

Power Optimization for Energy Efficiency in Cell-Free Massive MIMO with ZF Receiver

Yao Zhang, Haotong Cao, Meng Zhou and Longxiang Yang*

Department of Communication and Information Engineering
Nanjing University of Posts and Telecommunications, 210003, Nanjing, China
E-mail: {2017010201, 1015010309, 2018010202, yanglx}@njupt.edu.cn

Abstract—In this paper, a pilot-contaminated uplink cell-free massive multiple-input multiple-output (mMIMO) system with zero-forcing (ZF) receiver is considered. Then this paper derives a novel lower-bound expression for uplink energy efficiency (EE), which enables us to propose a resource optimization problem to maximize the total EE. This considered power optimization scheme takes into account the quality-of-service (QoS) requirement, each user power constraint and power consumption. Specifically, the power consumption model includes circuit power, data transmission power and backhaul power. Since the proposed power control problem is non-convex, a novel path-following approximation algorithm is characterized to tackle this problem. Simulation results indicate that the proposed algorithm only needs a few iterations to converge. Moreover, compared with equal power control (EPC) scheme, our algorithm can significantly improve the total EE.

Keywords—cell-free massive MIMO, zero-forcing, power optimization, energy efficiency, QoS requirement, circuit power consumption.

I. INTRODUCTION

In recent years, energy consumption raises a lot of concern as more and more communication resources are consumed to meet the rapid development of network and the increasing growth of information [1]. As a result, the EE metric has gradually become an important criterion for designing a green communication system in the upcoming 5G era [2][3]. At this time, an emerging network structure, called cell-free mMIMO [4-6], is regarded as a pivotal technology in 5G communication since it can significantly improve the spectral efficiency (SE) and EE.

Cell-free mMIMO system has been studied in many researches since 2015. In such a system, a great many access points (APs) distribute in a geographical large cell and simultaneously provide service to a small number of users. It absorbs the concepts of distributed mMIMO and network MIMO, hence, promising huge macro diversity and higher coverage. In cell-free system, a central processing unit (CPU) is designed to connect to each AP over high-speed, error-free fiber network and coordinate the cooperation between APs. Each AP only performs simple signal processing and leaves the complex, cumbersome operations to the CPU for processing. This configuration in line with the trend of green communication because the AP can be equipped with cheap, low-power circuit units.

Among many methods of maximizing the total EE in cell-free mMIMO, power optimization technique is always practical and economical. It aims to achieve the desired purposes (i.e., maximizing the total SE [7] or maximizing the total EE [8]) by adaptively adjusting the weight of the users or APs transmit power. Despite its merits, however, there is fairly little work on total EE maximization in cell-free mMIMO. In [8], the authors introduce a well-established downlink power consumption model and propose an iterative sub-optimal total EE maximization algorithm with conjugate beamforming (CB) precoder. This proposed algorithm is an application of the sequential convex approximation (SCA) method. Then, the authors in [9] use the same power consumption model and investigate the total EE maximization problem with ZF precoder. And a novel path-following algorithm is proposed to solve this optimization problem. In addition, in [10], the authors give a new power consumption model and formulate a new total EE maximization problem with CB precoder. The final solution is then determined by solving a sequence of fractional programming. It is demonstrated that all proposed power optimization algorithms in [8-10] can improve the total downlink EE significantly.

However, previous work is mainly about the downlink EE maximization and there is no research on the uplink side. Hence, this paper derives a new closed-form uplink EE expression with ZF receiver and formulates a total EE maximization problem with multiple constraints, such as QoS constraints and each user power constraints. The motivation behind utilizing ZF receiver is that ZF can mitigate the inter-user interference and improve the system performance, compared with CB receiver. Then a low-complexity path-following algorithm is proposed for solving this problem. Simulation results reveal that this proposed algorithm only needs a few iterations to converge. Moreover, compared with the EPC scheme in [4], our algorithm can significantly improve the total EE.

This paper is structured as follows: After the introduction, the system model is presented in Section II. Section III derives the achievable uplink SE and gives the uplink power consumption model. Section IV formulates the total EE maximization problem and Section V validates the proposed algorithm. Finally, Section VI concludes this paper.

Notation: Upper-case boldface letter denotes matrix and vector is represented by lower-case boldface letter. $\mathbf{A}^H$, $\mathbf{A}^T$ and $\mathbf{A}^*$ denote the conjugate transpose, transpose and conjugate of

the matrix $\mathbf{A}$, respectively. $\mathbb{C}$ represents the set of complex numbers and $\mathbf{I}_N$ means the $N \times N$ identity matrix. $|\cdot|$ and $\mathbb{E}\{\cdot\}$ indicate absolute operator and expectation operator. Finally, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ means a complex Gaussian random vector, whose mean is zero and covariance matrix is $\mathbf{I}_N$.

II. CELL-FREE MMIMO MODEL

This paper considers a cell-free system where MN -antenna APs distributed in an area serving K single-antenna users, $M > K$. Each user can be served by many APs at the same time-frequency block. In this paper, it assumes that the whole system is operated in time division duplex (TDD) model, which enables us to take full advantages of channel reciprocity. The $N \times 1$ dimension channel coefficient vector from user k to AP m can be model by

$$\mathbf{g}_{mk} = \sqrt{\beta_{mk}} \mathbf{h}_{mk}, \quad (1)$$

where β_{mk} denotes the effects of path loss and shadow fading, which is called large scale fading coefficient. What's more, $\mathbf{h}_{mk}$ represents small scale fading vector. It is assumed that $[\mathbf{h}_{mk}]_n \sim \mathcal{CN}(0, 1)$.

A. Uplink Pilot Training

In this stage, each user synchronously sends their pilot signal for estimating the uplink channel state information (CSI). The pilot allocated to user k is $\phi_k \in \mathbb{C}^{\tau \times 1}$, where $\|\phi_k\|^2 = 1$ and τ is the length of training phase which cannot be larger than the span of coherence interval, T . In fact, when $\tau < K$, there is no guarantee that the pilots assigned to each user are orthogonal. The reused pilots will reduce the system signal-to-interference-plus-noise ratio (SINR), which is called pilot contamination [4]. Therefore, this paper assumes that the CPU randomly allocates pilot to each user.

After K users transmitting the pilot signals, AP m receives

$$\mathbf{y}_{m,p} = \sqrt{\tau \rho_p} \sum_{k=1}^K \mathbf{g}_{mk} \phi_k^H + \mathbf{n}_{m,p}. \quad (2)$$

In (2), ρ_p is the transmit signal-to-noise ratio (SNR) and $\mathbf{n}_{m,p}$ is an $N \times \tau$ noise matrix whose elements are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ RVs.

In order to estimate $\mathbf{g}_{mk}$, AP m de-spreads $\mathbf{y}_{m,p}$ as

$$\tilde{\mathbf{y}}_{mk,p} = \mathbf{y}_{m,p} \phi_k = \sqrt{\tau \rho_p} \sum_{i=1}^K \mathbf{g}_{mi} \phi_i^H \phi_k + \mathbf{n}_{m,p} \phi_k. \quad (3)$$

Then the MMSE channel estimation of $\mathbf{g}_{mk}$ is [11]

$$\begin{aligned} \hat{\mathbf{g}}_{mk} &= \mathbb{E}\{\mathbf{g}_{mk} \tilde{\mathbf{y}}_{mk,p}^H\} / \left(\mathbb{E}\{|\tilde{\mathbf{y}}_{mk,p}|^2\} \right) \cdot \tilde{\mathbf{y}}_{mk,p} \\ &= c_{mk} \tilde{\mathbf{y}}_{mk,p}, \end{aligned} \quad (4)$$

where

$$c_{mk} = \frac{\sqrt{\tau \rho_p} \beta_{mk}}{\tau \rho_p \sum_{i=1}^K \beta_{mi} |\phi_i^H \phi_k|^2 + 1}. \quad (5)$$

The corresponding estimation error vector can be expressed as

$$\tilde{\mathbf{g}}_{mk} = \mathbf{g}_{mk} - \hat{\mathbf{g}}_{mk}. \quad (6)$$

The statistics of (4) and (6) are:

$$\hat{\mathbf{g}}_{mk} \sim \mathcal{CN}(\mathbf{0}, \alpha_{mk} \cdot \mathbf{I}_N), \quad (7)$$

$$\tilde{\mathbf{g}}_{mk} \sim \mathcal{CN}(\mathbf{0}, (\beta_{mk} - \alpha_{mk}) \cdot \mathbf{I}_N), \quad (8)$$

where

$$\alpha_{mk} = \frac{\tau \rho_p \beta_{mk}^2}{\tau \rho_p \sum_{i=1}^K \beta_{mi} |\phi_i^H \phi_k|^2 + 1}. \quad (9)$$

B. Uplink Data Transmission

Denote the data signal transmitted from user k by s_k , where $\mathbb{E}\{|s_k|^2\} = 1$. Then AP m can see,

$$\mathbf{y}_{m,d} = \sqrt{\rho_u} \sum_{k=1}^K \mathbf{g}_{mk} \sqrt{\eta_k} s_k + \mathbf{n}_{m,d}, \quad (10)$$

where ρ_u represents the SNR related to the data signal, $0 \leq \eta_k \leq 1$ denotes the power control coefficient and $\mathbf{n}_{m,d}$ is a vector of AWGN at AP m . Then the obtained $\mathbf{y}_{m,d}$ will be delivered to the CPU through the backhaul link. Define $\hat{\mathbf{G}} = [\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \dots, \hat{\mathbf{g}}_K] \in \mathbb{C}^{MN \times K}$, where $\hat{\mathbf{g}}_k = [\hat{g}_{1k}, \hat{g}_{2k}, \dots, \hat{g}_{Mk}]^T$. The CPU utilizes ZF receiver $\mathbf{A} = \hat{\mathbf{G}} (\hat{\mathbf{G}}^H \hat{\mathbf{G}})^{-1}$ to detect the data signal of user k , as

$$\begin{aligned} Y_k &= \sqrt{\rho_u} \mathbf{a}_k^H \sum_{i=1}^K \sqrt{\eta_i} \mathbf{g}_i s_i + \mathbf{a}_k^H \mathbf{n} \\ &= \sqrt{\rho_u \eta_k} s_k + \sqrt{\rho_u} \sum_{i \neq k} \sqrt{\eta_i} \mathbf{a}_k^H \tilde{\mathbf{g}}_i s_i + \mathbf{a}_k^H \mathbf{n}, \end{aligned} \quad (11)$$

where $\mathbf{a}_k$ is the k -th column of $\mathbf{A}$, $\mathbf{g}_k = [\mathbf{g}_{1k}, \mathbf{g}_{2k}, \dots, \mathbf{g}_{Mk}]^T$, $\tilde{\mathbf{g}}_k = [\tilde{g}_{1k}, \tilde{g}_{2k}, \dots, \tilde{g}_{Mk}]^T$ and $\mathbf{n} = [\mathbf{n}_{1,d}, \mathbf{n}_{2,d}, \dots, \mathbf{n}_{M,d}]^T$.

Then, s_k will be detected from Y_k .

III. ACHIEVABLE UPLINK SPECTRAL EFFICIENCY AND POWER CONSUMPTION MODEL

In this section, it firstly derives the lower-bound uplink SE expression. Then, the total uplink power consumption model is characterized.

A. Achievable Uplink Spectral Efficiency

It is clear that the second term in (11) represents estimation error, which is caused by pilot contamination and training noise. The mean-square of the estimation errors is:

$$\begin{aligned} &\mathbb{E}\left\{\left|\sqrt{\rho_u} \sum_{i=1}^K \sqrt{\eta_i} \mathbf{a}_k^H \tilde{\mathbf{g}}_i s_i\right|^2\right\} \\ &= \rho_u \sum_{i=1}^K \eta_i \mathbb{E}\left\{|\mathbf{a}_k^H \tilde{\mathbf{g}}_i|^2\right\} \\ &= \rho_u \sum_{m=1}^M \sum_{n=1}^N |\mathbf{a}_{mk}|_n^2 \sum_{i=1}^K \eta_i (\beta_{mi} - \alpha_{mi}). \end{aligned} \quad (12)$$

where $\mathbf{a}_{mk} = [[\mathbf{a}_{mk}]_1, [\mathbf{a}_{mk}]_2, \dots, [\mathbf{a}_{mk}]_N]^T$. Then, according to [12,

Table 2.3], the ergodic achievable uplink SE of user k can be obtained:

$$Se_k^{ZF}(\{\eta_k\}) = \left(1 - \frac{\tau}{T}\right) \mathbb{E} \left\{ \log_2 (1 + \text{SINR}_k) \right\}, \quad (13)$$

where

$$\text{SINR}_k = \frac{\rho_u \eta_k}{\rho_u \sum_{m=1}^M \sum_{n=1}^N \|\mathbf{a}_{mk}\|_n^2 \sum_{i=1}^K \eta_i (\beta_{mi} - \alpha_{mi}) + \|\mathbf{a}_k\|^2}, \quad (14)$$

and where the outer expectation in (13) is with respect to $\mathbf{a}_k$. In fact, it is hard to calculate the exact uplink SE in (13) since the ‘ergodic’ means that the CPU needs to know all realizations of $\mathbf{a}_k$. Next, this paper derives an approximate closed-form achievable uplink SE expression for user k , which is the function of β_{mk} only.

Theorem 1: Define $\boldsymbol{\eta} = [\eta_1, \eta_2, \dots, \eta_K]^T$, the approximate achievable uplink per-user SE with ZF receiver for any M, K , and N , is formulated as

$$\hat{Se}_k(\boldsymbol{\eta}) = \left(1 - \frac{\tau}{T}\right) \log_2 \left(1 + \frac{\rho_u \psi(k) \eta_k}{\rho_u \sum_{i=1}^K \eta_i c_i + 1} \right), \quad (15)$$

where

$$c_i = \arg \max_m (\beta_{mi} - \alpha_{mi}), \quad (16)$$

$$m_{j*} = \arg \max_m \beta_{mj}, \quad (17)$$

$$\psi(k) = \frac{\sum_{m \in \mathcal{M}_k} \alpha_{mk}^2}{\sum_{m \in \mathcal{M}_k} \alpha_{mk}} \left(\frac{N \left(\sum_{m \in \mathcal{M}_k} \alpha_{mk} \right)^2}{\sum_{m \in \mathcal{M}_k} \alpha_{mk}^2} - 1 \right), \quad (18)$$

and

$$\mathcal{M}_k = \{m \neq m_{j*} \mid j \neq k, \forall m\}, \quad (19)$$

Proof: Please refer to [11, Appendix A]. ■

It remarks that (15) is the function of β_{mk} only. Since β_{mk} varies slowly and remains unchanged during about 40 coherent intervals [4], (15) is more tractable in the real-life scenario.

The sum SE is

$$Se(\boldsymbol{\eta}) = \sum_{k=1}^K \hat{Se}_k(\boldsymbol{\eta}), \quad (20)$$

B. Uplink Power Consumption

From [13], the uplink power consumption can be expressed as:

$$P_{total} = \sum_{k=1}^K P_k + N \sum_{m=1}^M P_{tc,m} + \sum_{m=1}^M P_{bh,m}, \quad (21)$$

where P_k is the circuit and amplifier power consumption at user k , $P_{tc,m}$ reflects the power required to operate the circuit components at AP m while $P_{bh,m}$ represents the backhaul link power from AP m to the CPU. More specifically, P_k is modeled by:

$$P_k = \frac{1}{\mu_k} \rho_u N_0 \eta_k + P_{tc,k}, \quad (22)$$

where μ_k is the power amplifier efficiency and $P_{tc,k}$ denotes the circuit components power requirement at user k , N_0 represents the noise power. What's more, $P_{bh,m}$ can be written as:

$$P_{bh,m} = B \cdot Se(\boldsymbol{\eta}) \cdot P_{bt,m} + P_{0,m}, \quad (23)$$

where B is the bandwidth, $P_{bt,m}$ reflects the load-independent power and $P_{0,m}$ represents the fix backhaul link power consumption.

In summary, by defining $P_{fix} = \sum_{k=1}^K P_{tc,k} + \sum_{m=1}^M (NP_{tc,m} + P_{0,m})$, P_{total} is recast as

$$P_{total} = \rho_u N_0 \sum_{k=1}^K \frac{1}{\mu_k} \eta_k + B \sum_{m=1}^M Se(\boldsymbol{\eta}) \cdot P_{bt,m} + P_{fix}, \quad (24)$$

The total EE is then defined as the ratio of (20) and (24), as

$$Ee(\boldsymbol{\eta}) = \frac{B \cdot Se(\boldsymbol{\eta})}{\rho_u N_0 \sum_{k=1}^K \frac{1}{\mu_k} \eta_k + B \sum_{m=1}^M Se(\boldsymbol{\eta}) \cdot P_{bt,m} + P_{fix}}. \quad (25)$$

IV. POWER OPTIMIZATION FOR TOTAL ENERGY EFFICIENCY

In this section, it aims to maximize (25) by finding the optimal power control coefficients under each user QoS constraint and power constraint. Mathematically, the total EE maximization problem has the following form:

$$\max_{\{\eta_k\}} \frac{B \cdot Se(\boldsymbol{\eta})}{\rho_u N_0 \sum_{k=1}^K \frac{1}{\mu_k} \eta_k + B \sum_{m=1}^M Se(\boldsymbol{\eta}) \cdot P_{bt,m} + P_{fix}} \quad (26a)$$

$$\text{s.t. } \hat{Se}_k(\boldsymbol{\eta}) \geq \overline{Se}_k, \quad \forall k \quad (26b)$$

$$0 \leq \eta_k \leq 1, \quad \forall k \quad (26c)$$

where $\overline{Se}_k$ denotes the k -th user's QoS threshold. By dividing the numerator and the dominator of (26a) by $B \cdot Se(\boldsymbol{\eta})$, it becomes

$$\max_{\{\eta_k\}} \frac{1}{\frac{\rho_u N_0 \sum_{k=1}^K \frac{1}{\mu_k} \eta_k + P_{fix}}{B \cdot Se(\boldsymbol{\eta})} + \sum_{m=1}^M P_{bt,m}}. \quad (27)$$

Since the second term of the denominator of (27) doesn't dependent on η_k , it can only minimize the first term of the denominator of (27) to maximize $Ee(\boldsymbol{\eta})$. In order to facilitate the following quadratic convex transformation, this paper

defines $\eta_k = \zeta_k^2$, $P_{abs} = \rho_u N_0 \sum_{k=1}^K \zeta_k^2 / \mu_k + P_{fix}$ and $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_K]^T$. Thus, the optimization problem (26) is equivalent to

$$\max_{\{\zeta_k, \theta\}} \frac{B(T-\tau)}{T \ln 2} \theta \quad (28a)$$

$$\text{s.t. } \theta \leq \frac{1}{P_{abs}} \sum_{k=1}^K \ln \left(1 + \frac{\rho_u \psi(k) \zeta_k^2}{\rho_u \sum_{i=1}^K \zeta_i^2 c_i + 1} \right) \quad (28b)$$

$$\hat{S}e_k(\zeta) \geq \overline{S}e_k, \quad \forall k \quad (28c)$$

$$0 \leq \zeta_k \leq 1, \quad \forall k \quad (28d)$$

It is easy to find that the second term of the right-hand-side (RHS) of the constraint (28b) is troublesome and non-convex, so fractional programming in [9] can't be applied in this paper. To tackle this problem, a path-following iteration method is proposed, which has low computational complexity and can replace the constraint (28b) with a convex constraint at each iteration.

Recall the following convex inequality [9],

$$\frac{\ln(1+1/y)}{x} \geq 2 \frac{\ln(1+1/\bar{y})}{\bar{x}} + \frac{1}{\bar{x}(\bar{y}+1)} - \frac{y}{(\bar{y}+1)\bar{y}\bar{x}} - \frac{\ln(1+1/\bar{y})}{\bar{x}^2} x$$

for $\forall y > 0, \bar{y} > 0, x > 0, \bar{x} > 0$, (29)

by treating $1/y$ as y and $1/\bar{y}$ as $\bar{y}$, (29) can be recast as

$$\frac{\ln(1+y)}{x} \geq 2 \frac{\ln(1+\bar{y})}{\bar{x}} + \frac{\bar{y}}{\bar{x}(\bar{y}+1)} - \frac{\bar{y}^2}{\bar{x}(\bar{y}+1)y} - \frac{\ln(1+\bar{y})}{\bar{x}^2} x. \quad (30)$$

Using (30), it has

$$\frac{\ln(1+\text{SINR}_k)}{\rho_u N_0 \sum_{i=1}^K \frac{1}{\mu_i} \zeta_i^2 + P_{fix}} \geq a_k^{(n)} - b_k^{(n)} \left(\frac{\sum_{i=1}^K \zeta_i^2 c_i}{\psi(k) \zeta_k^2} + \frac{1}{\rho_u \psi(k) \zeta_k^2} \right) - c_k^{(n)} \left(\rho_u N_0 \sum_{i=1}^K \frac{1}{\mu_i} \zeta_i^2 + P_{fix} \right), \quad (31)$$

where

$$a_k^{(n)} = 2 \frac{\ln(1+\text{SINR}_k^{(n)})}{P_{abs}^{(n)}} + \frac{\text{SINR}_k^{(n)}}{P_{abs}^{(n)} (\text{SINR}_k^{(n)} + 1)}, \quad (32)$$

$$b_k^{(n)} = \frac{(\text{SINR}_k^{(n)})^2}{P_{abs}^{(n)} (\text{SINR}_k^{(n)} + 1)}, \quad (33)$$

and

$$c_k^{(n)} = \frac{\ln(1+\text{SINR}_k^{(n)})}{(P_{abs}^{(n)})^2}. \quad (34)$$

The $\text{SINR}_k^{(n)}$ and $P_{abs}^{(n)}$ are given by:

$$\text{SINR}_k^{(n)} = \frac{\rho_u \psi(k) (\zeta_k^{(n)})^2}{\rho_u \sum_{i=1}^K (\zeta_i^{(n)})^2 c_i + 1}, \quad (35)$$

and

$$P_{abs}^{(n)} = \rho_u N_0 \sum_{i=1}^K \frac{1}{\mu_i} (\zeta_i^{(n)})^2 + P_{fix}, \quad (36)$$

where $\{\zeta^{(n)}\}$ is a feasible solution determined from the n -th iteration of (28).

After doing the above simple algebraic substitutions, there is still non-convex term in the RHS of (31), e.g., $(\sum_{k=1}^K \zeta_k^2 c_k) / (\psi(k) \zeta_k^2)$. Next, this paper recalls a new convex inequality [14]:

$$\frac{y^2}{x} \geq 2 \frac{\bar{y}\bar{y}}{\bar{x}} - \frac{\bar{y}^2}{\bar{x}^2} x, \quad \forall y \geq 0, \bar{y} \geq 0, x \geq 0, \bar{x} \geq 0. \quad (37)$$

Inequality (37) deserves special attention. In fact, the sign '=' in (37) always happens when $\bar{x} = x$ and $\bar{y} = y$ at the optimal point. Thus, this paper uses the identity for (37) to exactly approximate (31).

By utilizing (37), the non-convex term in (31) can be recast as:

$$\frac{\sum_{i=1}^K \zeta_i^2 c_i}{\psi(k) \zeta_k^2} \geq 2 \frac{\sum_{i=1}^K \zeta_i^{(n)} \zeta_i c_i}{\psi(k) (\zeta_k^{(n)})^2} - \frac{\sum_{i=1}^K (\zeta_i^{(n)})^2 c_i}{(\psi(k) (\zeta_k^{(n)})^2)^2} \psi(k) \zeta_k^2. \quad (38)$$

Finally, by introducing another convex inequality [14]:

$$y^2 \geq 2y\bar{y} - \bar{y}^2, \quad \forall y \geq 0, \bar{y} \geq 0, 2y \geq \bar{y}, \quad (39)$$

it obtains

$$\frac{\psi(k) \sum_{i=1}^K (\zeta_i^{(n)})^2 c_i}{(\psi(k) (\zeta_k^{(n)})^2)^2} \zeta_k^2 \geq \frac{\psi(k) \sum_{i=1}^K (\zeta_i^{(n)})^2 c_i}{(\psi(k) (\zeta_k^{(n)})^2)^2} \left(2\zeta_k \zeta_k^{(n)} - (\zeta_k^{(n)})^2 \right). \quad (40)$$

Therefore, the constraint (28b) at the $(n+1)$ -th iteration can be replaced with

$$\theta \leq \sum_{k=1}^K \left(a_k^{(n)} - \frac{b_k^{(n)}}{\rho_u \psi(k) \zeta_k^2} - 2b_k^{(n)} \frac{\sum_{i=1}^K \zeta_i^{(n)} \zeta_i c_i}{\psi(k) (\zeta_k^{(n)})^2} - c_k^{(n)} \left(\rho_u N_0 \sum_{i=1}^K \frac{1}{\mu_i} \zeta_i^2 + P_{fix} \right) + b_k^{(n)} \frac{\sum_{i=1}^K (\zeta_i^{(n)})^2 c_i}{(\psi(k) (\zeta_k^{(n)})^2)^2} \psi(k) \left(2\zeta_k \zeta_k^{(n)} - (\zeta_k^{(n)})^2 \right) \right). \quad (41)$$

Since (28c) can be represented as a second order cone (SOC) [15], it is equivalent to:

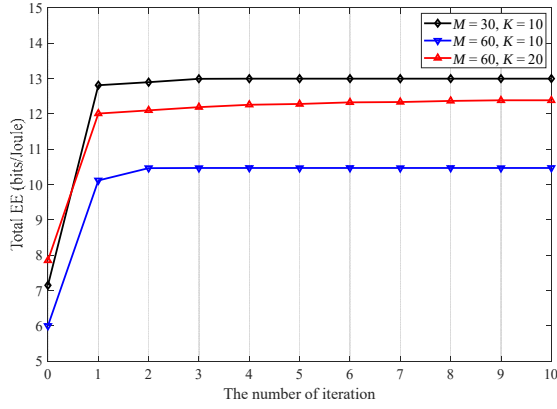


Figure 1. The convergence of Algorithm 1 with $N = 5$ and $\overline{S}e_k = 0.5$

$$\left(\sqrt{\rho_u \psi(k)} \zeta_k\right) \geq \left(2^{T \overline{S}e_k / (T-\tau)} - 1\right) \left(\rho_u \sum_{i=1}^K \zeta_i^2 c_i + 1\right), \quad (42)$$

Hence, the $(n+1)$ -th iteration of (28) can be reformulated as:

$$\max_{\{\zeta_k, \theta\}} \frac{B(T-\tau)}{T \ln 2} \theta \quad (43a)$$

$$\text{s.t.} \quad (41), (42), (28d) \quad (43b)$$

$$2\zeta_k \geq \zeta_k^{(n)}, \forall k \quad (43c)$$

Note that (43c) is due to $2y \geq \bar{y}$ in (39).

The problem (43) involves $K+1$ scalar real variables, $K+1$ quadratic constraints and K linear constraints, the complexity of each iteration is $\mathcal{O}((K+1)^{4.5} + (K+1)^{3.5})$.

The algorithm for solving (28) is summarized as follows.

Algorithm 1 Path-following algorithm for solving (28)

1. Choose a feasible solution $\{\zeta_k^{(0)}, t^{(0)}\}$, define the error tolerance ε and the number of iterations $\mathcal{N}$. Set $n = 0$.
2. **while** $n \leq \mathcal{N}$ or $\left|\sum_{k=1}^K (t_k^{(n+1)} - t_k^{(n)})\right| < \varepsilon$
3. Solve (43) to obtain the solution $\{\zeta_k^{(n+1)}, t^{(n+1)}\}$
4. Reset $n := n + 1$.
5. **end while**

Next, this paper analyzes the convergence of Algorithm 1. Suppose $\{\zeta\}$ is feasible to (43). By using (31), (38) and (40), $\{\zeta\}$ still satisfies (28b), which implies that $\{\zeta\}$ is feasible to (28). What's more, since $\{\zeta^{(n)}\}$ is an optimal solution to the n -th iteration of (28), it is also feasible to (28). It should be noticed that the sign '=' in (31), (38) and (40) will happen when $\zeta = \zeta^{(n)}$. Thus, an optimal solution of the (n) -th iteration is also a feasible solution to the $(n+1)$ -th iteration. Hence, the sequence generated by Algorithm 1 is monotonically increasing. In summary, the objective of Algorithm 1 is guaranteed to converge.

V. SIMULATION AND RESULTS

In this section, it performs a series of simulations to

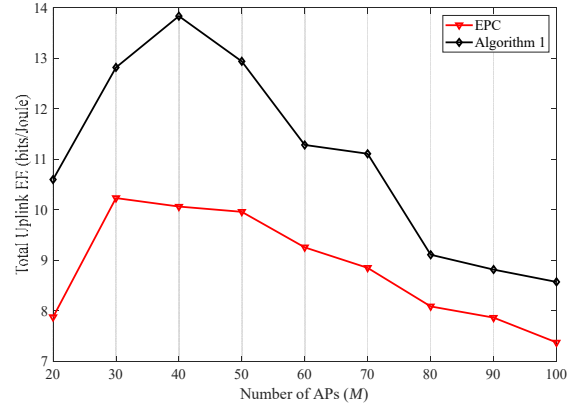


Figure 2. The total uplink EE versus the number of APs (M) with $K = 20$, $N = 5$ and $\overline{S}e_k = 0.5$ bit/s/Hz.

investigate the EE performance and validates the effectiveness of Algorithm 1.

A. Simulation Parameters

The simulation area is assumed to be an $R \times R$ square area. This square area is wrapped-around to imitate an infinite network environment.

The large-scale fading coefficient is model by [7]

$$\beta_{mk} = \text{PL}_{mk} \cdot z_{mk}, \quad (44)$$

where PL_{mk} is the pathloss between AP m and user k , z_{mk} denotes the shadow fading, which is assumed to be uncorrelated and has a standard deviation σ_{sh} . For all example, this paper adopts the three-slope pathloss model in [4]. Table I enumerates the system parameters appeared in this paper.

For comparison, this paper considers the case without power optimization, i.e., EPC scheme in [4]. In this case, each user uses its full power to transmit the data signal, e.g., $\eta_k = 1$, $\forall k$.

B. Performance Evaluation

First of all, this paper evaluates the effectiveness of the proposed Algorithm 1 in maximizing the total uplink EE. Fig.1

TABLE I. SYSTEM PARAMETERS

Parameter	Value
σ_n^2 (noise variance at the receiver)	$290 \times \kappa \times B \times NF$
κ	Boltzmann constant
B	20MHz
NF	9dB
T	200
τ	$K/2$
ρ_u, ρ_p	0.1W, 0.1W
σ_{sh}	8dB
R	1km
$P_{tc,k}, P_{tc,m}$	0.1W, 0.2W
$P_{0,m}, P_{bt,m}$	0.2W, 0.25W/(Gbits/s)
μ_k	0.388

depicts numerically the total uplink EE versus the number of iterations, with $N = 5$, $\overline{Se}_k = 0.5$ bits/s/Hz and different M and N . It can be easily concluded that Algorithm 1 has a fast convergence rate and the total uplink EE tends to a stable value after 4 iterations. Therefore, in the following simulations, this paper sets $N = 4$.

Next, this paper investigates the impact of different M on the total uplink EE with and without power optimization. Fig.2 shows the impact of different M on the total uplink EE, with $K = 20$, $N = 5$ and $\overline{Se}_k = 0.5$ bits/s/Hz. As one can see, Algorithm 1 can significantly improve the total EE, compared with EPC scheme. More specifically, when $M = 40$, the total uplink EE obtained via Algorithm 1 is 13.84 bits/s/Hz, which is 38.1% higher than the EPC scheme. In addition, Fig.2 reveals that when the number of APs (M) beyond some optimal points, the total EE will decrease as M increases. This is because increasing M will increase the value of $\sum_{m=1}^M (NP_{tc,m} + P_{0,m})$, which grows faster than $Se(\eta)$.

VI. CONCLUSIONS AND FUTURE WORKS

This paper investigates the total uplink EE of pilot-contaminated cell-free massive MIMO with ZF receiver. A new path-following approximation algorithm for maximizing the total EE is proposed, which satisfies the QoS requirements and each user power constraints. Simulation results indicate that the proposed algorithm is more tractable and has a fast converge rate. Moreover, the results also validate the effectiveness of the proposed algorithm.

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Yao Zhang received the B.S. degree in College of Computer Science & Technology from Qingdao University, China, in 2016. He is currently pursuing his Ph.D. degree at the Department of Communication and Information Engineering in Nanjing University of Posts and Telecommunications. His research interests include massive MIMO, especially power optimization in cell-free massive MIMO.



Haotong Cao (S'17) received the B.S. degree in communication engineering from the Nanjing University of Posts and Telecommunications, Nanjing, China, in 2015, where he is currently pursuing the Ph.D. degree with the Key Laboratory of Broadband Wireless Communication and Sensor Network Technique, Ministry of Education. He is currently a visiting student with the Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Loughborough, U.K. His research interests include next-generation network, wireless communication theory, resource allocation in wired and wireless networks, and unmanned aerial vehicle data ferrying. He has served as the TPC member of multiple IEEE international conferences, such as IEEE ICC and IEEE VTC. He is also serving as the reviewer of multiple IEEE journals, such as the IEEE TRANSACTIONS ON NETWORK AND SERVICE MANAGEMENT, the IEEE TRANSACTIONS ON GREEN COMMUNICATIONS AND NETWORKING, and the IEEE ACCESS.



Meng Zhou received the M.S. degree from Northwest Normal University, Lanzhou, China, in 2018. He is presently pursuing the Ph.D. degree with the College of Communication and Information Engineering from the Nanjing University of Posts and Telecommunications (NJUPT), Nanjing, China. His broad interests in wireless communications, with a special interest of massive MIMO, physical layer security, HetNets in 5G.



Longxiang Yang is with the College of Telecommunications and Information Engineering, Nanjing University of Posts and Telecommunications (NJUPT), Nanjing, China. He is a Full Professor and a Doctoral Supervisor of NJUPT, where he is the Vice Head of the College of Telecommunications and Information Engineering. He has fulfilled multiple National Natural Science Foundation projects of China. He has authored and co-authored over 100 technical papers published in various journals and conferences. His research interests include cooperative communication, network coding, wireless communication theory and Internet of Things.