skew matrix of the compensator, C , is 2 since $1 - (\epsilon_g^2/4 + \epsilon_p^2/4) > 0$. This implies that SVD of C is possible with

$$C = U \cdot \Sigma \cdot V^*. \quad (7)$$

, where Σ is a diagonal matrix of 2×2 while U and V are 2×2 unitary matrices corresponding to rotation and they are all unique [5]. We can see that Σ is a skew matrix with only a gain imbalance as

$$\Sigma = g \cdot \begin{bmatrix} 1 + \rho/2 & 0 \\ 0 & 1 - \rho/2 \end{bmatrix} \quad (8)$$

Then, V^* can be thought of the I/Q imbalance rotator with φ enforcing the phase mismatch to zero as shown in Figure 5. Above all, the mathematical equivalence between the widely used skew matrix of (2a) and SVD based compensator model confirms that the valid range of skew matrix representation is no more confined for $|\epsilon_g|, |\epsilon_p| \ll 1$ since the SVD is always available so far as $|\epsilon_g/2|^2 + |\epsilon_p/2|^2 < 1$.

A. Estimate for SVD based compensator

The estimation formula itself is the same as that of the conventional single skew matrix form, however, only the phase update rule is applied to the preceding I/Q imbalance rotator, V^* , not concerning the gain mismatch although affected by the rotate operation.

$$\hat{\phi}[n] = \hat{\phi}[n-1] + \mu \cdot \Delta \hat{\epsilon}_p \quad (9a)$$

$$V^*[n] = \begin{bmatrix} +\cos \hat{\phi}[n] & -\sin \hat{\phi}[n] \\ +\sin \hat{\phi}[n] & +\cos \hat{\phi}[n] \end{bmatrix} \quad (9b)$$

The estimate obtained with $u[n]$ can be either the newly proposed L_1 -norm or conventional L_2 -norm. The gain, g , obtained with $\text{tr}(\Sigma)/2$ and U are just a scale and rotation of the restored signal respectively, thus they can be compensated by the following linear adaptive equalizer before the de-mapper not contributing to the residual IRR at all. Consequently, only two parameters are necessary and sufficient for complete

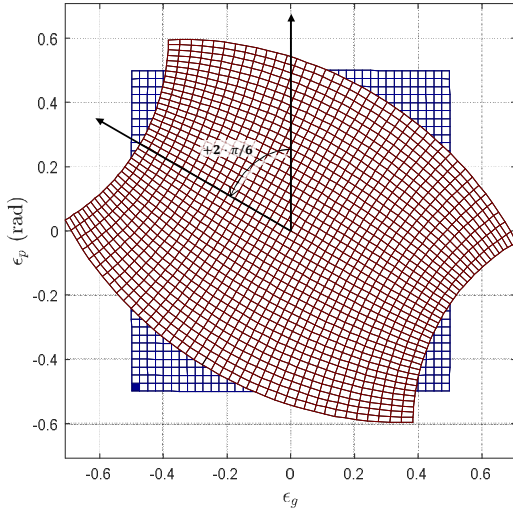


Figure 6. Mapping of (δ_g, δ_p) in square to warped (ϵ_g, ϵ_p) caused by the mismatch between the imbalance model and compensator.

compensation of all sorts of I/Q imbalance model whether they are represented with either (ϵ_g, ϵ_p) in single skew matrix form or (ρ, φ) in SVD based form.

B. Effects of imbalance model discrepancy

The symmetry and simplicity of the imbalance matrix, $\mathbf{D}$, came from the assumption of $|\delta_g|, |\delta_p| \ll 1$. However, accurate modeling of the real system requires even the frequency independent I/Q imbalance model to be represented by a 2×2 matrix with the nonlinear functions, i.e.

$$\mathbf{D} = \begin{bmatrix} f_{11}(\delta_g, \delta_p) & f_{12}(\delta_g, \delta_p) \\ f_{21}(\delta_g, \delta_p) & f_{22}(\delta_g, \delta_p) \end{bmatrix} \quad (10)$$

As explained before, a unique (ρ, φ) can be obtained using SVD based model from $\mathbf{D}(\delta_g, \delta_p)$. Then, (ϵ_g, ϵ_p) for compensation matrix, $\mathbf{C}$, is obtained with $(\rho \cdot \cos \varphi, \rho \cdot \sin \varphi)$ satisfying

$$\left\{ \frac{\mathbf{1}}{\text{tr}(\mathbf{C} \cdot \mathbf{D})} \cdot \mathbf{U}^{-1} \right\} \cdot \mathbf{C}(\epsilon_g, \epsilon_p) \cdot \mathbf{D}(\delta_g, \delta_p) = \mathbf{I}. \quad (11)$$

If the imbalance model and compensator are matched, then $(\epsilon_g, \epsilon_p) = (\delta_g, \delta_p)$. It is noted that the discrepancy between the model and compensator, which is inevitable even in small quantity, does not mean that there is no (ϵ_g, ϵ_p) cancelling the image. Instead, there exists (ϵ_g, ϵ_p) satisfying (11), but $(\epsilon_g, \epsilon_p) \neq (\delta_g, \delta_p)$ when they are not matched with each other.

Figure 6 shows the plane of (ϵ_g, ϵ_p) for (δ_g, δ_p) in $[-0.5, +0.5]$ which spans more than half of the range both in gain/phase mismatch. The assumption of small perturbation does not hold anymore. The nonlinear function used for the generation of the mapping in (12) is from [15], precisely modeling the impairment to its best effort as in other works

$$\mathbf{D} = \begin{bmatrix} +(1 - \delta_g/2) \cdot \cos \delta_p/2 & -(1 - \delta_g/2) \cdot \sin \delta_p/2 \\ -(1 + \delta_g/2) \cdot \sin \delta_p/2 & +(1 + \delta_g/2) \cdot \cos \delta_p/2 \end{bmatrix} \quad (12)$$

We also added an arbitrary imbalance rotator of $+\pi/6$ resulting in the rotation of (ϵ_g, ϵ_p) , but keeping the residual image to zero.

V. CONCLUSIONS

In this paper, we have explained the useful property in I/Q imbalance model of the RF transceiver, the duality between gain mismatch and phase mismatch. This finding gave us the insight to reformulate L_1 -norm and the corresponding I/Q imbalance estimation algorithm. Furthermore, applied to the compensation model itself, it was shown that the duality can be generalized to an SVD based model proving the existence of a unique estimate even in the presence of mismatch between the model and compensator.

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