

Cross-correlation index and multiple-access performance of Gold codes in a spread-spectrum system

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Abstract— Spread-spectrum system is a well-researched topic in literature. However, the system loading capacity with respect to its spreading codes is not well known. In this paper, a technique for determining the loading capacity is developed by the author. The concept of cross-correlation index is first introduced for the codes. Using this concept, a technique for predicting the system loading capacity is then developed for a set of Gold codes. This work involves the proposal of a mathematical model for the cross-correlation index, and the use of the same for the prediction of the loading capacity and system bit-error-rate performance. The model was tested for various set of Gold codes. Results for a set of 255-chip Gold codes are presented here. The results show that the codes have the capacity to support a maximum of about 15 users, above which bit-error-rate increases rapidly, ultimately resulting in emergence of error floor. The point at which the cross-correlation index equals to auto-correlation index marks the turning point around which the system performance revolves.

Keywords— Spread-spectrum system; Gold codes; cross-correlation index; multiple-access performance; bit-error-rate; error floor.

I. INTRODUCTION

The emergence of spread-spectrum techniques dates back to the 1940s. It was originally invented for the military but has grown to become an important worldwide technique in commercial and civil applications [1-3]. Properties of spreading codes are known to be central to the performance on spread-spectrum systems. Code properties are known to be a cause of multiple-access interference, an important limiting factor on the system performance. The significance of spreading codes has made the investigation of code properties, code design [4-7], code performance [8-12], and the search for better spreading codes [13-15] ongoing research activities.

Gold codes are a type of binary sequences used in wireless telephony and satellite navigation. The codes, whose history dates back to the sixties [16-18], have become important elements in spread-spectrum systems and other modern applications. The codes are, for example, used as spreading sequences for multi-user separation in code-division multiple-access (CDMA) systems, as access codes in the global positioning system (GPS), in tomographic imaging [19], in digital terrestrial television [20], in anti-jam underwater communication [21] and in encryption of medical images [22].

Gold codes are known to have bounded small cross-correlations within a set, making them useful as access codes for multiple users transmitting over the same frequency band. The cross-correlation between any pair of members a set of Gold codes are known to be triple-valued, but this knowledge is inadequate for determining the multiple-access performance of the codes. The same fact applies to the well-known value of peak cross-correlation between the code pairs. Knowledge of entire cross-correlation spectrum of the codes is necessary to determine their multiple-access performance. Unfortunately, generating the entire cross-correlation spectrum is difficult and intractable. This work provides an effective alternative to this problem.

In this paper, the author introduces the concept of cross-correlation index for the estimation of multiple-access performance of the codes. Starting from basic principles, this paper develops a mathematical model for the codes' cross-correlation index. This is then used as a tool for predicting system performance. This paper is part of a larger study. Some aspects of the work has been present in conferences [23][24] and published in journal articles [12][25].

The rest of this paper is organized as follows. Mathematical model for the system is developed in Section II. Research methods used for this work are outlined in Section III. Results are presented in Section IV. Finally, this paper concludes with a summary of important deductions.

II. THEORY

Consider a direct-sequence spread-spectrum (DSSS) system. Spread spectrum signal transmitted by a user k can be expressed as

$$s_k(t) = A c_k(t) b_k(t) \cos(\omega_c t + \theta_k), \quad (1)$$

where $b_k(t)$ is the user binary data, $c_k(t)$ is the spreading code and ω_c is the carrier frequency. The spreading code $c_k(t)$ for the user can be denoted as

$$c_k(t) = \sum_{i=1}^N c_k^i P_c(t - iT_c), \quad c_k^i \in \{-1, +1\}, \quad (2)$$

where N is length of the code, and P_c is a rectangular pulse having a duration T_c . Let the wireless communication channel

be represented by multiple paths having a real positive gain β_l , propagation delay τ_l and phase shift γ_l , where l is the path index. The channel impulse response $h_k(t)$ for L independent paths can be modelled as

$$h_k(t) = \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} \delta(t - \tau_{kl}) \quad (3)$$

At the receiving end, the received signal $r_k(t)$ for the user is obtained by convolving $s_k(t)$ with $h_k(t)$:

$$r_k(t) = \int_{-\infty}^{\infty} s_k(\tau) h_k(t - \tau) d\tau \quad (4)$$

$$r_k(t) = \sum_{l=1}^L A \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \quad (5)$$

$$r(t) = \sum_{k=1}^K \sum_{l=1}^L A \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \quad (6)$$

$$z_1(m) = \int_{mT_b}^{(m+1)T_b} (r(t) + n(t)) c_1(t - \tau_{k1}) \cos(\omega_c t - \theta_{k1}) dt \quad (7)$$

$$z_1(m) = \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=1}^K \sum_{l=1}^L A \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \right\} c_1(t - \tau_{k1}) \cos(\omega_c t - \theta_{k1}) dt \\ + \int_{mT_b}^{(m+1)T_b} n(t) c_1(t - \tau_{k1}) \cos(\omega_c t - \theta_{k1}) dt \quad (8)$$

$$z_1(m) = \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{l=1}^L A \beta_{1l} e^{j\gamma_{1l}} c_1(t - \tau_{1l}) b_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{1l}) \right\} c_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{1l}) dt \\ + \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=2}^K \sum_{l=1}^L A \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \right\} c_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{k1}) dt \\ + \int_{mT_b}^{(m+1)T_b} n(t) c_1(t - \tau_{k1}) \cos(\omega_c t - \theta_{k1}) dt \\ = z_{11} + z_{12} + z_{13} \quad (9)$$

Substituting the expressions for $s_k(t)$ and $h_k(t)$ into this integral, and using relevant properties of the Dirac delta function $\delta(t)$ gives (5). For a multi-user system comprising K users, the received signal $r(t)$ is a linear superposition of the signals for the users, and is given by (6).

Let user-1 be the reference user. Assuming coherent demodulation, the receiver output $z(m)$ for m^{th} bit during the bit duration T_b of the user is given by (7), where $n(t)$ is receiver noise. Substituting for $r(t)$ gives (8). Define $k = 1$ for the reference user. Using this in (8) gives (9), where z_{11} represents the desired signal for the reference user, z_{12} is interference term, and z_{13} is noise term.

Consider the mathematical expression for z_{11} of (9). In this expression, signal recovery at the receiver involves matrix multiplication of code sequences, such that the term $c_1^2(t - \tau_{11})$ represents the inner product of the first user-assigned code $c_1(t)$ with itself. Now, the user's code is a sequence of chip elements (+1,-1). Using this fact, we can show that $c_1^2(t - \tau_{11})$ is equal to the length N of the code. Also, let ω_c be chosen such that $f_c = \frac{n}{T_b}, n \in \mathbb{Z}$. By substituting these, we can show that the expression for z_{11} reduces to:

$$z_{11} = \frac{AN}{2} \sum_{l=1}^L \beta_{1l} e^{j\gamma_{1l}} \left\{ \int_{mT_b}^{(m+1)T_b} b_1(t - \tau_{11}) dt \right\} \quad (10)$$

If $b_1(t - \tau_{11}) = 1$ in the period T_b , then

$$z_{11} = \frac{AN}{2} \sum_{l=1}^L \beta_{1l} e^{j\gamma_{1l}} T_b \quad (11)$$

For a Gaussian channel, number of paths $L = 1$. Consequently, there is no multipath fading. Therefore, $\beta_{11} = 1$ and $\gamma_{11} = 0$. Hence, we have

$$z_{11} = \frac{ANT_b}{2} \quad (12)$$

Similarly, we can show that the interference term z_{12} reduces to:

$$z_{12} = \pm \frac{A}{2} \sum_{k=2}^K \int_{mT_b}^{(m+1)T_b} d_{1k} dt, \quad (13)$$

where $d_{1k} = c_1(t - \tau_{11})c_k(t - \tau_{k1})$.

Consider the noise term z_{13} :

$$z_{13} = \int_{mT_b}^{(m+1)T_b} n(t) c_1(t - \tau_{k1}) \cos(\omega_c t - \theta_{k1}) dt \quad (14)$$

In this equation, it can be seen that the spreading code $c_1(t - \tau_{k1})$ multiplies the receiver noise $n(t)$. Effect of this is to spread out the noise. Hence the noise is suppressed greatly. Bit-error-rate (or bit-error-probability) P_e can now be calculated using:

$$P_e = Q \left(\sqrt{\frac{E_b}{N_o + I_o}} \right) = Q \left(\sqrt{\frac{|z_{11}|^2}{N_o + |z_{12}|^2}} \right) \\ = Q \left(\sqrt{\frac{\left| \frac{ANT_b}{2} \right|^2}{N_o + \left| \frac{A}{2} \sum_{k=2}^K \int_{mT_b}^{(m+1)T_b} d_{1k} dt \right|^2}} \right) \quad (15)$$

where I_o is interference term. This expression shows that bit-error-rate (BER) depends on code length N . Since the Q -function is a monotonically decreasing function, it implies that as N increases, BER decreases. That is to say, longer codes give better BER. The equation also shows that BER depends on the cross-correlation index d_{1k} . This implies that BER for a multi-user system is affected by cross-correlation index of the codes.

Consider a set of Gold code sequences consisting of $2^n - 1$ sequences, each one with a period $N = 2^n - 1$, where n is the length of the linear feedback shift register used for generating the code. Gold codes exhibit triple-valued cross-correlation function [16, 26, 27] with values $\{-1, -t(n), t(n)-2\}$, where

$$t(n) = \begin{cases} 2^{(n+1)/2} + 1, & n \text{ odd} \\ 2^{(n+2)/2} + 1, & n \text{ even} \end{cases} \quad (16)$$

III. METHODOLOGY

Graphs of cross-correlation indices were generated for the codes using direct software implementation of the mathematical model that was presented above. The model was tested for different sets of Gold codes. The outcome of this was compared with results of simulations of the system performance.

For the software simulations, various sets of Gold codes were generated from appropriate combinations of preferred pairs of m -sequences, using linear feedback shift registers.

IV. RESULT AND DISCUSSION

The outcome of the investigation are hereby presented for a set of 255-chip Gold codes, obtained using the following generator polynomials:

$$P_1^8(x) = x^8 + x^7 + x^6 + x + 1 \\ P_2^8(x) = x^8 + x^7 + x^5 + x^3 + 1$$

Figure 1 shows cross-correlation index for the codes. This results show that cross-correlation index D_{1k} is a linearly increasing function of number of users. Clearly, autocorrelation index $D_{11} = D_{1k}$ when number of users is 16. Therefore the system is expected to saturate when number of users approaches 16.

Now consider the system bit-error-rate (BER) performance as shown by Figure 2. This figure shows the system performance for 1 to 257 users. The results were obtained from software simulations for the transmission of typically about one million random QPSK symbols through a Gaussian channel of zero mean and unit variance Gaussian noise. From this figure, it can be seen that there is no obvious error floor

for 10 users. In contrast, the onset of error floor can be seen for 20 users. This emergence of error floor is an indication of system saturation. In other words, system saturation sets in in-between 10 and 20 users, say 15 users on the average. This deduction is in agreement with the prediction obtained from the cross-correlation index of Figure 1 (As explained previously, the cross-correlation index of Figure 1 predicts that system saturation should set in when the number of users increases beyond 16, and this is in close agreement with the system BER performance of Figure 2).

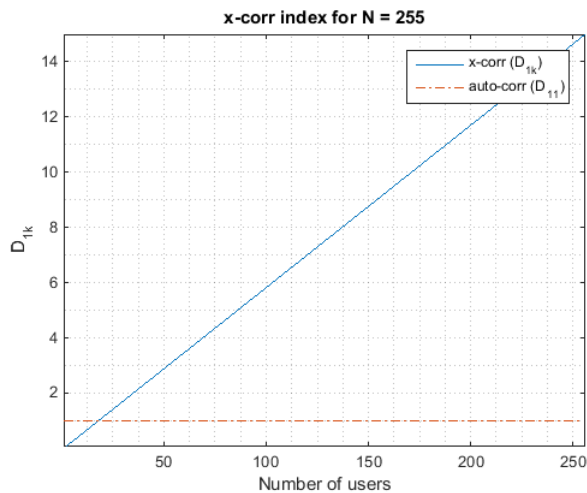


Figure 1. Cross-correlation index for $N = 255$

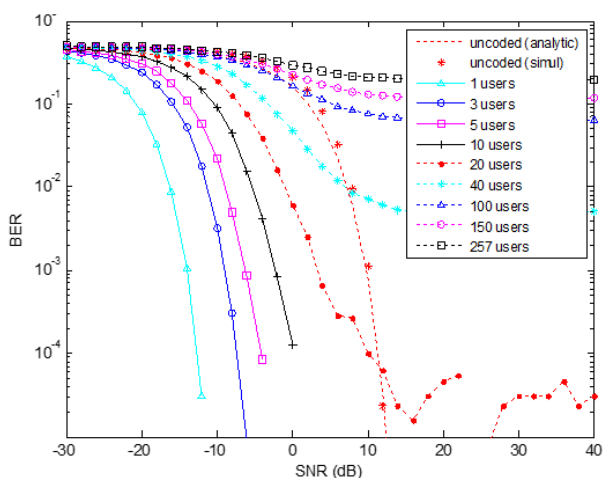


Figure 2. System loading performance for one to 257 users.

V. CONCLUSIONS

This paper proposed a mathematical model for cross-correlation index of Gold codes. The model was tested for a set of 255-chip Gold codes and was found to agree with results of the system performance. The results showed that the cross-correlation index is an accurate tool for predicting level

of multiple-access interference experienced by a spread-spectrum signal and for the estimation of the system loading capacity. Further results on other code lengths will be presented in future communications.

ACKNOWLEDGMENT

The author wishes to thank the Faculty Research Committee of the Faculty of Agriculture, Science and Technology, North-West University for supporting this work.

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