

Orthogonal Polynomials Based Complex Gaussian Processes of Nonlinear Power Amplifier for 5G Wireless Communication Systems

Ditthawat Songratthaset* and Suwat Pattaramalai*

*Electronic and Telecommunication Engineering Department,
King Mongkut's University of Technology Thonburi, Thailand
ditthawat.s@mail.kmutt.ac.th, suwat.pat@kmutt.ac.th

Abstract— In this paper, an orthogonal polynomial based complex Gaussian process of nonlinear power amplifier for the filter bank multicarrier modulation (FBMC) systems is proposed. One of the most challenging problems for the FBMC systems is a non-linear distortion caused by a high power amplifier (HPA). Due to the signals for the FBMC communication scenario are modeled as a complex-Gaussian distribution, an analytical expression of the HPA characteristic based on an orthogonal polynomial method by using an upper triangle solution for complex-Gaussian process is derived in this paper. To ensure the robustness of the proposed orthogonal polynomials, the different input distribution such as an exponential and Rayleigh distribution is investigated. In the simulation, a normalized mean squared error (NMSE) and the probability of error performances (BER) in the additive white Gaussian noise (AWGN) channel and the frequency-selective Rayleigh fading channel of the proposed orthogonal polynomial method is determined. Simulation results show that the proposed orthogonal polynomial method significantly outperforms the conventional polynomial method. Furthermore, the proposed orthogonal polynomial based complex-Gaussian input distribution is superior to the exponential, and Rayleigh input distribution in terms of both NMSE and BER performance.

Keywords— Orthogonal Polynomial, Nonlinear Distortion, FBMC, HPA, BER

I. INTRODUCTION

Nowadays, the filter bank multicarrier (FBMC) technique has been proposed as an enhancement to the orthogonal frequency division multiplex (OFDM) technique for 5G future broadband communication systems [1]. Due to the multicarrier techniques-based communication schemes, the FBMC signals exhibit a large peak to average power ratio (PAPR) resulting in a significant nonlinear distortion when the signal is passed through a nonlinear high power amplifier (HPA). This leads to a performance degradation of the system [2]. To overcome the nonlinear distortion

problem, many HPA linearization methods has been proposed such as a Volterra series, the rational function, Wiener-Hammerstein systems, and memory polynomial model [3]. Moreover, the effect of the HPA nonlinear distortions on BER performance in MIMO system and on FBMC-based multi-cellular network are investigated in [4] and [5], respectively. In [6], an analytical expression for bit error rate performance is derived for a nonlinear power amplifier based on the polynomial approximation method. In [7], [8] a theoretical characterization of a soft envelope limiter (SEL), a solid state power amplifier (SSPA), and a travelling wave tube amplifier (TWTA) HPA model has been investigated by using a polynomial fitting method. Although the polynomials method is widely used to model the nonlinear power amplifier, the basic polynomial approximation could not be well suited comparing with other mathematic method [9]. In [10], an orthogonal polynomial-based analytical expression of the HPA characteristic by using an orthogonal polynomial basis function for baseband uniform input distribution is proposed. The results show that the HPA modeling error of the orthogonal polynomial method is superior to the conventional polynomial method. In [11], the theoretical performance analysis of nonlinear power amplifier FBMC systems based on orthogonal polynomial method for baseband uniform, exponential, and Rayleigh input distribution is proposed. The study indicates that the theoretical and simulation results are in good agreement. However, the signals for the FBMC communication scenario are modeled as a

complex-Gaussian distribution. This fact motivates us to analyse the HPA characteristic based on the orthogonal polynomial method by using an orthogonal polynomial basis function for complex-Gaussian process. To ensure the robustness of the proposed orthogonal polynomials, the different input distribution which is not complex-Gaussian is investigated. The contributions of this paper are to propose the HPA characteristic expression based on the orthogonal polynomial method including the complex-Gaussian based-orthogonal polynomial method, the exponential based-orthogonal polynomial method, and the Rayleigh based-orthogonal polynomial method. Moreover, the NMSE and BER performance comparison of the proposed method at different IBO is determined. The organization of this paper is as follows. In section II the FBMC system model, HPA models and the nonlinear HPA characteristics are described. In Section III, we present the derivation of the proposed orthogonal polynomial method. In Section IV and V, the simulation results and the conclusion are given, respectively.

II. SYSTEM MODEL

In this section, the FBMC model and the HPA characteristic of the FBMC systems are described.

A. FBMC Model

Based on the FBMC systems, the input modulated symbol in the frequency domain is converted to the time domain signal and passed through the digital filter in order to suppress the sidelobes of the FBMC signals, resulting in a strictly band limitation. The typically FBMC signal is defined by [11]

$$z(t) = \sum_{p=0}^{P-1} \sum_{q=0}^{N-1} a_{p,q} g\left(t - \frac{pT}{2}\right) e^{\frac{j2\pi qt}{T}} e^{j\varphi_{p,q}} \quad (1)$$

where N is the number of subcarriers, P is the number of symbols and T is a symbol period. $g(t)$ is the time and frequency shifted prototype filter impulse response. $a_{p,q}$ are real-valued symbols at the p th symbol and q th subcarrier. After that, the baseband FBMC signals $z(t)$ is amplified by a nonlinear HPA, which leads to a substantial amount of signal distortion.

B. HPA Model and Nonlinear HPA Characterization

In general, the output signal of the nonlinear HPA $y(t)$ can be modeled by the amplitude modulation/amplitude modulation (AM/AM) characteristic ($F_a(\rho)$) and the amplitude modulation/phase modulation (AM/PM) characteristic ($F_p(\rho)$) of the HPA as [8]

$$\begin{aligned} y(t) &= F_a(\rho) \exp(jF_p(\rho)) \exp(j\theta) \\ &= S(\rho) \exp(j\theta) \end{aligned} \quad (2)$$

where ρ, θ are the signal input modulus and phase, respectively. $S(\rho) = F_a(\rho) \exp(jF_p(\rho))$ is the complex soft envelop of the amplified signal $y(t)$.

According to the Bussgang theorem [12], the nonlinear HPA output $y(t)$ can be written as

$$y(t) = G(t)z(t) + d(t) \quad (3)$$

where $G(t)$ is a complex gain and $d(t)$ is an additive zero mean noise uncorrelated from $z(t)$. For the sake of simplicity, the time variable (t) is discarded. The complex gain G can be determined by

$$G = \frac{1}{2} \mathbb{E} \left[\frac{\partial S(\rho)}{\partial \rho} + \frac{S(\rho)}{\rho} \right] \quad (4)$$

where $\mathbb{E}[\cdot]$ is the expectation operator.

Theoretically, the baseband output $y(t)$ of a polynomial model based on a (quasi-) memoryless HPA can be written as [13]

$$\begin{aligned} y(t) &= \sum_{k=1}^K a_k |z(t)|^{k-1} z(t) \\ &= \sum_{k=1}^K a_k \phi_k(z(t)) \end{aligned} \quad (5)$$

where $\phi_k(z(t)) = |z(t)|^{k-1} z(t)$ is the conventional polynomial basis function. $z(t)$ is the HPA input and K is the polynomial order. a_k is the conventional polynomial coefficient at order k , which can be investigated by the least-squares (LS) scheme [14]. By using the polynomial approximation approach, the complex gain of the polynomial method can be obtained by using the derivation of the moment-generation function (MGF), which is [8]

$$G = a_1 + \sqrt{\frac{\pi}{8}} \sum_{k=2, k \text{ even}}^K (k+1)a_k \sigma^{k-1} \prod_{i=0}^{\frac{k-1}{2}} (2i+1) + \frac{1}{2} \sum_{k=3, k \text{ odd}}^K (k+1)a_k (\sqrt{2}\sigma)^{k-1} \left(\frac{k-1}{2}\right)! \quad (6)$$

Basically, a normalized mean squared error (NMSE) is employed in order to verify the quantitative measure HPA modeling error defined as [15]

$$NMSE(dB) = 10 \log_{10} \left[\frac{\sum_{n=1}^N |y(t_n) - \hat{y}(t_n)|^2}{\sum_{n=1}^N |y(t_n)|^2} \right] \quad (7)$$

where N is the number of input samples. $y(t_n)$ is the measured HPA output and $\hat{y}(t_n)$ is the HPA modeled output.

III. THE PROPOSED METHOD

According to aforementioned researches, the NMSE of the orthogonal polynomial method by using an orthogonal polynomial basis function for baseband uniform input distribution is proposed in [10, 11]. Although the orthogonal polynomial method is superior to the conventional polynomial method, the signals for the FBMC communication scenario are modeled as a complex-Gaussian distribution. In this paper, an analytical expression of the complex gain of HPA characteristic based on an orthogonal polynomial method for the complex-Gaussian, exponential and Rayleigh input distribution is proposed.

A. The Orthogonal Polynomial for Complex-Gaussian Process

The HPA output $y(t)$ based on the orthogonal polynomial method for complex-Gaussian input distribution is modified as

$$y(t) = \sum_{k=1}^K \mu_{2k+1} \varphi_{2k+1}(z(t)) \quad (8)$$

where μ_k is the orthogonal polynomial coefficient at order k , which can be calculated by using the least-squares (LS) solution. K is the polynomial order and φ_k is the orthogonal polynomial basis function with polynomial coefficient order k . To obtain the closed-form expressions for the orthogonal polynomials, an upper triangular matrix

for the complex-Gaussian distribution (Δ) could be applied [16].

Therefore, the new orthogonal polynomial basis function can be examined by

$$\varphi_{2k+1}(z(t)) = \sum_{l=1}^k \Delta_{lk} \varphi_{2l+1}(z(t)) \quad (9)$$

where the upper triangular matrix with k th-order orthogonal polynomial is given by

$$\Delta_{lk} = \sum_{l=1}^k (-1)^{l-k} \frac{\sqrt{l+1}}{(k+1)!} \binom{l}{k}, k \leq l \quad (10)$$

The baseband output $y(t)$ in (8) can be investigated by substituting (10) into (9), which is

$$y(t) = \sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} \varphi_{2l+1}(z(t)) \quad (11)$$

Based on the analytical approach in [8], the HPA output in (11) can be modeled as

$$y(t) = e^{j\theta} \left[\sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} \rho^{2k} \right] = e^{j\theta} S(\rho) \quad (12)$$

where $S(\rho)$ of the orthogonal polynomial expression is

$$S(\rho) = \sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} \rho^{2k} \quad (13)$$

The HPA gain based on the orthogonal polynomial method can be determined by substituting (13) into (4), which is

$$\begin{aligned} G &= \frac{1}{2} \mathbb{E} \left[\frac{\partial}{\partial \rho} \left(\sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} \rho^{2k} \right) + \left(\frac{\sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} \rho^{2k}}{\rho} \right) \right] \\ &= \frac{1}{2} \mathbb{E} \left[\sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} (2k \rho^{2k-1}) + \sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} (\rho^{2k-1}) \right] \\ &= \frac{1}{2} \sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} \mathbb{E} [2k(\rho^{2k-1}) + (\rho^{2k-1})] \quad (14) \end{aligned}$$

Finally, the HPA gain expression based on the orthogonal polynomial method for complex-Gaussian input distribution is given by

$$G = \frac{1}{2} \sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} (2k+1) \mathbb{E}[(\rho^{2k-1})] \quad (15)$$

B. The Orthogonal Polynomial for Exponential Process

To ensure the robustness of the orthogonal polynomials when the input signal is not complex-Gaussian distributed, the exponential based-orthogonal polynomial model is derived. Typically, the expectation is related to the probability density functions (PDF) of the signal. Hence, the expectation term (ρ^{2k}) in (15) for the exponential input distribution can be analysed by using the m th-order moment scheme with its PDF, which is [11]

$$\mathbb{E}[\rho^{2k}] = 2k! \lambda^{2k} \quad (16)$$

Finally, the HPA gain expression of the orthogonal polynomial method for the exponential distribution is mathematically derived by substituting (16) into (15), which is

$$G = \frac{1}{2} \sum_{k=1}^K \mu_{2k+1} \sum_{l=1}^k \Delta_{lk} (2k+1) (2k! \lambda^{2k}) \quad (17)$$

C. The Orthogonal Polynomial for Rayleigh Process

Similarly, the expectation term for the Rayleigh distribution investigated based on the k th-order moment approach is equivalent to [11]

$$\mathbb{E}[\rho^{2k}] = (\sqrt{2}\sigma)^{2k} (2k/2)! + \sqrt{\pi/2} \sigma^{2k} 1 \cdot 3 \cdot 5 \dots k \quad (18)$$

By substituting (18) into (15), the nonlinear HPA gain of the Rayleigh based-orthogonal polynomial model is defined by

$$G = \frac{1}{2} \sum_{k=1}^K \mu_{2k+1} \left[\sum_{l=2, l \text{ even}}^k \Delta_{lk} (2k+1) ((\sqrt{2}\sigma)^{2k} (2k/2)!) + \sum_{l=1, l \text{ odd}}^k \Delta_{lk} (2k+1) \left(\sqrt{\frac{\pi}{2}} \sigma^{2k} 1 \cdot 3 \cdot 5 \dots k \right) \right] \quad (19)$$

IV. SIMULATION RESULTS

In this section, the NMSE and BER performance is investigated in order to evaluate the derived complex gain of the HPA characterization without

noise distortion consideration. In our analysis, the commonly used TWTA model with input saturation voltage $A_{\text{sat}} = 1$ is employed as the HPA output $y(t_n)$ for investigating the NMSE [17]. In the simulations, a total of 5000 input and output samples is employed. For BER performance, the average SNR is ranging from 1–40 dB is applied and the received signal (s) is given by the following equation [11].

$$s = hzG + hd + w \quad (20)$$

where w and h indicate noise component and the channel coefficient, respectively. In the simulation, an AWGN channel and frequency-selective Rayleigh channel are used in the BER investigation. Figure 1, 2 and 3 presents the NMSE comparison between the conventional polynomial method and the orthogonal polynomial method including the Gaussian based-orthogonal polynomial method, the exponential based-orthogonal polynomial method and the Rayleigh based-orthogonal polynomial method with different polynomial order k at IBO = 4, 6, 8, respectively. From Figure 1 at the polynomial order 11, the NMSE of the Gaussian, the exponential and the Rayleigh based-orthogonal polynomial method is equal to -9 dB, -5 dB and -5.5 dB, respectively. For the conventional polynomial method, the NMSE is equal to -2 dB. From the figure, it clearly seen that the NMSE of the orthogonal polynomial method is superior to the conventional polynomial method. Moreover, the NMSE of the Gaussian based-orthogonal polynomial method outperforms the other methods for full polynomial order ranges. Additionally, the NMSE becomes progressively smaller when additional higher order polynomial terms are applied. At the polynomial order 11 as shown in Figure 2, the NMSE of the conventional polynomial method, the exponential and the Rayleigh based-orthogonal polynomial method is equal to -2.7 dB, -6 dB, -6.5 dB, respectively. From Figure 3, the NMSE of the conventional polynomial method, the exponential and the Rayleigh based-orthogonal polynomial method is equal to -4 dB, -7.5 dB and -7.9 dB, respectively. For the Gaussian based-orthogonal polynomial method, it offers the same result at all IBO. As a result, it is clearly seen that the Gaussian based-orthogonal polynomial method provides the lowest NMSE. Moreover, it is worth

noticing that the exponential and the Rayleigh based-orthogonal polynomial method provide lower NMSE when IBO value increase, meanwhile, the Gaussian based-orthogonal polynomial method do not introduce a different result for any IBO values. The BER performance comparison among the proposed orthogonal polynomial method is presented for AWGN channel and frequency-selective Rayleigh channel. In our simulation, the subcarrier's channel is generated following the Jake's model [18]. From Figure 4, it is clearly seen that the BER performance in AWGN channel of the Gaussian based-orthogonal polynomial method is better than that of the exponential and the Rayleigh based-orthogonal polynomial method. Similarly, we could observe that the BER in frequency-selective Rayleigh channel of the proposed the Gaussian based-orthogonal polynomial method yields less distortion than that of the exponential and the Rayleigh based-orthogonal polynomial method for a given SNR as shown in Figure 5.

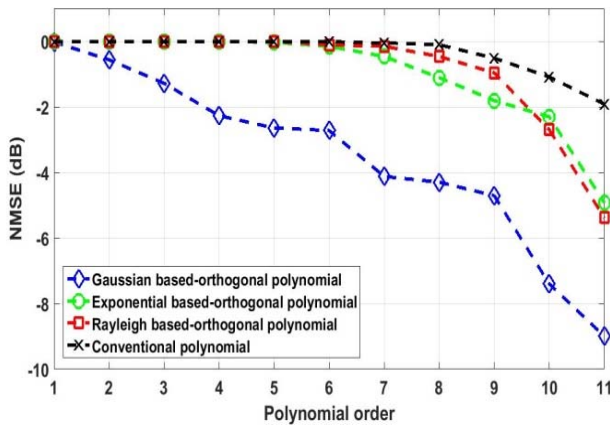


Figure 1. The NMSE comparison at IBO = 4

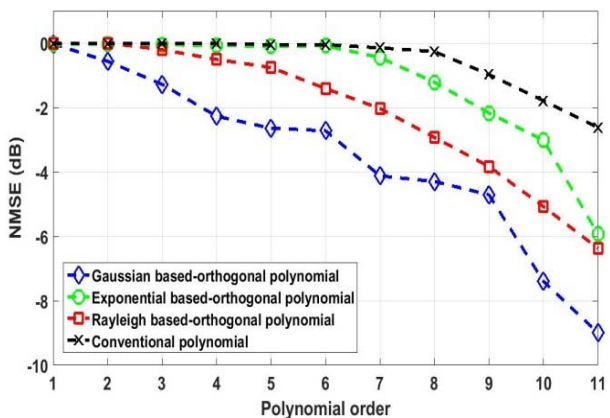


Figure 2. The NMSE comparison at IBO = 6

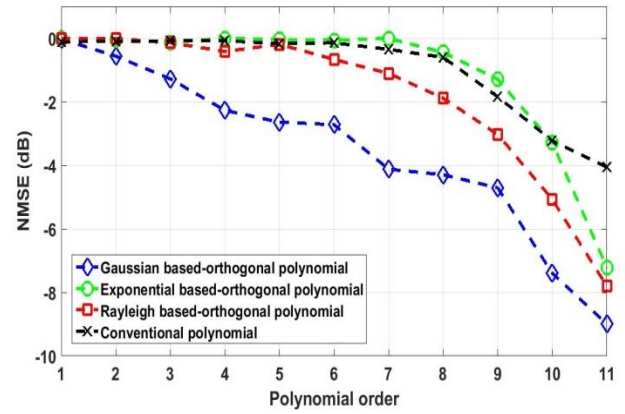


Figure 3. The NMSE comparison at IBO = 8

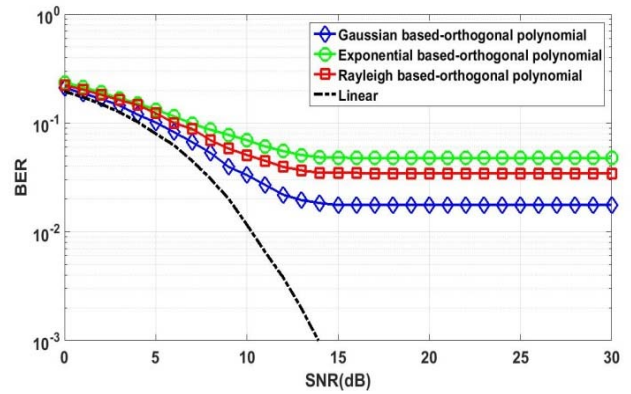


Figure 4. BER performance comparison at IBO=8, AWGN channel

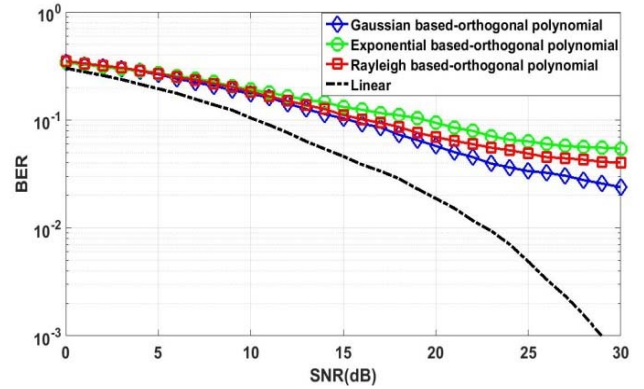


Figure 5. BER performance comparison at IBO=8, frequency-selective Rayleigh channel

V. CONCLUSIONS

In this paper, an analytical expression of the nonlinear power amplifier based on orthogonal polynomials is proposed by using an upper triangular matrix for the complex-Gaussian distribution for 5G wireless communication systems. Moreover, the exponential and Rayleigh based-

orthogonal polynomial method is investigated in order to ensure the robustness of the proposed orthogonal polynomials method. The analytical computation has been investigated through simulation results by using different input back-off (IBO) and the signal to nonlinear noise ratio (SNR). From the simulation result, it is worth noticing that the NMSE of the orthogonal polynomial method is superior to the conventional polynomial method. Moreover, the Gaussian based-orthogonal polynomial method provides the lowest NMSE compared with the exponential and Rayleigh based-orthogonal polynomial method. Additionally, the exponential and the Rayleigh based-orthogonal polynomial method provide lower NMSE when IBO value increase, meanwhile, the Gaussian based-orthogonal polynomial method do not introduce a different result for any IBO values. Furthermore, the BER performance of the Gaussian based-orthogonal polynomial method outperforms the exponential and Rayleigh based-orthogonal polynomial method in both AWGN and frequency-selective Rayleigh channel. In the future work, other stochastic mathematic method could be further studied in order to analyse the non-linear distortion of the HPA in the FBMC systems.

REFERENCES

- [1] B. F. Boroujeny, "Filter bank multicarrier modulation: a waveform candidate for 5G and beyond," *Hindawi journal on Advances in Electrical Engineering*, vol. 2014, December 2014, pp. 1-25.
- [2] D. Qu, S. Lu and T. Jiang, "Multi-block joint optimization for the peak-to-average power ratio reduction of FBMC-OQAM signal," *IEEE Trans. Signal Processing*, vol. 61, no. 7, pp. 1605-1613, Apr. 2013.
- [3] R. Zayani et al., "Adaptive predistortion techniques for non-linearly amplified FBMC-OQAM signals", *Proceeding of IEEE International Conference on Vehicular Technology (VTC Spring)*, Seoul, May 2014, pp. 1-5.
- [4] M. Cherif and R. Bouallegue, "The effect of high power amplifier nonlinearity on MU-Massive MIMO system performance over Rayleigh fading channel," in *15th IEEE Int. Conf. Wireless Communications & Mobile Computing Conference (IWCMC)*, Morocco, Jun. 2019
- [5] B. Elmaroud, A. Faqihi and D. Aboutajdine, "Sensitivity analysis of FBMC-based multi-cellular networks to synchronization errors and HPA nonlinearities," *EURASIP J. Advances in Signal Processing*, DOI: 10.1186/s13634-016-0441-0, Mar. 2017.
- [6] G. U. Maheswari, A. Govindasamy, and S. J. Thiruvengadam, "Performance analysis of filter bank multicarrier system with non-linear high power amplifiers for 5G wireless networks," *IET Signal Processing*, vol. 11, no. 1, February 2017, pp. 66-72.
- [7] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems," *IEEE Trans on Communications*, vol. 48, no. 10, October 2000, pp. 1755-1764.
- [8] H. Shaiek et al. "Analytical analysis of SER for beyond 5G post-OFDM waveforms in presence of high power amplifiers", *IEEE Access*, vol. 7, 2019, pp.29441-29452.
- [9] R. Dallinger et al., "Adaptive pre-distortion techniques based on orthogonal polynomials signals", *Proceeding of IEEE International Asilomar Conference on Systems and Computers (ASILOMAR)*, Canada, November 2010, pp. 1945-1950.
- [10] D. Songratthaset and S. Pattaramalai, "Nonlinear power amplifier characteristic based on orthogonal polynomials method for FBMC systems," in *Proc. 11th IEEE Int. Conf. IFIP Wireless and Mobile Network Conference (WMNC)*, Czech Republic, Sep. 2018, pp. 1-6.
- [11] D. Songratthaset and S. Pattaramalai, "An orthogonal polynomial-based analytical expression of nonlinear power amplifier for FBMC systems," *IEEE Access*, vol. 7, 2019, pp.107072-107082.
- [12] H. E. Rowe, "Memoryless Nonlinearity with Gaussian Inputs: Elementary Results," *Bell Syst. Tech. J.* vol.61, no.7, pp. 1519-1526, Sep. 1982.
- [13] R. Raich et al, "Orthogonal polynomials for power Amplifier modeling and predistorter design," *IEEE Trans. on Vehicular Technology*, vol. 53, no. 5, September 2004, pp. 1468-2797.
- [14] G. U. Maheswari, et al, "Performance analysis of filter bank multicarrier system with non-linear high power amplifiers for 5G wireless network," *IET J. Advances in Signal Processing*, vol. 11, no. 1, pp. 66-72, Feb. 2017.
- [15] R. Raich, H.Qian and G.T. Zhou, "Orthogonal polynomials for power amplifier modeling and predistorter design," *IEEE Trans. Vehicular Technology*, vol. 53, no. 5, pp. 1468-1479, Sep. 2004.
- [16] R. Raich et al, "Orthogonal polynomial for complex Gaussian process," *IEEE Trans. Signal Processing*, vol. 52, no. 10, pp. 2788-2796, Oct. 2004.
- [17] A. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT Amplifiers," *IEEE Trans. on Communication*, vol.29, 1981, pp.1715-1720.
- [18] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005



Ditthawat Sonratthaset (M'19) received the M.Econ. degree from Kasetsart University, Bangkok, Thailand, in 2005. He is currently pursuing the Ph.D. degree with the Department of Electronic and Telecommunication Engineering, King Mongkut's University of Technology Thonburi, Thailand.



Suwat Pattaramalai received the B.Eng. degree from Chulalongkorn University, Bangkok, Thailand, in 1992, and the M.Eng. and Ph.D.Eng degrees from Florida Atlantic University, in 1996 and 2007, respectively. He is currently a Lecturer with the Electronic and Telecommunication Engineering Department, King Mongkut's University of Technology Thonburi, Thailand.