

Power Allocation for Multi-Relay AF Cooperative System with Maximum System Capacity

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Abstract— In order to maximize the capacity of the cooperative communication system, a power allocation scheme for multi-relay AF cooperative system based on blind relays is presented in this paper. Under the premise that the source power is known, the power allocation problem is modeled to maximize the capacity of the system and transformed into the standard convex optimization. The power allocation factors of each relay is obtained by solving the Karush-Kuhn-Tucker (KKT) condition. Finally, the capacity and outage probability of the proposed scheme are provided for the proposed power allocation scheme. Compared with the typical equal power allocation, the proposed convex optimization-based power allocation scheme achieves better performance.

Keywords— Cooperative communication; Power allocation; Amplify-and-forward; Convex optimization theory; Maximize capacity

I. INTRODUCTION

Nowadays, cooperative communication systems have attracted the great attentions for their attained better performance gains compared with direct transmission. Therefore, it is considered as one of the most important techniques for 5G wireless communications [1]. In recent years, different cooperative relay strategies, such as amplify-and-forward (AF) and decode-and-forward (DF) protocol [2], have been proposed. In the AF protocol, the information at the relay node is amplified before being forwarded to the destination node. While in the DF protocol, the coded information sent to each relay node is firstly decoded, then re-encoded and forwarded. Previous research works have studied different relay selection schemes for cooperative communication systems, such as hybrid relay selection (HRS). Its basic idea is to choose the best relay in either AF or DF protocols. The selection of the best relay is commonly based on the signal-to-noise ratio (SNR) [3]-[4]. Moreover, it has been shown that increasing the number of the relays can improve the diversity gains and offer more flexibility of the network. However, some new problems arise [5], such as the optimal relay selection and power allocation for the relays.

Power allocation is a main issue in cooperative communication system design and many power allocation strategies have been studied for different cooperative transceiver structures and protocols. The authors in [6] studied

a simple power allocation scheme for a single relay AF cooperative communication system that minimizes the outage performance. In [7], a power allocation scheme has been derived for dual-hop AF network with an arbitrary number of relays over the Rayleigh fading channel. A power allocation technique has been proposed for a multi-hop cooperative relay network and the network scenarios based on the location of the relays in the network have been briefly discussed in [8]. The authors in [9] proposed a power allocation strategy for AF relay which requires only the knowledge of the mean fading coefficients of the channels, aiming to optimize the outage performance and achieve the maximum SNR gains to some extent. The distributed power allocation strategy for parallel relay networks with limited channel state information (CSI) at the source and the relay nodes were studied in [10], where a distributed decision mechanism for each relay was presented to individually decide the cooperative schemes. An optimal power allocation scheme with two kinds of CSI, complete CSI and statistical CSI of the relay nodes, in conventional AF scheme were considered [11]. The statistical power allocation scheme for MIMO cooperative scheme was introduced in [12] based on the opportunistic relaying [13]. In [14], two power allocation schemes for a multi-user AF wireless network were proposed, where the source and the destination appeared in pairs. In [15], a relay selection and power allocation scheme is proposed to minimize the system outage probability by maximizing the system capacity for each channel realization with power constraint. [16] extends the conventional power allocation scheme to multi-user wireless cooperative system. [17] investigated the simultaneous wireless information and power transfer (SWIPT) amplify-and-forward relay systems, in which the relay nodes are equipped with radio frequency energy harvesters. Both the power splitting (PS) ratio and adaptive power control are jointly optimized for the SWIPT multiple-relay systems. [18] proposed the power allocation strategy of the cooperative relay networks with non-orthogonal multiple access (NOMA). The above works mainly consider the single user scenario with multiple relays or the selection of certain single relay for cooperation to simplify the system analysis.

This paper introduces a power allocation scheme with multiple relay nodes on the premise of known source power to

maximize the system capacity based on the convex optimization of the optimal relay power allocation scheme. The capacity and the outage probability of the proposed scheme are provided and compared with the popular equal power allocation scheme. The simulation results show that the system performance of the proposed power allocation scheme based on convex optimization is better than that based on the equal power allocation scheme.

The rest of this paper is organized as follows. Section II comprises of multi-relay AF cooperative system model. Section III explains the optimal power allocation scheme based on convex optimization. Section IV contains simulation results and comparative analysis between equal power allocation scheme and the optimal power allocation scheme. Finally, Section V concludes this paper.

II. MULTI-RELAY AF COOPERATIVE SYSTEM MODEL

In this section, we consider an AF cooperative system which consists of a source node (S), a destination node (D) and M relay nodes ($R_1, R_2, \dots, R_M$), as shown in Fig. 1. Each node only has equipped with one antenna and the channel between any two nodes follows a flat Rayleigh fading allocation. The channel fading coefficients from the source node to the destination node, from the source node to the i -th relay node, and from the i -th relay node to the destination node can be presented as h_{sd} , h_{si} , and h_{id} , respectively.

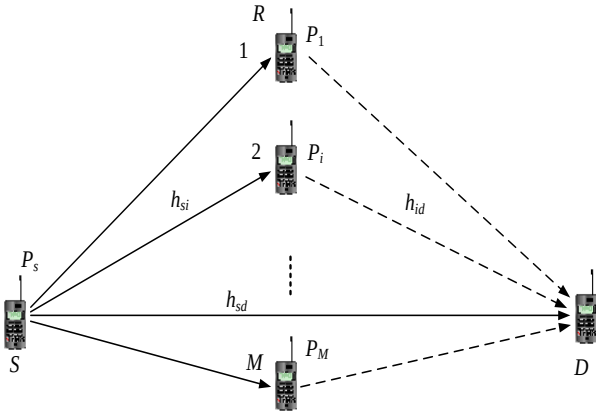


Figure 1. Multi-relay AF cooperative system model

Each transmission process in the multi-relay AF cooperative system can be divided into two time slots. At the first time slot, the source sends the information signal to the destination node and M relay nodes work in the form of broadcast. Therefore, the information received at the i -th relay and the destination node can be written respectively as

$$y_i = \sqrt{P_s} h_{si} x_s + n_{si} \quad (1)$$

$$y_{d1} = \sqrt{P_s} h_{sd} x_s + n_{sd} \quad (2)$$

where P_s is the power of the transmitted signal at the source, and x_s is the signal from the source. n_{sd} and n_{si} are the additive white Gaussian noise (AWGN) of the destination node and the relay node, respectively, which are independent of each other and have the same distribution as $N_0(0, 1)$.

At the second time slot, the relay will adopt AF protocol to forward the received signal. Therefore, the destination node receives the transmitted signal from the relay i as follows

$$\begin{aligned} y_{d2} &= \sqrt{P_i} h_{id} A y_i + n_{id} \\ &= \sqrt{P_s P_i} A h_{si} h_{id} x_s + \sqrt{P_i} A h_{id} n_{si} + n_{id} \end{aligned} \quad (3)$$

where P_i is the power of the i -th relay, A is the amplification factor of the relay with $A = \sqrt{1/(CN_0)}$, C is a constant, and n_{id} is the AWGN which also has the same distribution and independent of the AWGN of the destination node and the relay.

Finally, the destination node merges the information of both time slots as

$$y = y_{d1} + y_{d2} \quad (4)$$

The SNR of the destination can be expressed as

$$\gamma_e = \sum_{i=1}^M \frac{\gamma_{1i} \gamma_{2i}}{\gamma_{2i} + C} \quad (5)$$

where $\gamma_{1i} = \frac{P_s h_{si}^2}{N_0}$, $\gamma_{2i} = \frac{P_i h_{id}^2}{N_0}$ are the instantaneous SNR from the source to the relay and from the relay to the destination node, respectively.

III. OPTIMAL POWER ALLOCATION SCHEME BASED ON CONVEX OPTIMIZATION

In this section, we discuss the optimal power allocation scheme at the relays. Here, we use the convex optimization to derive the optimal relay power allocation on the basis of ensuring the maximum system capacity.

According to the available knowledge of the destination SNR, the capacity of the system can be expressed as

$$I = \frac{1}{2} \log_2 (1 + \gamma_e) \quad (6)$$

Assuming the power P_s of the source node and total power P are already known, the power at the relay is set as $P_r = P - P_s$. Therefore, the problem of the power allocation focuses on how to distribute the power P_r among the relay nodes to maximize the capacity of the system. In the following, we provide the detailed derivation process.

Since maximizing the system capacity is equivalent to maximizing the SNR, the mathematical model of the power allocation of the multi-relay AF cooperative system can be expressed as

$$\begin{aligned} \maximize \quad & \gamma_e = \sum_{i=1}^M \frac{P_s P_i h_{si}^2 h_{id}^2}{CN_0^2 + N_0 P_i h_{id}^2} \\ \text{subject to} \quad & \sum_{i=1}^M P_i \leq P_r, \\ & P_i \geq 0, i = 1, \dots, M \end{aligned} \quad (7)$$

Then, converting (7) into the standard form of the convex optimization, it can be rewritten as

$$\begin{aligned} \maximize \quad & f(P_i) = -\sum_{i=1}^M \frac{P_s P_i h_{si}^2 h_{id}^2}{CN_0^2 + N_0 P_i h_{id}^2} \\ \text{subject to} \quad & \sum_{i=1}^M P_i \leq P_r, \\ & P_i \geq 0, i=1, \dots, M \end{aligned} \quad (8)$$

Since (8) is a typical convex optimization problem, we can use KKT condition to solve it. Here, the optimal value of the relay power is set to be P_i^* . For the inequality constraints, the Lagrangian coefficient can be set as λ_i^* , $i=1, \dots, M$. For the equality constraints, the Lagrangian coefficient is set to be v^* . Therefore, the Lagrangian function is further expressed as

$$L = f(P_i^*) - \sum_{i=1}^M \lambda_i^* P_i^* + v^* \left(\sum_{i=1}^M P_i - P_r \right), \quad (9)$$

and the KKT condition is

$$P_i^* \geq 0, i=1, \dots, M \quad (10a)$$

$$\sum_{i=1}^M P_i^* = P_r \quad (10b)$$

$$\lambda_i^* \geq 0, i=1, \dots, M \quad (10c)$$

$$\lambda_i^* P_i^* = 0, i=1, \dots, M \quad (10d)$$

$$\frac{\partial f}{\partial P_i} \bigg|_{P_i=P_i^*} - \lambda_i^* + v^* = 0, i=1, \dots, M \quad (10e)$$

According to (10c), we can reformulate (10e) as

$$v^* - \frac{P_s h_{si}^2 h_{id}^2 CN_0^2}{(CN_0^2 + N_0 P_i h_{id}^2)} \geq 0, i=1, \dots, M. \quad (11)$$

Also, based on (10), v^* has non-negative value. First, let us consider the case of $v^* < \frac{P_s h_{si}^2 h_{id}^2}{CN_0^2}, i=1, \dots, M$, which does not hold at $P_i^*=0$. According to (10-4), we can get $\lambda_i^*=0$. In this case, the optimal relay power obtained by (10e) is calculated as

$$P_i^* = \frac{\sqrt{\frac{P_s h_{si}^2 h_{id}^2 CN_0^2}{v^*} - CN_0^2}}{N_0 h_{id}^2}, \quad i=1, \dots, M. \quad (11)$$

In the following, we will consider the case of $v^* \geq \frac{P_s h_{si}^2 h_{id}^2}{CN_0^2}, i=1, \dots, M$ with $P_i^*=0$. Based on the above analysis, the optimal power of the relay nodes becomes

$$P_i^* = \max \left(0, \frac{\sqrt{\frac{P_s h_{si}^2 h_{id}^2 CN_0^2}{v^*} - CN_0^2}}{N_0 h_{id}^2} \right), \quad i=1, \dots, M. \quad (12)$$

Then, we can get the value of v^* that satisfies (10-2).

Since $\sum_{i=1}^M P_i^*$ decreases with the increase of v^* , which belongs to the quasi-convex optimization problem. Hence, we can use the dichotomy method to solve and get the value of v^* . Assuming $f(v^*) = \sum_{i=1}^M P_i^*$, then we can make the following process.

- Assume that the initial value of v^* is $l = 0$, the maximum is $u = 1$, and the tolerable error is $\varepsilon > 0$.
- If $f(u) > P_r$, then $l = u, u = 2u$.
- Execute the loop process

Let $mid = (u + 1) / 2$,

if $f(mid) > P_r$, then $l = mid$,

otherwise, $u = mid$,

until $f(l) - f(u) \leq \varepsilon$.

- $v^* = mid$.

After the steps above, we can calculate the optimal power of the relays.

IV. SIMULATION RESULTS AND ANALYSIS

In this section, according to the proposed power allocation scheme of the multi-relay AF cooperative system, the simulation results of the system capacity and the outage probability in Rayleigh fading channel are presented and compared with those of the traditional equal power allocation scheme. In this multi-relay AF cooperative network, we assume that the CSI is completely known by the destination node and the relay node. The total power of the system is 500w, the source power is 200w, the tolerable error is $\varepsilon = 0.1$, and the constant C in the amplification factor is $C=1.70$.

Fig. 2 shows the capacity comparison of the system under equal power allocation and optimal power allocation for different number of the relays. For equal power allocation scheme, it is a suboptimal power allocation scheme and the same total relay transmission power is distributed equally among all the relay nodes. Here, we consider that the system total power and the source power are fixed. Hence, the total relay power is also kept as a constant. As shown in Fig.2, the capacity of both power allocation schemes increase with the increase of the relay number. However, the optimal power allocation outperforms the traditional power allocation by improving the capacity of the system. It is seen from Fig. 2 that the required relay number of the proposed scheme is 2 fewer than the average power allocation scheme with the capacity being 5.8bit/s/Hz. With the increase of the relay number, the difference between the two schemes becomes more significant.

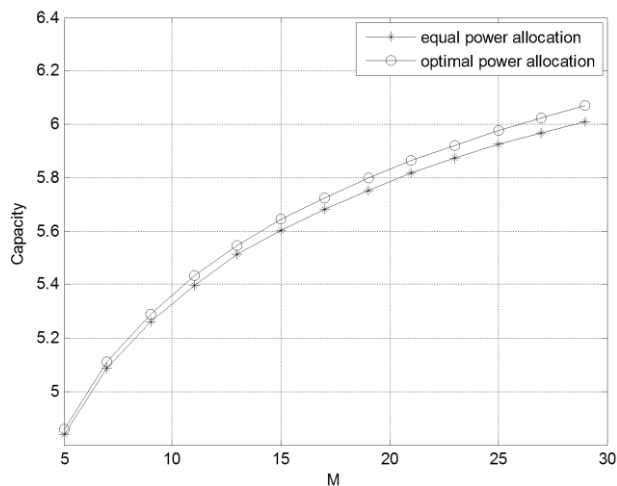


Figure 2. Comparison of equal power and optimal power allocation capacity

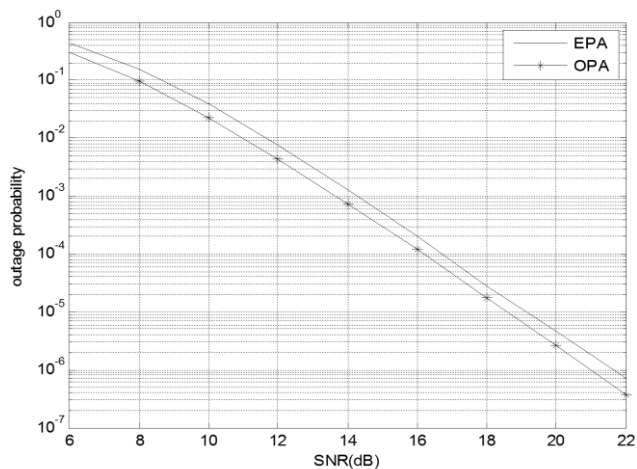


Figure 3. Comparison of outage probability between equal power and optimal power allocation

Fig. 3 shows the comparison of the outage probability of the multi-relay AF cooperative system with the optimal power allocation scheme and also the equal power allocation with different SNR. It is assumed that the spectrum utilization $R = 0.5$ bit/s/Hz and the relay number is 3. As shown in Fig. 3, the proposed optimal power allocation scheme attained a better outage probability performance. For example, when the outage probability is 10^{-3} , the proposed scheme can obtain about 1.2 dB SNR gains. In addition, with the increase of SNR, the outage probability performance of both the power allocation schemes gradually decreases.

V. CONCLUSION

This paper aims at maximizing the capacity of the multi-relay AF cooperative communication system. The optimal power allocation among the multiple relays is derived based on the convex optimization method. Furthermore, the capacity and the outage probability of the proposed optimal power allocation scheme are provided and compared with the typical equal power allocation scheme. Simulation results show that the proposed power allocation scheme achieves a larger

capacity and better outage performance compared with the equal power allocation scheme.

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