

Cross-correlation index and multiple-access performance of even-degree Gold codes

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Abstract - It is well known that for any particular code of a spread-spectrum system, there is a maximum number of users that would be tolerated and that this number would be well below nominal full load. However, the proportion of the full code set that are actually available for use before the onset of system saturation is not well known. In this work, the author presents a simple but effective means for determining performance limits of spreading sequences. This work involves the development of a mathematical model, the concept of cross-correlation index and its use for the determination of the system loading capacity. The model was developed and tested for different sets of Gold codes, the outcome of which was found to agree with results of software simulations for the system. Results shows that cross-correlation index is very effective for determining upper limit of simultaneous users that a spread-spectrum system can support in practice with respect to its spreading sequences.

Keywords: Spread-spectrum system; analytic model; Gold codes; cross-correlation index; loading capacity; bit-error-rate.

I. INTRODUCTION

Spread spectrum communication is a tried and tested technique developed originally for military applications as far back as the nineteen-forties. The technique later became a significant worldwide modality in the larger society, not only as a means of communication, but as a modality in other areas of application including the global positioning system (GPS), tomography imaging [1], digital terrestrial television [2], anti-jam underwater communication [3] and encryption of medical images [4].

Spreading codes are known to play a critical role in determining the system performance. Performance of any spread-spectrum system is affected by properties of spreading codes. It is well known that for any particular code, there is a maximum number of users that would be tolerated and that number would be well below the full load. However, the proportion of the full code set that are actually available for use before the onset of system saturation is not well known. This work seeks to fill this gap. In this work, the author develops a mathematical model for determining performance limits of spreading sequences. In a previous work, the model was tested successfully for a set of 255-chip Gold codes [5]. Similar results are presented in this paper for another set of

Gold codes. Results show that the model is equally valid for these new sets of codes.

There exist other advanced hybrids of spread-spectrum systems involving some channel enhancement techniques [6-10], but these are not our present focus. This work is part of a larger study. The rest of this paper is organized as follows. Starting from basic principles in Section II, a mathematical model is derived for the system. Validation method adopted for this work is presented in Section III. Results of the analysis are contained in Section IV, and conclusion in Section V.

II. SYSTEM MODEL

Consider a direct-sequence spread-spectrum (DSSS) system. Spread spectrum signal transmitted by a user k can be expressed as

$$s_k(t) = \frac{A}{N} c_k(t) b_k(t) \cos(\omega_c t + \theta_k), \quad (1)$$

where $b_k(t)$ is the user binary data, $c_k(t)$ is the spreading code, ω_c is the carrier frequency and N is length of the code. Here, N serves as a normalization factor to keep bit-energy constant regardless of the length of the spreading sequence. The spreading code $c_k(t)$ for the user can be denoted as

$$c_k(t) = \sum_{i=1}^N c_k^i P_c(t - iT_c), \quad c_k^i \in \{-1, +1\}, \quad (2)$$

where P_c is a rectangular pulse having a duration T_c . Let the wireless communication channel be represented by multiple paths having a real positive gain β_l , propagation delay τ_l and phase shift γ_l , where l is the path index. The channel impulse response $h_k(t)$ for L independent paths can be modelled as

$$h_k(t) = \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} \delta(t - \tau_{kl}) \quad (3)$$

At the receiving end, the received signal $r_k(t)$ for the user is obtained by convolving $s_k(t)$ with $h_k(t)$:

$$r_k(t) = \int_{-\infty}^{\infty} s_k(\tau) h_k(t - \tau) d\tau \quad (4)$$

Substituting the expressions for $s_k(t)$ and $h_k(t)$ into this integral, and using relevant properties of the Dirac delta function $\delta(t)$ gives (5). For a multi-user system comprising K users, the received signal $r(t)$ is a linear superposition of the signals for the users, and is given by (6).

Let user-1 be the reference user. Assuming coherent demodulation, the receiver output $z(m)$ for m^{th} bit during the bit duration T_b of the user is given by (7), where $n(t)$ is receiver noise. Substituting for $r(t)$ in (7) gives (8). Define $k = 1$ for the reference user. Using this in (8) gives (9), where z_{11} represents the desired signal for the reference user, z_{12} is interference term, and z_{13} is noise term. Consider z_{11} (10). Using the fact that the term $c_1^2(t - \tau_{1l})$ represents the inner product of the first user-assigned code $c_1(t)$ with itself, and

$$r_k(t) = \frac{A}{N} \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \quad (5)$$

$$r(t) = \frac{A}{N} \sum_{k=1}^K \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \quad (6)$$

$$z_1(m) = \int_{mT_b}^{(m+1)T_b} (r(t) + n(t)) c_1(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) dt \quad (7)$$

$$\begin{aligned} z_1(m) = \frac{A}{N} \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=1}^K \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \right\} c_1(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) dt \\ + \int_{mT_b}^{(m+1)T_b} n(t) c_1(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) dt \end{aligned} \quad (8)$$

$$\begin{aligned} z_1(m) = \frac{A}{N} \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{l=1}^L \beta_{1l} e^{j\gamma_{1l}} c_1(t - \tau_{1l}) b_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{1l}) \right\} c_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{1l}) dt \\ + \frac{A}{N} \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=2}^K \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) \right\} c_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{kl}) dt \\ + \int_{mT_b}^{(m+1)T_b} n(t) c_1(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) dt \\ = z_{11} + z_{12} + z_{13} \end{aligned} \quad (9)$$

$$\begin{aligned} z_{11} = \frac{A}{N} \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{l=1}^L \beta_{1l} e^{j\gamma_{1l}} c_1(t - \tau_{1l}) b_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{1l}) \right\} c_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{1l}) dt \\ = \frac{A}{2N} \sum_{l=1}^L \beta_{1l} e^{j\gamma_{1l}} \int_{mT_b}^{(m+1)T_b} c_1^2(t - \tau_{1l}) b_1(t - \tau_{1l}) [1 + \cos 2(\omega_c t - \theta_{1l})] dt \end{aligned} \quad (10)$$

choosing ω_c such that $f_c = \frac{n}{T_b}$ ($n \in \mathbb{Z}$), we can show that z_{11} reduces to:

$$z_{11} = \frac{A}{2} \sum_{l=1}^L \beta_{1l} e^{j\gamma_{1l}} \left\{ \int_{mT_b}^{(m+1)T_b} b_1(t - \tau_{1l}) dt \right\}. \quad (11)$$

Assume that $b_1(t - \tau_{1l}) = 1$ in the period T_b . For a Gaussian channel, number of paths $L = 1$. Consequently, there is no multipath fading. Therefore, $\beta_{1l} = 1$ and $\gamma_{1l} = 0$. With these, we can show that for a Gaussian channel, the last equation reduces to:

$$z_{11} = \frac{AT_b}{2}. \quad (12)$$

Similarly, we can show that the interference term z_{12} will reduce to:

$$z_{12} = \pm \frac{A}{2N} \sum_{k=2}^K \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} \left\{ \int_{mT_b}^{(m+1)T_b} d_{1k} dt \right\}, \quad (13)$$

where $d_{1k} = c_1(t - \tau_{1l})c_k(t - \tau_{kl})$, the cross-correlation between the code sequence for user-1 and user- k . For a Gaussian channel, $L = 1$, $\beta_{kl} = 1$ and $\gamma_{kl} = 0$, so that:

$$z_{12} = \pm \frac{A}{2N} \sum_{k=2}^K \int_{mT_b}^{(m+1)T_b} d_{1k} dt. \quad (14)$$

Define cross-correlation index D_{1k} as:

$$D_{1k} = \pm \frac{A}{2N} \sum_{k=2}^K \int_{mT_b}^{(m+1)T_b} d_{1k} dt \quad (15)$$

As we shall see shortly, the cross-correlation index is an important parameter affecting the system performance. It is a simple but powerful tool for predicting loading capacity in a multi-user spread-spectrum system.

III. METHODOLOGY

To validate the mathematical model, cross-correlation indices for various sets of Gold codes were obtained by direct software implementation of equation 15. Bit-error-rate (BER) curves were also generated for the system using software simulations, with the codes used as spreading sequences. The simulations involved transmission of typically about one million random QPSK symbols through a Gaussian channel of zero mean and unit variance Gaussian noise. The outcome of the direct implementation of the mathematical model was compared with results of the software simulations.

IV. RESULTS AND DISCUSSION

Results are presented here for a set of degree-six Gold codes, having a code length of 63-chips. The results include graphs of cross-correlation index D_{1k} , generated by direct implementation of (15) at zero lag. For the purpose of comparison, the value of auto-correlation index D_{11} was also obtained and plotted. Both quantities were normalised by code length.

Figure 1 shows the graph of cross-correlation index for the codes. This figure shows that cross-correlation index increases linearly with number of users, reaching a peak value of 15 at maximum loading (63 users) for the code. Figure 1 also shows that $D_{1k} = D_{11}$ for four users. Therefore, the system is expected to saturate when number of users is greater than four. These predictions agree with results obtained for the system performance. This is explained as follows.

Consider the system BER for the codes as shown by Figure 2. This graph was obtained from software simulations for the transmission of random QPSK symbols through the communication channel. The figure shows a plot of BER for increasing number of users in suitable steps. The figure shows that for three users or less, the system BER is well-behaved, having the usual characteristic waterfall shape. For five users, the BER curve can be seen to manifest anomalous behavior, indicating the onset of system saturation. This observation agrees with the earlier prediction obtained from the cross-correlation index (Figure 1): the cross-correlation index predicts that system saturation should set in when the number of users is higher than four. The situation becomes more obvious as the number of users increases. For ten users, the appearances of error floor is obvious, indicating a well-developed system saturation. As shown by the figure, the system BER and error floor becomes worse as the number of users increase from 10 to 20 to 30, etc. This indicates worsening system saturation.

Another observation relates to the severity of the system saturation. Figure 1 shows that above four users, cross-correlation index becomes increasingly higher than autocorrelation index, reaching a peak value of 15 at maximum loading (63 users) for the code. The meaning of this is that interference becomes increasingly higher the desired transmitted signal, up to about fifteen times higher at maximum system loading. As a result, the transmitted signal becomes swamped by interference, and this gives the reason for the worsening BER and the high level of error floor when the system is heavily loaded.

In an earlier paper [5], results for another set of code was presented. The deductions from the previous paper agree closely with those of the current paper, and this generalizes the validity of the model proposed by this work.

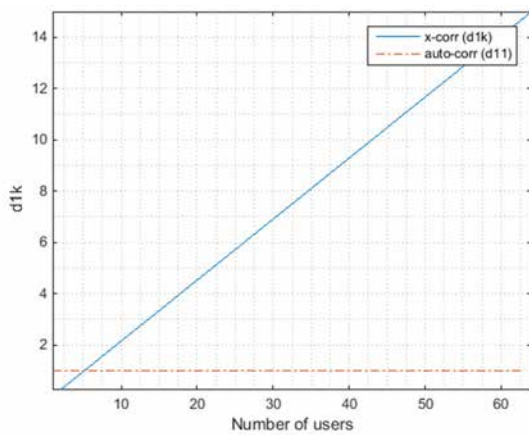


Figure 1. Cross-correlation index for a set of 63-chip Gold codes

V. CONCLUSION

This work proposed an analytic model for predicting the performance limits of a spread-spectrum system. The work involved the development of a concept of cross-correlation index for spreading sequences. The model was tested for different sets of Gold codes, and was found to agree closely with results of software simulation for the system. The results confirm the model is a simple but powerful tool for predicting the onset of system saturation, bit-error-rate behavior, system loading capacity and the emergence of error floor.

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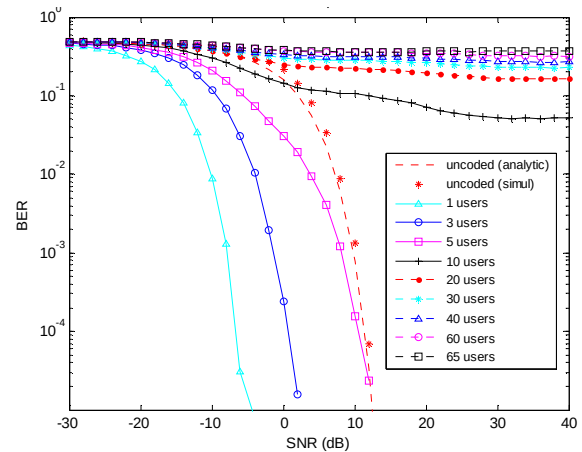


Figure 2. System bit-error-rate performance for the codes

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