

TED: A Tetrominoes based edge descriptor

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Abstract—A Tetrominoes based Edge Descriptor (TED) is proposed in this paper. First of all, the relationship between the sparsity of tetrominoes and human visual system are discussed. Secondly, the spatial and frequency response of tetrominoes are considered. And then the new structure of tetrominoes based edge detection structure is proposed. The experimental work results show the better results both in subjective and objective aspect than conventional methods.

Keywords— Edge detection, Tetrominoes, Optimization, Denoising, Combination.

I. INTRODUCTION

Image edge information is one of the primary means of human perception of the world, so the edge detection has been one of the basic tools for image processing, image understanding and computer vision[1]. In recent years, the combination with the characteristics of human cognitive and computer vision algorithms attracted extensive attentions. Latest research of cognitive characteristics of the human visual cortex shows that the sparsity is consistent with the human visual system. Although the neurons of the neocortex are highly connected, there are only a fraction of neurons in the active state simultaneously due to the inhibitory neurons[2]. Therefore, information is always by active cells which are a small part of the huge number of neural cellular expressed. This encoding method is called sparse discrete characterization.

Human visual system is consistent with the sparse discrete characterization. Considering a digital image with size of 256×256 , and each pixel point in the range 0-255, then there may be $256 \times 256 \times 256$ possible images, apparently for humans only a small part (sparse) of them are meaningful as shown in the left part of Fig. 1; and others are noise as shown in a right half part of Fig. 1. The 2-D points are mapped into a series of units to form a digital image, therefore, only a set of points can be considered meaningful to human, particularly adjacent pixels, which is the standard structure of current 2-D filters.

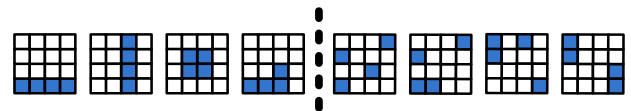


Figure 1. The tetrominoes structure VS random structure

This paper utilized the 2-order high-pass tetrominoes with four adjacent pixels to obtain the images in different directions. And then these images are fused together based on the self-similarity of the image structure to achieve edge detection result.

II. TETROMINOES

Tetrominoe is proposed by Golomb[3]. A tetromino is a geometric shape composed of several squares, connected orthogonally. A popular use of tetrominos is in the video game Tetris, where they are often called Tetriminos. Fig. 2 shows the 2-4 order tetrominoes contain 1,2 and 5 different tetrominoes. Tetrominoe can also be considered as a subset of the cliques of Markov Random Field. Tetrominoe are formed by joining unit squares along their edges. A free tetromino is considered up to congruence. That is, two free polyominoes are the same if there is a combination of translations, rotations, and reflections that turns one into the other[4].

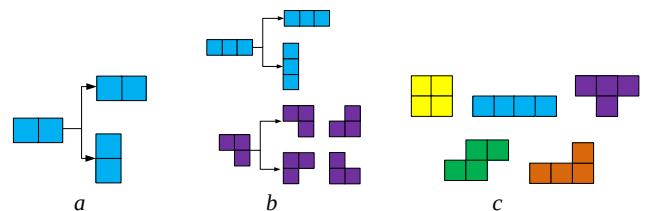


Figure 2. The free tetrominoes

- a 2-order tetrominoe kernel with its rotations
- b 3-order tetrominoes kernels with their rotations
- c 4-order tetrominoes kernels (O-I-T-S-L)

Based on the theory of the tetrominoe, [5] proposed a localized adaptive algorithm named Tetrolet. The main principle is that in each of the image block of 4×4 , combined Haar wavelet fast algorithm to calculate the 117 possible 4-

order tetrominoes and choose the sparsest combination of forms, for image representation.

III. OPTIMAL EDGE-MATCHING TETROMINOES

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The coefficients in Tetrolet are ± 1 [5] to achieve the low-pass and band-pass results. In order to achieve optimal image edge representation we introduced the optimal edge-matching filter-based edge detector in [6] to form the 2-D tetrominoes high-pass filter which only contains two coefficients $-1/6$ and $1/2$. The proof is in Appendix A. By extending the 1D edge-matching filter in 2D, the optimal edge-matching tetrominoes are obtained which is shown in Fig. 3.

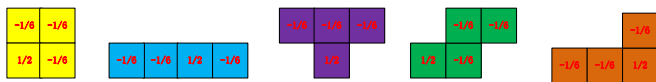


Figure 3. The 2-D filter characteristics in TED

It can be found that the frequency domain of the 2-D tetrominoes filter represents different direction of the image which can be realized as high-pass information, corresponding to the edges of the image. The rotation of the tetrominoes can represent more edge information of the images.

The direct 2-D convolution operation is applied in space domain and thus the optimal combination in [5] is avoided. As tetrominoes structures are limited to a small number of possible combinations, the sparsity of the image representation is guaranteed. Consider the 4×4 pixel cells in Fig.1, if only four pixels are applied for filtering operation each time, obviously, there are $16 \times 15 \times 14 \times 13 = 43680$ possibilities. However it will be much sparser in tetrominoes structure, for example "I" only need to consider 8 cases as shown in Fig. 4.

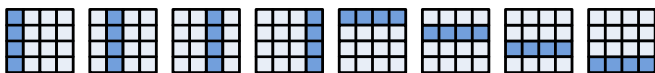


Figure 4. possible structure of tetrominoes "I"

Table I shows all the combination of 4-order tetrominoes. There are 5 tetrominoes in 4-order case named O, I, T, S, L. The total tetrominoes need to consider is 35 which is only 0.08% of random 43680 combinations.

TABLE 1. NO. OF 4-ORDER TETROMINOES COMBINATIONS

O	I	T	S	L	Tetrominoes	Random
9	8	6	6	6	35	43680

IV. EDGE DESCRIPTOR

Adaptive Tetrolet Algorithm

Input: Image a

1. 2-D high-pass tetrominoes filter for the i -th channel to obtain $b(i)$

2. $b(i)$ is binarized to obtain an edge image $c(i)$

3. Do "OR" operation to all the image $c(i)$ to obtain the final edge image d

Output: Edge detection result d

$Out = \bigcup T(In)$

(1)

T is 2-D high-pass tetrominoes filter, $\bigcup$ is "OR" operation.

V. EXPERIMENTAL

Figure 5 shows a series of visual appearance of the natural image. The top row is the original image, and the 2-4 rows are the edge detection results by canny, sobel and TED method respectively. The left 5 columns kept the 20% of the most significant coefficient and the 6 and 7 columns kept 10% and 20% of them in the same scene respectively. It can be concluded that the results of canny, and sobel contains obvious details information. However a sharpen edges of the proposed TED algorithm is achieved.

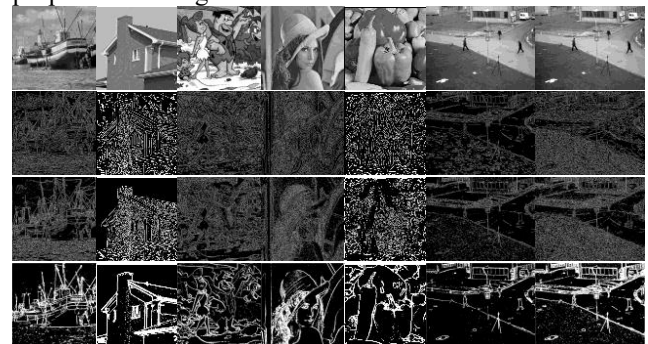


Figure 5. Edge detection results of natural images

Objective evaluation is done under Drive database which contains the manually delineated edge detection results. Fig. 6 shows the comparisons among several methods. Image a_1 is the original eye bowl, and image a_2 is the manually delineated edge detection. The rest images are the result of detecting the edge of the eye. Among them b_1 and b_2 are canny, c_1 and c_2 are sobel, d_1 and d_2 are prewitt, e_1 and e_2 are roberts, f_1 and f_2 are log, and g_1 and g_2 are the proposed TED method respectively. Image b_1 - g_1 retain 10% of the significant coefficients, and it can be seen that the results contain a lot of detail information except the image g_1 . To highlight the visual appearance a morphological dilation with a 2×2 kernel is applied to b_1 - g_1 and the results are shown b_2 - g_2 . It can be seen that the main lines is highlighted but the details part are also enhanced thus the false edge is increased. In contrast, the TED method gives a balanced result which contains main lines and

detailed information and it is most similar than the manual results.

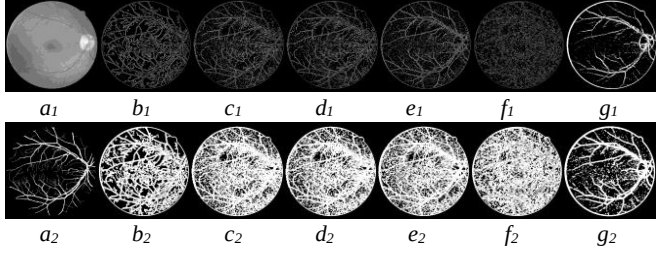


Figure 6. Comparisons of edge detection results of eye bowl image

Figure 7 shows the receiver operating characteristic (ROC) curve of the eye bowl edge detection according to retain different number of significant coefficients. Fig. 7 (a) shows the direct method without dilation. As the most area of the results are detailed, conventional methods are unable to obtain the high TPR (true positive rate) in any case. The proposed TED method obtains 80% of TPR in the case of 20% FPR (false positive rate). Fig. 7 (b) shows the results with dilation operation and the ROC curve is much better than the previous one. All of the methods are significantly improved and the results are quite similar especially in the FPR less than 10%. However, TED method is still significantly better than others when FPR is greater than 10%.

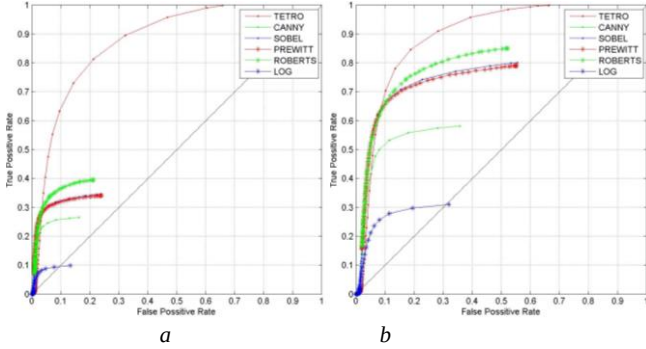


Figure 7. ROC curve of edge detection results of eye bowl image

VI. CONCLUSIONS

The tetrominoes based edge detection method (TED) is proposed in this paper which can highlight the edge detection results in both natural images and medical images. Experimental results achieved better results both in visual performance and objective measurement than the conventional methods.

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VII. APPENDIX

Theorem. Under the constraint $\sum_{n=-N}^{N-1} h(n) = 1$, the impulse response of the edge matching filter is given by

$$h(n) = \begin{cases} \frac{1}{\sqrt{2N}}, & n = 0 \\ \frac{1}{2N + \sqrt{2N}}, & 1 \leq n \leq N \end{cases} \quad (1)$$

which can maximize the value

$$\rho = \frac{h(0)}{\sum_{n=-N}^{N-1} h^2(n)} \quad (2)$$

Which will yield a maximum difference between the outputs at the positions $n = -1$ and $n = 0$ and also make the value of the output noise variance as small as possible.

Proof. Based on the Lagrangian method, we define a function

$$L(h, b) = \frac{h(0)}{\sum_{n=-N}^{N-1} h^2(n)} + b \left(\sum_{n=-N}^{N-1} h(n) - 1 \right) \quad (3)$$

where b is a constant to be determined. Let

$$\frac{\partial L(h, b)}{\partial h(0)} = \frac{\sum_{n=-N}^{N-1} h^2(n) - 2h^2(0)}{\left(\sum_{n=-N}^{N-1} h^2(n) \right)^2} + b = 0 \quad (4)$$

$$\frac{\partial L(h, b)}{\partial h(n)} = \frac{-2h(0)h(n)}{\left(\sum_{n=-N}^{N-1} h^2(n) \right)^2} + b = 0 \quad (5)$$

and

$$\frac{\partial L(h, b)}{\partial b} = \sum_{n=-N}^{N-1} h(n) - 1 = 0 \quad (6)$$

From (5) we can get $h(n)$ is the same for each n . So we set $h(n)$ to be h .

Then we obtain

$$\begin{cases} \frac{(2N-1)h^2 + h^2(0) - 2h^2(0)}{M} + b = 0 \end{cases} \quad (7)$$

$$\begin{cases} \frac{-2h(0)h}{M} + b = 0 \end{cases} \quad (8)$$

$$\begin{cases} (2N-1)h + h(0) - 1 = 0 \end{cases} \quad (9)$$

(7)-(8) to obtain

$$(2N-1)h^2 - h^2(0) + 2h(0)h = 0 \quad (10)$$

It can be drawn that from (9)

$$h(0) = 1 - (2N-1)h \quad (11)$$

Put (11) into (10) to obtain

$$4N^2h^2 - 2Nh^2 - 4Nh + 1 = 0 \quad (12)$$

And solve (12) to get

$$h = \frac{4N \pm \sqrt{8N}}{2(4N^2 - 2N)} \quad (13)$$

$$\text{That is } h = \frac{2N \pm \sqrt{2N}}{4N^2 - 2N} \quad (14)$$

Put (14) into (11) to obtain

$$h(0) = 1 - (2N-1) \frac{2N \pm \sqrt{2N}}{2N(2N+1)} = \mp \frac{1}{\sqrt{2N}} \quad (15)$$

$$\text{We set } h(0) = \frac{1}{\sqrt{2N}} \quad (16)$$

Put (16) into (13) to obtain

$$h = \frac{1 - h(0)}{2N-1} = \frac{1}{2N + \sqrt{2N}} \quad (17)$$

Hence, the theorem is proven.

Set $N=2$, we get

$$h(-2) = \frac{1}{6}, h(-1) = \frac{1}{6}, h(0) = \frac{1}{2}, h(1) = \frac{1}{6} \quad (18)$$

And the corresponding high-pass filter t is

$g-h$

$$\text{where } g(n) = \begin{cases} 1, & n=0 \\ 0, & 1 \leq n \leq N \end{cases} \quad (19)$$

$$t(-2) = -\frac{1}{6}, t(-1) = -\frac{1}{6}, t(0) = \frac{1}{2}, t(1) = -\frac{1}{6} \quad (20)$$



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