

Day-Ahead Electricity Market State-Space Model and Its Power Production, Demand and Price Forecasting Algorithm Using H-infinity Filter

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Abstract—Development of an electricity market model is very important step of forecasting power of generators and client demand. This paper proposes a day-ahead state-space power system model which is obtained by a set of partial differential equations. After simplifications, the 4th order user-friendly state-space power system model is obtained where the measurements are obtained by a set of sensors. Secondly, we proposed an H-infinity based power system states forecasting algorithm where process and measurement noise covariances are not need to know. In each iteration, the residual error between true and forecasted states are minimised lead to an accurate forecasted system states. Numerical simulation illustrates that the proposed scheme can able to forecast the system states within 1-12 seconds.

Index Terms—Day-ahead electricity market, energy consumption, H-infinity filter, price forecasting, power generation, power market, state-space electric market.

I. INTRODUCTION

An electricity market is a real-time place of high revenue to the investors where they are used different algorithms for predicting the market conditions. To begin with, the extended Kalman filter (EKF) for the Japan electricity market price forecasting is proposed in [1]. In this approach, a time-varying autoregressive model is adopted for price forecasting [2]. Moreover, the derivative-free EKF scheme is used for price forecasting of electricity market [3], [4], [5]. It uses the Black-Scholes electricity market model for price forecasting. However, the forecasting algorithm requires a transformation of the initial price dynamics into a state-space model of the linear canonical form. Generally speaking, the electricity market assets value and volatility are not directly measurable [6]. Therefore, by forecasting the electricity market value of the utility company, it becomes possible to forecast the firm's distance to default [6]. Then the forecasted electricity market value of the utility company is used to determine bankruptcy risk and the probability of default [6].

Furthermore, the day-ahead electricity price forecasting using the KF is presented in [7]. In this scheme, the state transition matrix of the electricity market model is obtained using the regression approach. Unfortunately, the training data is based on the America PJM historical data which may be

unreliable or inaccurate [8]. Additionally, the H-infinity and KF algorithms are adopted for short-term electricity market price forecasting where the system model is designed based on regression analysis using the California independent system operator (ISO) data [9], [10]. For short-term electricity price forecasting, the unscented KF is used in [11], [12]. For application point of view, the KF algorithm is used for cyber physical systems [13], [14]. The reliable and secure communication requirement of smart grid is presented in [15], [16].

In order to deal uncertainty, the Bayesian learning process is used [17], where two-player stochastic games are utilised. When the system parameters are frequently varies over time, this method cannot perfectly predict the system states [18]. To address the uncertainties of electricity consumption and renewable energy output power [19], a two-stage optimization algorithm is proposed and verified. The occupational complexity of this approach is very high, and the linearization is very difficult. For prediction of wind speed and solar power, long short-term memory network with peephole and wavelet decomposition is presented in [20], [21]. It provides an effective forecasting technique to predict the wind farm considering wind speed and solar power as uncertainties. To remove uncertainty in the electricity load data, an empirical mode decomposition based extreme learning technique is proposed and verified. It is very difficult to find suitable active functions such as tanh, sigmoid, softmax, and scaled exponential linear unit in this method [22]. For house hold energy consumption, an intelligent data mining model is proposed to identify energy consumption patterns [23]. The uncertainty is not considered in this time-series data. For short term load clustering, autoregressive integrated moving-average model based support vector regression approach is proposed and verified [24]. A hybrid machine learning technique based on linear regression and random forest is developed in [25].

From the machine learning point of view, the multi-layer back-propagation and neuro fuzzy schemes are adopted for predicting electricity market clearing price [26], [27]. Combining with the support vector machine and wavelet transform, the designed scheme is used for predicting the electricity

price. For day-ahead electricity price forecasting, a deep leaning based forecasting algorithm is developed in [28]. The method requires significant amount of training and testing data. Moreover, the Kalman filter based load-forecasting algorithm is presented in [29]. The algorithm is applied to the virtual market where state-space model is assumed to be identity matrix. The mixed-integer linear programming based energy scheduling algorithm for microgrid is presented in [30]. It requires linearization of nonlinear wind turbine model, and the algorithm is high computational intensive. Likewise, energy forecasting using the long short-term memory and gated recurrent unit (deep neural network staffs) is proposed in [31], [32]. Using spark cluster, the algorithm is tested, and it shows that the gated recurrent unit provides better load forecasting accuracy compared with the long short-term memory. Finally, evolutionary ensemble long short-term memory machine learning method is proposed for customer peak electricity demand prediction [33], [34]. Basically, the testing, training, and validation datasets are used in this work based on empirical experimental results.

This paper proposes a 4th order power system state-space model, and an optimal H-infinity algorithm is designed for forecasting it system states. The key contributions of the article are emphasized as follows:

- Proposed a day-ahead state-space power system model which is obtained by a set of partial differential equations. After simplifications, the 4th order user-friendly state-space power system model is obtained where the measurements are obtained by a set of sensors.
- Proposed an H-infinity based power system states forecasting algorithm where process and measurement noise covariances are not need to know. In each iteration, the residual error between true and forecasted states are minimised lead to an accurate forecasted system states.
- Numerical simulation is conducted considering proposed model, and it shows that the proposed scheme can able to forecast the system states within 1-12 seconds.

II. POWER SYSTEM STATE-SPACE REPRESENTATION

For power generation and load forecasting, it requires a suitable state-space framework. This model is obtained through a set of differential equations, which involve power generation, demand and price. It involves some controlling parameters. According to Alvarado scheme, the differential equations of electricity market are given by [35], [5], [15]:

$$\dot{P}_g = (\lambda - c_g P_g - h P_e - b_g) / \tau_g. \quad (1)$$

$$\dot{P}_d = (c_d P_d + b_d - \lambda) / \tau_d. \quad (2)$$

$$\dot{P}_e = P_g - P_d. \quad (3)$$

$$\dot{\lambda} = -P_e / \tau_\lambda. \quad (4)$$

Here, P_g , P_d , and P_e are the generated power, demand power, and difference between power supply and demand, λ is the electricity price, τ_g , τ_d , τ_λ are the supply, demand, and price controlling parameters, c_g , b_g , c_d , and b_d are the time constants related to power supply and consumption, and $h P_e$ is the

additional cost paid by the supplier when there is excess supply or demand [36], [37].

After simplifications, it can be written as follows:

$$\dot{\mathbf{x}} = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u}. \quad (5)$$

Here, $\mathbf{x} = [P_g, P_d, P_e, \lambda]'$ is the power system states, $\mathbf{u} = [b_g, b_d, 0, 0]'$ is the system input, the state matrix $\mathbf{A}_c$, and the system input matrix $\mathbf{B}_c$ are given

$$\text{by: } \mathbf{A}_c = \begin{bmatrix} -c_g/\tau_g & 0 & -h/\tau_g & 1/\tau_g \\ 0 & c_d/\tau_d & 0 & -1/\tau_d \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1/\tau_\lambda & 0 \end{bmatrix}, \quad \mathbf{B}_c = \text{diag}[-1/\tau_g, 1/\tau_d, 0, 0].$$

Using sampling interval μ , (5) can be written as follows:

$$\mathbf{x}(t+1) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{n}(t). \quad (6)$$

Here, $\mathbf{A} = \mathbf{I} + \mu \mathbf{A}_c$, $\mathbf{B} = \mu \mathbf{B}_c$, t is the discrete time, and $\mathbf{n}$ is the process noise whose covariance is $\mathbf{Q}$. The measurements from the independent system operator are obtained as follows:

$$\mathbf{y}(t+1) = \mathbf{C} \mathbf{x}(t) + \mathbf{w}(t). \quad (7)$$

Here, $\mathbf{H}$ is the measurement matrix, and $\mathbf{w}$ is the measurement noise with $\mathbf{R}$ is the covariance matrix.

III. H-INFINITY ALGORITHM FOR POWER SYSTEM STATES FORECASTING

H-infinity is an iterative algorithm to forecast the power system states without know the noise statistics. The following are the steps for H-infinity forecasting process [38], [39]:

$$\hat{\mathbf{x}}(t+1) = \mathbf{A} \hat{\mathbf{x}}(k) + \mathbf{B} \mathbf{u}(t) + \mathbf{G}(t)[\mathbf{y}(t) - \mathbf{C} \hat{\mathbf{x}}(t)]. \quad (8)$$

Here, $\hat{\mathbf{s}}(t)$ and $\hat{\mathbf{s}}(t+1)$ are the prior and posterior state freacsting, and the H-infinity filter gain $\mathbf{G}(t)$ is given by:

$$\mathbf{G}(t) = \mathbf{P}(t)[\mathbf{I} - \theta \bar{\mathbf{M}}(t) \mathbf{P}(t) + \mathbf{C}' \mathbf{R}^{-1}(t) \mathbf{C} \mathbf{P}(t)]^{-1} \mathbf{C}' \mathbf{R}^{-1}(t). \quad (9)$$

Here, $\mathbf{P}(t)$ is the prior error covariance, θ is the user defined bound and $\bar{\mathbf{M}}(t) = \mathbf{L}(t) \mathbf{M}(t) \mathbf{L}'(t)$, where $\mathbf{L}(t)$ as well as $\mathbf{M}(t)$ are the user defined performance variables. The forecasted error covariance matrix $\mathbf{P}(t+1)$ is determined by:

$$\mathbf{P}(t+1) = \mathbf{A} \mathbf{P}(t)[\mathbf{I} - \theta \bar{\mathbf{M}}(t) \mathbf{P}(t) + \mathbf{C}' \mathbf{R}^{-1}(t) \mathbf{C} \mathbf{P}(t)]^{-1} \mathbf{A}' + \mathbf{Q}(t). \quad (10)$$

In each iteration, the following condition is hold:

$$\mathbf{P}^{-1}(t) - \theta \bar{\mathbf{M}}(t) + \mathbf{C}' \mathbf{R}^{-1}(t) \mathbf{C} > \mathbf{0}.$$

Using the state-space model of power system, the performance of the H-infinity forecasting algorithm is demonstrated in the following section.

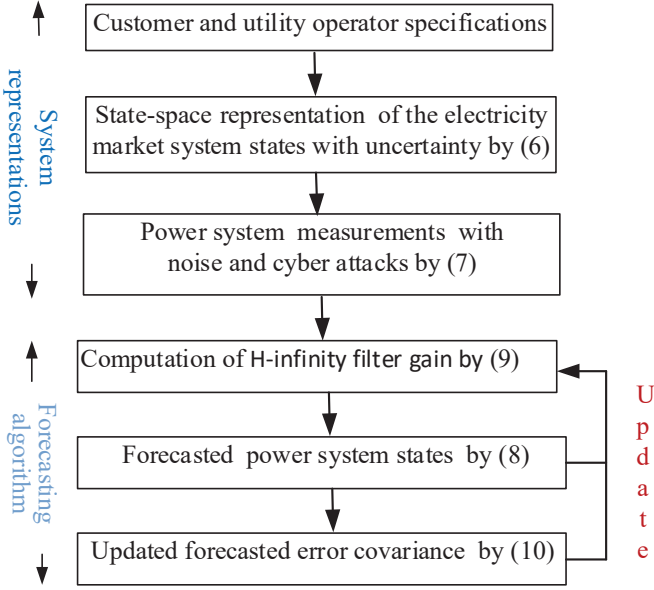


Fig. 1: Step by step process of the proposed H-infinity based power system states forecasting algorithm.

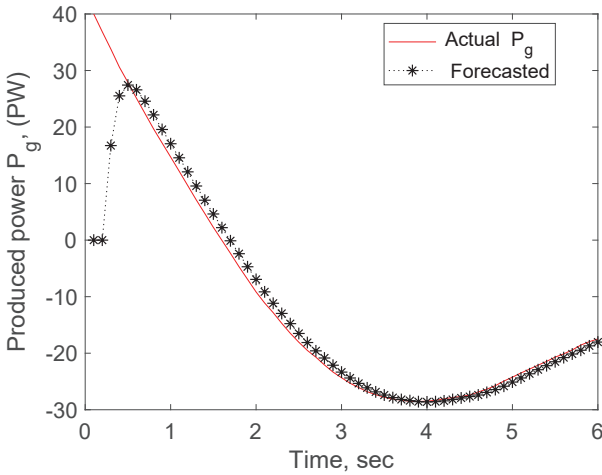


Fig. 2: Actual power production and it forecasting one.

IV. NUMERICAL RESULTS: ACTUAL AND FORECASTED SYSTEM STATES

The numerical simulation is conducted using state-space model with and without cyber attacks. The Matlab is used for simulation. Overall, it can be seen that the proposed algorithm can forecast the power system states within 1-12 seconds. The whole simulation process is described in Fig. 1. After simplification the power system differential equations (1)-(4), the state-space framework is obtained in (6), which is polluted by uncertainty. The measurements in (7) are obtained by a set of sensors, which are manipulated by uncertainty. Based on the mean squared error principle, the H-infinity gain is computed by (9). Basically, it can minimize the error dynamic leads

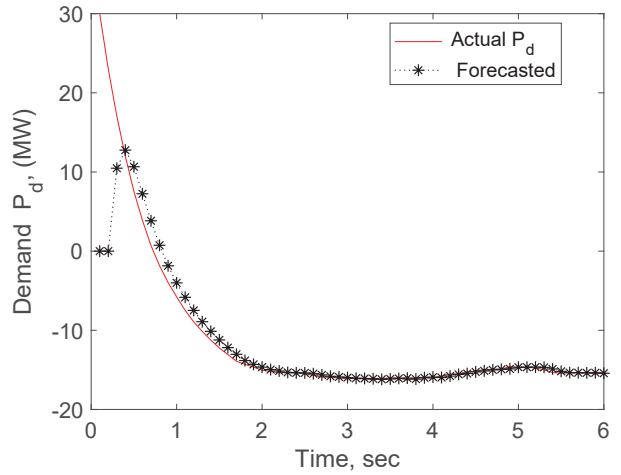


Fig. 3: Actual power demand and it forecasting one.

TABLE I: Numerical simulation parameters.

Symbols	Values	Symbols	Values
τ_g	0.3	c_g	0.2
b_g	3	τ_d	0.2
c_d	-0.3	b_d	11
τ_λ	110	h	0.2
$\mathbf{Q}$	$0.01 \cdot \mathbf{I}$	$\mathbf{R}$	$0.012 \cdot \mathbf{I}$
μ	0.1 sec		

to an accurate forecasted system states. Afterwards, the final forecasted state is obtained through (8). Lastly, the estimated error covariance is computed by (10). In each iteration, this error dynamic is minimised leads to an accurate forecasted system states.

For simulation, the numerical parameters are described in Table I [35], [15]. In this table, the process and measurement noises are follow Gaussian distributions with covariances are $10^{-2}\mathbf{I}$ and $12 \times 10^{-2}\mathbf{I}$. The sampling time is μ is 0.1 seconds. The other power generation, consumption and price controlling parameters are described in this Table.

Based on the noisy measurements, the numerical results are graphically illustrated in Figs. 2-4. Basically, Fig. 2 demonstrates the power production versus it forecasted state. Clearly, The proposed scheme can able to properly forecast this state within 3 seconds. Because, the developed algorithm can find suitable gain to minimise error dynamic. Furthermore, Fig. 3 shows the actual power demand versus predicted one. Obviously, it needs around 2 seconds to properly know the power demand. Finally, the electricity price and its predicted state is presented in Fig. 4. Most importantly, the proposed algorithm provides consistency forecasting results over time.

V. CONCLUSION AND FUTURE WORK

Development of an electricity market model is very important step of forecasting power of generators and client demand. This paper proposes an electricity market state-space model where measurements are obtained by a set of

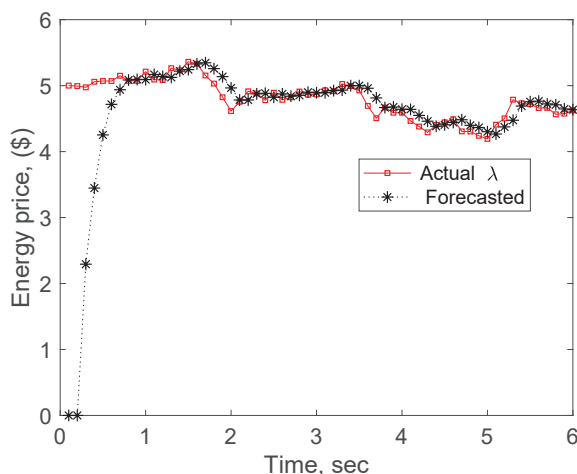


Fig. 4: Actual price and it forecasting one.

sensors. These measurement units are corrupted by noises. Then the power consumption, generation, and price forecasting algorithm is proposed. The developed algorithm is predicted and corrected the electricity market system states over time. Numerical results show that the proposed method can properly forecast electricity market states within 1-12 seconds. This kind of system model and method can assist to develop the electricity market simulator and help investor to participate energy economy area. In future, we will develop distributed electricity market state forecasting algorithm considering cyber attacks and packet loss [40], [41], [42].

REFERENCES

- [1] K. Ofuji and S. Kanemoto, "Price forecasting of Japan electric power exchange using time-varying AR model," in *Proc. of the International Conference on Intelligent Systems Applications to Power Systems*, 2007, pp. 1–6.
- [2] J. Lago, F. De Ridder, P. Vrancx, and B. De Schutter, "Forecasting day-ahead electricity prices in Europe: The importance of considering market integration," *Applied energy*, vol. 211, pp. 890–903, 2018.
- [3] G. Rigatos, "A Kalman filtering approach to the detection of option mispricing in electric power markets," in *Proc. of the Symposium on Computational Intelligence Applications in Smart Grid*, 2014, pp. 1–6.
- [4] G. G. Rigatos, "Kalman filtering approach to the detection of option mispricing in elaborated PDE finance models," in *State-Space Approaches for Modelling and Control in Financial Engineering*, 2017, pp. 141–152.
- [5] M. M. Rana and R. Bo, "Kalman filter based electricity market states forecasting: A state-space framework," in *Proc. of the Annual IEEE International Conference on CYBER Technology in Automation, Control, and Intelligent Systems*, to appear in 2019.
- [6] G. Rigatos and P. Siano, "Forecasting of power corporations' default probability with nonlinear Kalman filtering," *IEEE Systems Journal*, vol. 12, no. 2, pp. 1099–1107, 2018.
- [7] L. Ding and Q. Ge, "Electricity market clearing price forecast based on adaptive Kalman filter," in *Proc. of the Symposium on International Conference on Control, Automation and Information Sciences*, 2018, pp. 417–421.
- [8] S. Nikam, L. Ragha, and S. Kulkarni, "Neuro fuzzy approach for financial forecasting," *International Journal of Research in Advent Technology*, vol. 1, no. 2, pp. 49–65, 2013.
- [9] A. A. Arabali, E. Chalko, M. Etezadi-Amoli, and M. S. Fadali, "Short-term electricity price forecasting," in *Proc. of the Proc. of the Power & Energy Society General Meeting*, 2013, pp. 1–5.
- [10] Y. Okawa and T. Namerikawa, "Dynamic electricity pricing via the H-infinity control considering uncertainties in market participants behavior," in *Proc. of the European Control Conference*, 2015, pp. 1064–1069.
- [11] S. K. Dash, R. Bisoi, and P. Dash, "A hybrid functional link dynamic neural network and evolutionary unscented Kalman filter for short-term electricity price forecasting," *Neural Computing and Applications*, vol. 27, no. 7, pp. 2123–2140, 2016.
- [12] R. A. Chinnathambi, A. Mukherjee, M. Campion, H. Salehfar, T. Hansen, J. Lin, and P. Ranganathan, "A multi-stage price forecasting model for day-ahead electricity markets," *Forecastinsr*, vol. 1, no. 1, pp. 26–46, 2018.
- [13] M. M. Rana, L. Li, and S. W. Su, "Distributed dynamic state estimation considering renewable generation and packet losses," in *Proc. of the International Conference on Control, Automation, Robotics and Vision*, 2016, pp. 1–6.
- [14] Z. Xing, Y. Xia, L. Yan, K. Lu, and Q. Gong, "Multisensor distributed weighted Kalman filter fusion with network delays, stochastic uncertainties, autocorrelated, and cross-correlated noises," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 5, pp. 716–726, 2018.
- [15] H. Li, L. Lai, and W. Zhang, "Communication requirement for reliable and secure state estimation and control in smart grid," *IEEE Transactions on Smart Grid*, vol. 2, no. 3, pp. 476–486, 2011.
- [16] M. M. Rana and L. Li, "An overview of distributed microgrid state estimation and control for smart grids," *Sensors*, vol. 15, no. 2, pp. 4302–4325, 2015.
- [17] H. Ding, C. Di, and L. Shenghong, "Bayesian method-based learning automata for two-player stochastic games with incomplete information," in *International Conference in Communications, Signal Processing, and Systems*. Springer, 2018, pp. 25–33.
- [18] X. Ren, X. Shao, and S. Li, "Stochastic optimization based on principal-agent problem," in *International Colloquium on Computing, Communication, Control, and Management*, vol. 2, 2009, pp. 176–179.
- [19] O. D. Melgar-Dominguez, M. Pourakbari-Kasmaei, and J. R. S. Mantovani, "Adaptive robust short-term planning of electrical distribution systems considering siting and sizing of renewable energy based dg units," *IEEE Transactions on Sustainable Energy*, vol. 10, no. 1, pp. 158–169, 2018.
- [20] T. Yang, H. Wang, S. Aziz, H. Jiang, and J. Peng, "A novel method of wind speed prediction by peephole LSTM," in *International Conference on Power System Technology (POWERCON)*, 2018, pp. 364–369.
- [21] M. Chai, F. Xia, S. Hao, D. Peng, C. Cui, and W. Liu, "PV power prediction based on LSTM with adaptive hyperparameter adjustment," *IEEE Access*, vol. 7, pp. 115473–115486, 2019.
- [22] M. M. Ali, M. I. Babar, M. Hamza, M. Jehanzeb, S. H. Habib, and M. S. Khan, "Industrial financial forecasting using long short-term memory recurrent neural networks," vol. 10, no. 4, 2019, pp. 88–99.
- [23] S. Singh and A. Yassine, "Big data mining of energy time series for behavioral analytics and energy consumption forecasting," *Energies*, vol. 11, no. 2, p. 452, 2018.
- [24] K. Gajowniczek and T. Zabkowski, "Simulation study on clustering approaches for short-term electricity forecasting," *Complexity*, vol. 2018, 2018.
- [25] L. Massidda and M. Marrocu, "Smart meter forecasting from one minute to one year horizons," *Energies*, vol. 11, no. 12, p. 3520, 2018.
- [26] L. Zhang, F. Wang, B. Xu, W. Chi, Q. Wang, and T. Sun, "Prediction of stock prices based on LM-BP neural network and the estimation of overfitting point by RDCI," *Neural Computing and Applications*, vol. 30, no. 5, pp. 1425–1444, 2018.
- [27] A. Kablan, "Adaptive neuro fuzzy inference systems for high frequency financial trading and forecasting," in *Proc. of the International Conference on Advanced Engineering Computing and Applications in Sciences*, 2009, pp. 105–110.
- [28] A. Brusaferrri, M. Matteucci, P. Portolani, and A. Vitali, "Bayesian deep learning based method for probabilistic forecast of day-ahead electricity prices," *Applied Energy*, vol. 250, pp. 1158–1175, 2019.
- [29] Z. Zheng, H. Chen, and X. Luo, "A kalman filter-based bottom-up approach for household short-term load forecast," *Applied Energy*, vol. 250, pp. 882–894, 2019.
- [30] A. A. Bashir, M. Pourakbari-Kasmaei, J. Contreras, and M. Lehtonen, "A novel energy scheduling framework for reliable and economic operation of islanded and grid-connected microgrids," *Electric Power Systems Research*, vol. 171, pp. 85–96, 2019.

- [31] S. Kumar, L. Hussain, S. Banarjee, and M. Reza, "Energy load forecasting using deep learning approach-LSTM and GRU in spark cluster," in *Proc. of the International Conference on Emerging Applications of Information Technology*, 2018, pp. 1–4.
- [32] C. Monteiro, I. Ramirez-Rosado, L. Fernandez-Jimenez, and P. Conde, "Short-term price forecasting models based on artificial neural networks for intraday sessions in the Iberian electricity market," *Energies*, vol. 9, no. 9, p. 721, 2016.
- [33] S. Ai, A. Chakravorty, and C. Rong, "Evolutionary ensemble LSTM based household peak demand prediction," in *Proc. of the International Conference on Artificial Intelligence in Information and Communication*, 2019, pp. 1–6.
- [34] —, "Household power demand prediction using evolutionary ensemble neural network pool with multiple network structures," *Sensors*, vol. 19, no. 3, p. 721, 2019.
- [35] J. Nutaro and V. Protopopescu, "The impact of market clearing time and price signal delay on the stability of electric power markets," *IEEE Transactions on power systems*, vol. 24, no. 3, pp. 1337–1345, 2009.
- [36] M. Roozbehani, M. A. Dahleh, and S. K. Mitter, "Volatility of power grids under real-time pricing," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1926–1940, 2012.
- [37] H. Li, A. Dimitrovski, J. B. Song, Z. Han, and L. Qian, "Communication infrastructure design in cyber physical systems with applications in smart grids: A hybrid system framework," *IEEE Communications Surveys & Tutorials*, vol. 16, no. 3, pp. 1689–1708, 2014.
- [38] M. Rana, "Architecture of the internet of energy network: An application to smart grid communications," *IEEE Access*, vol. 5, pp. 4704–4710, 2017.
- [39] D. Simon, *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. New Jersey: John Wiley and Sons, 2006.
- [40] Y. Zhang, L. Wang, W. Sun, R. C. Green II, and M. Alam, "Distributed intrusion detection system in a multi-layer network architecture of smart grids," *IEEE Transactions on Smart Grid*, vol. 2, no. 4, pp. 796–808, 2011.
- [41] T. Irita and T. Namerikawa, "Decentralized fault detection of multiple cyber attacks in power network via Kalman filter," in *Proc. of the European Control Conference*, 2015, pp. 3180–3185.
- [42] Y. Okawa and T. Namerikawa, "Slam problem via H-infinity filter with compensation for intermittent observation," in *Proc. of the International Workshop on Advanced Motion Control*, 2014, pp. 386–391.



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