

Massive MIMO in Fixed-Point Arithmetic

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Abstract—Massive MIMO base stations are expensive to build due to the requirement for a large number of RF transceivers and high-resolution analog-to-digital converters. A way to reduce the implementation cost is to build the base stations with inexpensive hardware, resulting in the received signals to be coarsely quantized. To implement the data detection and decoding process in real time, fixed-point arithmetic with reduced precision is used. This article reports the minimum wordlength needed to maintain the Bit-Error Rate (BER) at acceptable levels. Specifically, the eigenvalue decomposition, which is the most computationally demanding portion of the receiver algorithm, can be calculated with wordlengths of 7 and 10 bits for eigenvectors and eigenvalues, respectively.

Index Terms—Massive MIMO, fixed-point arithmetic

I. INTRODUCTION

The number of Receive Antennas (Receivers), R , in a Massive Multiple-Input Multiple-Output (MMIMO) base station is much larger than the number of Transmit Antennas (Terminals), T . Since large-scale computation is needed for data detection and decoding based on measurements from high-resolution Analog-to-Digital Converters (ADC) connected to highly-linear RF front-end amplifiers for each antenna, the cost of a MMIMO base station is very high. A way to reduce the cost of the implementation is to build the base stations with inexpensive low-end hardware [1], such as simple RF receivers feeding the received signals to coarse (1-, 2-, or 3-bit) ADCs [2], [3].

The equations of the detection process are given below, where $\mathbf{X}$ is the complex $T \times 1$ transmitted signal vector, $\mathbf{Y}$ is the $R \times 1$ received signal vector, $\mathbf{H}$ is the $R \times T$ channel transfer matrix, where each entry is modeled as an independent identically distributed as complex circularly symmetric Gaussian random value, $\mathbf{N}$ is an $R \times 1$ complex noise vector, and $\mathbf{N}'$ is a $T \times 1$ noise vector.

$$\mathbf{Y} = \mathbf{H} \cdot \mathbf{X} + \mathbf{N} \Rightarrow \mathbf{H}^H \cdot \mathbf{Y} = \underbrace{(\mathbf{H}^H \cdot \mathbf{H})}_{\mathbf{\Theta}} \cdot \mathbf{X} + \underbrace{\mathbf{H}^H \cdot \mathbf{N}}_{\mathbf{N}'} \quad (1)$$

The multiplication of both sides by $\mathbf{H}^H$ converts the $R \times T$ system to a $T \times T$ system, which is less expensive to solve. The estimation of $\mathbf{X}$ through the solution of this linear system is the most computationally demanding portion of the receiver algorithm for reasonably high values of T . In this process, two major operations need to be performed: (i) calculation of the Hermitian matrix $\mathbf{\Theta} = \mathbf{H}^H \cdot \mathbf{H}$, and (ii) EigenValue Decomposition (EVD) [4] of matrix $\mathbf{\Theta} = \mathbf{Q} \cdot \mathbf{\Lambda} \cdot \mathbf{Q}^H$ followed by solving the linear system by matrix multiplications.

The challenge is to perform these linear-algebra operations in real time on inexpensive hardware. To achieve such performance, only fixed-point arithmetic will be used [5], [6]. The channel matrix, $\mathbf{H}$, and the received signal vector, $\mathbf{Y}$, will be coarsely quantized with $B = 1$, $B = 2$, or $B = 3$ bits. This article reports the minimum wordlength in a fixed-point arithmetic implementation of the data detection and decoding process, which is needed to maintain the Bit-Error Rate (BER) at acceptable levels. Our contributions are as follows.

- 1) Assessment of the minimum wordlength needed to represent the Hermitian matrix $\mathbf{\Theta} = \mathbf{H}^H \cdot \mathbf{H}$.
- 2) Assessment of the minimum wordlength needed to implement the QR decomposition of matrix $\mathbf{\Theta}$.
- 3) Assessment of the minimum wordlength needed to complete the eigenvalue decomposition of matrix $\mathbf{\Theta}$ through the Francis-Kublanovskaya algorithm.

The article is organized as follows. Background information is presented in Section II. The detection process implemented in fixed-point arithmetic is presented in Section III. Section IV provides conclusions and closing remarks.

II. BACKGROUND

This section provides the fundamentals of MMIMO iterative channel estimation and simulation software for coarsely quantized MMIMO communications used in our previous work.

A. Iterative Channel Estimation for MMIMO

In MMIMO a large number of sufficiently-spaced receive antennas results in a large probability of a high-capacity communications channel from each user's transmitter to the base station [7]. This allows MMIMO systems to communicate over random multipath propagation channels with transmission power nearly as low as ideal Additive White Gaussian Noise (AWGN) channels. To take advantage of this situation, the receiver requires accurate estimates of radio Channel State Information (CSI), which in turn leads to the requirement for a large number of pilot symbols to be used when standard channel estimation techniques are employed [8].

Recently we have shown that the channel state information can be accurately estimated from coarsely quantized measurements using an iterative procedure, where data symbols detected and decoded in previous iterations are used as additional (virtual) pilot symbols [9]. For the first iteration, only the pilot symbols are used to estimate the channel, since no prior knowledge of the transmitted data is available. For the second

and following iterations, channel estimation is enhanced by using the estimated data symbols from the previous iterations as virtual pilot symbols, to get a lower error estimate of the channel parameters. Our simulation results indicate that an accurate estimation is achieved in three iterations or less.

B. Simulation Software for Coarsely Quantized MMIMO

In our previous work [9] the linear system shown in Equation (1) was solved in double-precision (64-bit) floating-point arithmetic [10] using the standard MATLAB® linear solution mechanism, where $\tilde{\mathbf{X}} = \mathbf{H}^{-1} \cdot \mathbf{Y}$ is coded as $\tilde{\mathbf{X}} = \mathbf{H} \backslash \mathbf{Y}$, as shown in Equations (2) and (3). While these standard linear system solution computations give accurate results, direct implementation of these solutions for use in MMIMO receivers would require computational hardware with prohibitive cost in base stations. The backslash operator method was also applied for estimating the channel matrix, $\mathbf{H}$.

$$\mathbf{Y} = \mathbf{H} \cdot \mathbf{X} + \mathbf{N} \Rightarrow \tilde{\mathbf{X}} = \mathbf{H} \backslash \mathbf{Y} \quad (2)$$

$$\mathbf{H}^H \cdot \mathbf{Y} = \mathbf{\Theta} \cdot \mathbf{X} + \mathbf{N}' \Rightarrow \tilde{\mathbf{X}} = \mathbf{\Theta} \backslash (\mathbf{H}^H \cdot \mathbf{Y}) \quad (3)$$

To reduce the latency of estimating $\mathbf{X}$, the linear system is solved in fixed-point arithmetic with reduced precision [11] through an eigenvalue decomposition of the Hermitian matrix $\mathbf{\Theta}$, as shown in Equation (4). It should be observed that the eigenvalue decomposition is intrinsically stable, since unitary matrix $\mathbf{Q}$ is a rotation matrix, which means that all its elements range from -1.0 to $+1.0$.

$$\mathbf{\Theta} = \mathbf{Q} \cdot \mathbf{\Lambda} \cdot \mathbf{Q}^H \Rightarrow \tilde{\mathbf{X}} = \mathbf{Q} \cdot \mathbf{\Lambda}^{-1} \cdot \mathbf{Q}^H \cdot \mathbf{H}^H \cdot \mathbf{Y} \quad (4)$$

C. Coarse Quantization

When signal x is in the range of the quantization range i , $q_{min}^i \leq x < q_{max}^i$, its reported quantized value is $q(x) = q_i$. The values of q_i for all the intervals i , which minimize the mean square error, $E[|q(x) - x|^2]$, where $E[\cdot]$ is the expectation operator, are easily shown to be the mean value of the signal in each interval $q_i = E[x | q_{min}^i \leq x < q_{max}^i]$ [12].

An independent Additive White Gaussian Noise (AWGN) channel with mean $\mu = 0$ and standard deviation, $\sigma = 1$, is assumed for each receive antenna in this article. Table I presents the probabilities, quantization thresholds, and the corresponding bin-average values for $B = 1$ -bit, $B = 2$ -bit, and $B = 3$ -bit quantization processes for a Gaussian random variable with zero mean and unity standard deviation.

TABLE I
PROBABILITIES, QUANTIZATION THRESHOLDS, AND THE
CORRESPONDING BIN-AVERAGE VALUES.

B	Probabilities	Quantization thresholds	Bin-average values
1	50%	0	± 0.797884561
2	25%	0 ± 0.674489572	± 0.324662678 ± 1.271106444
3	12.5%	0 ± 0.318634923 ± 0.674489572 ± 1.150349346	± 0.157972012 ± 0.491353344 ± 0.895384581 ± 1.646828306

In fixed-point representation, it is beneficial to scale up the smallest magnitude of the bin-average values, since this will simplify the implementation and slightly increase the precision. For example, in a $B = 2$ quantization process, 0.324662678 is promoted to 1.000000000 , and 1.271106444 is promoted to $1.271106444/0.324662678 = 3.915160350$. The other quantization processes will follow a similar pattern. The updated values are presented in Table II.

TABLE II
QUANTIZATION THRESHOLDS, SCALE FACTORS, AND THE
CORRESPONDING BIN-AVERAGE VALUES.

B	Quantization thresholds	Scale factor	Bin-average values
1	0	0.797884561	± 1.000000000
2	0 ± 0.674489572	0.324662678	± 1.000000000 ± 3.915160350
3	0 ± 0.318634923 ± 0.674489572 ± 1.150349346	0.157972012	± 1.000000000 ± 3.110382264 ± 5.667995047 ± 10.424810617

It is apparent that the measurements of the channel output can take only a small number of different values (for example, ± 1.0000 and ± 3.9152 in a process with $B = 2$ quantization bits). The estimated channel coefficients, the entries of $\mathbf{H}$, are also coarsely quantized to reduce the computation cost in the receiver. Since the quantized values have equal probability, the entropy of the quantized signal is maximized, which in turn maximizes the upper bound of the mutual information between the received signal and the transmitted signal when their joint distribution is not known [13].

III. DETECTION IN FIXED-POINT ARITHMETIC

In this section the major operations of the receiver algorithm implemented in fixed-point arithmetic are discussed (calculating the Hermitian matrix $\mathbf{\Theta}$, its Hessenberg decomposition, and the eigenvalue decomposition through a numerically stable algorithm), and numerical results are presented.

A. Calculating $\mathbf{\Theta} = \mathbf{H}^H \cdot \mathbf{H}$ in Fixed-Point Arithmetic

The pattern for calculating the matrix $\mathbf{\Theta} = \mathbf{H}^H \cdot \mathbf{H}$ is sum-of-products, and the computation budget consists of T^2 inner products of R -element complex vectors. Due to their length and random nature, data signals of different users are likely to have very low correlation with each other. For this reason, $\mathbf{\Theta}$ is diagonal dominant. A sample of the reduced-wordlength matrix $\mathbf{\Theta}$ in a MMIMO with $R = 128$, $T = 16$, and $B = 1$ is shown below. It is observed that its diagonal elements are real-valued and positive; in comparison, the real and imaginary components of off-diagonal elements are significantly smaller in magnitude. Statistics collected over one million samples of matrix $\mathbf{\Theta}$ indicate that the largest off-diagonal magnitude has an average of 49 and a standard deviation of 6. This means that most off-diagonal values can be represented with 7 bits (sign bit included). The overflowing off-diagonal values will be saturated to ± 63 .

$$\Theta = \begin{pmatrix} 289 & 5+j & 4 & -11-j & 2 & 7+j & 6 & \dots \\ 5-j & 4 & 244 & 3+j & 1 & 6-j & 8 & \dots \\ -11+j & 2 & 3-j & 1 & 211 & -9-j & 13 & \dots \\ 7-j & 6 & 6+j & 8 & -9+j & 13 & 208 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

It is apparent that a wordlength of 9 bits provides sufficient precision to store each element of the matrix Θ . For higher quantization levels ($B = 2$ and $B = 3$) a longer wordlength may be required. However, if needed, a reduction of the wordlength could be performed in such cases by right shifting followed by truncation or rounding. The same symbol Θ will be used for the matrix with reduced wordlength.

B. Hessenberg Decomposition of Θ in Fixed-Point Arithmetic

A Hessenberg form is preserved during a similarity transformation [4]. For this reason, a conversion from a Hermitian form, Θ , to a Hessenberg form, A_{hess} , is beneficial to be performed prior to applying the Francis-Kublanovskaya recursion, since this will significantly reduce the operation count. It is observed that the reduced off-diagonal wordlength is highly beneficial for the fixed-point arithmetic since only off-diagonal elements are rotated in the Hessenberg conversion. For example, the first-column vector v (diagonal element is excluded), where $v^T = [5-j, 4, -11+j, 2, 7-j, 6, \dots]$, is converted into a vector w , where $w^T = [59-j, 62, 0, 0, \dots]$. This vector still has a small magnitude compared to diagonal elements, which means that the Hessenberg form is also diagonal dominant. This is an important observation for selecting the wordlength of the eigenvalue decomposition.

C. Francis-Kublanovskaya Algorithm in Fixed-Point Arithmetic

The Francis-Kublanovskaya (FK) recursion [14]–[16], in which each iteration is a similarity transformation based on the QR decomposition, is used to calculate the eigenvalue decomposition of the Hermitian matrix Θ (or of the Hessenberg matrix A_{hess} if a Hessenberg conversion is applied). At iteration k , the matrix $\Theta^{(k)}$ (where $\Theta^{(0)} = \Theta$) is decomposed into the product of a unitary matrix $Q^{(k)}$ and an upper triangular matrix $R^{(k)}$. By multiplying $R^{(k)}$ by $Q^{(k)}$, a similarity transformation of $\Theta^{(k)}$ is obtained:

$$\Theta^{(k+1)} = R^{(k)} \cdot Q^{(k)} = Q^{(k)H} \cdot \Theta^{(k)} \cdot Q^{(k)} \quad (5)$$

If $k \rightarrow \infty$, then the matrix $\Theta^{(k)}$ tends to a diagonal matrix, Λ , whose elements are the eigenvalues of the initial Θ :

$$\Theta = \underbrace{Q^{(n)H} \cdot \dots \cdot Q^{(k)H} \cdot \dots \cdot Q^{(0)H}}_{Q^H} \cdot \Lambda \cdot \underbrace{Q^{(0)} \cdot \dots \cdot Q^{(k)} \cdot \dots \cdot Q^{(n)}}_Q \quad (6)$$

The off-diagonal elements of matrix Θ (or the Hessenberg form A_{hess}) decrease in magnitude over FK iterations. As a result, there is no danger of overflow in any of those elements, and the wordlength needed to represent diagonal elements would suffice for representing the off-diagonal elements, too.

The unitary matrix Q of the eigenvalue decomposition of the Hermitian matrix Θ , and all unitary matrices $Q^{(k)}$ of the QR decompositions, are essentially rotation matrices. As a result, all their elements range between -1.0 and $+1.0$. In our fixed-point arithmetic implementation, a scale factor equal to 2^{qwl-1} is embedded into the matrix Q , so that its wordlength is $(qwl - 1)$ magnitude bits + 1 sign bit = qwl bits. The matrices Θ and Λ are already in fixed-point representation, their wordlengths being equal, $\theta wl = \lambda wl$. As a result, the eigenvalue decomposition can be written as:

$$\begin{aligned} \Theta &= Q \cdot \Lambda \cdot Q^H \Rightarrow \Theta = \frac{2^{qwl-1} Q}{2^{qwl-1}} \cdot \Lambda \cdot \frac{2^{qwl-1} Q^H}{2^{qwl-1}} \Rightarrow \\ &\Rightarrow 2^{2 \cdot (qwl-1)} \Theta = \underbrace{(2^{qwl-1} Q)}_{\underline{Q}} \cdot \Lambda \cdot \underbrace{(2^{qwl-1} Q^H)}_{\underline{Q}^H} \quad (7) \end{aligned}$$

where $\underline{Q}$ and $\underline{Q}^H$ is the unitary matrix and its Hermitian transpose in fixed-point representation (that is, with the scale factor embedded). Similarly, $\underline{\Theta}^{(k)}$ is the fixed-point representation of $\Theta^{(k)}$, and an FK iteration in the fixed-point domain is:

$$\begin{aligned} \left. \begin{aligned} \Theta^{(k)} &= Q^{(k)} \cdot R^{(k)} \Rightarrow \underline{\Theta}^{(k)} = \underline{Q}^{(k)} \cdot R^{(k)} \\ \underline{\Theta}^{(k+1)} &= R^{(k)} \cdot \underline{Q}^{(k)} \end{aligned} \right\} \Rightarrow \\ &\Rightarrow \underline{\Theta}^{(k+1)} = \underline{\Theta}^{(k+1)} \gg (qwl - 1) \quad (8) \end{aligned}$$

It is mentioned that the last right shift operation may be performed with or without rounding.

The challenge of designing a MMIMO base station is to minimize the wordlengths qwl and θwl while the BER is maintained at acceptable levels. Our simulation software used in prior work [9] has been converted to use only fixed-point arithmetic. The linear system shown in Equation (1) is solved through an eigenvalue decomposition rather than the MATLAB® backslash operator. The wordlength of matrix Q was set to $qwl = 6$ and $qwl = 7$ bits, and the wordlength of matrix Θ was set to $\theta wl = 9$, $\theta wl = 10$, and $\theta wl = 11$. The number of quantization bits were set to $B = 1$, $B = 2$, and then $B = 3$. Extensive simulation runs were attempted, and BER performance figures were recorded.

The system was evaluated where each user independently transmits a single-carrier signal with a modulation of order 4 QPSK using 256-symbol blocks with a cyclic prefix of 16 samples. An error correction code consisting of a rate $\frac{1}{2}$ convolutional code applied over 32 single-carrier blocks was used. The simulated random multipath fading channel had a maximum delay of 9 symbol periods. The channel state is assumed to be stable over each set of 32 single-carrier blocks, where for each set the user transmits 2 pilots per block. A joint channel estimation/data detection/data decoding algorithm as described in [9] is used to provide the final result.

The BER figures versus the wordlength for implementing the eigenvalue decomposition in fixed-point arithmetic with reduced precision are presented in Table III. For reference, the BER figures corresponding to a double-precision floating-point

implementation are also provided. For a coarse quantization with $B = 1$ bit, neither the floating-point nor the fixed-point implementations do not achieve a satisfactory BER in a small number of iterations. For a coarse quantization with $B = 2$ or $B = 3$, three to four iterations generally suffices to achieve a good BER. It is observed that an additional iteration is generally needed in fixed-point implementation over the floating-point implementation to match their BERs; for example, $\text{BER}(B = 2, qwl = 6, \theta wl = 9, i = 3) = 0.0259$, whereas $\text{BER}(B = 2, qwl = 64, \theta wl = 64, i = 2) = 0.0236$.

It is apparent that for a quantization with $B = 2$ bits, the matrix $\mathbf{Q}$'s wordlength, qwl , has little impact on BER. As an example, $\text{BER}(B = 2, qwl = 6, \theta wl = 9, i = 2) = 0.0823$, and $\text{BER}(B = 2, qwl = 7, \theta wl = 9, i = 2) = 0.0794$. On the other hand, the matrix $\mathbf{\Theta}$'s wordlength, θwl , has a much stronger impact on BER. For example, $\text{BER}(B = 2, qwl = 6, \theta wl = 10, i = 2) = 0.0395$, which a significant decrease from 0.0823. It should be observed that the BER does not improve significantly for $\theta wl \geq 11$. A MMIMO system with $B = 3$ is robust enough to operate with $\theta wl = 9$, but the cost of the implementation will be higher due to the increased complexity of the implementation in fixed-point arithmetic.

TABLE III
BER VERSUS WORDLENGTH FOR $R = 128$, $T = 16$, $E_b/N_0 = 5$ dB
(ONLY THE FIRST THREE ITERATIONS ARE SHOWN).

B	Wordlength		Iteration		
	qwl	θwl	$i = 1$	$i = 2$	$i = 3$
1	Double-Precision Floating-Point Arithmetic				
	64	64	0.4581	0.3235	0.2116
	Fixed-Point Arithmetic				
	6	9	0.4717	0.3990	0.2670
	6	10	0.4713	0.3443	0.2503
	6	11	0.4706	0.3461	0.2482
	7	9	0.4705	0.3930	0.2666
	7	10	0.4741	0.3442	0.2516
	7	11	0.4723	0.3520	0.2539
	Double-Precision Floating-Point Arithmetic				
2	64	64	0.2854	0.0236	0.0078
	Fixed-Point Arithmetic				
	6	9	0.3284	0.0823	0.0259
	6	10	0.3225	0.0395	0.0112
	6	11	0.3203	0.0311	0.0125
	7	9	0.3277	0.0794	0.0303
	7	10	0.3245	0.0399	0.0107
	7	11	0.3206	0.0315	0.0106
	Double-Precision Floating-Point Arithmetic				
	64	64	0.1849	0.0396	0.0066
3	Fixed-Point Arithmetic				
	6	9	0.2380	0.0379	0.0091
	6	10	0.2265	0.0389	0.0089
	6	11	0.2226	0.0453	0.0104
	7	9	0.2342	0.0354	0.0083
	7	10	0.2255	0.0371	0.0090
	7	11	0.2201	0.0436	0.0119

Based on these considerations, it is apparent that a good trade-off in implementing a MMIMO base station is a coarse quantization with $B = 2$ bits, a $\mathbf{Q}$'s wordlength $qwl = 7$ bits, and a $\mathbf{\Theta}$'s wordlength $\theta wl = 10$ bits. With such a reduced qwl , vector rotations can be easily implemented in inexpensive hardware, such as reasonably sized Field-Programmable Gate

Arrays (FPGA). A reduced θwl allows the extensive use of FPGA Digital-Signal Processing units to perform the reverse products in the FK recursion, as well as complete the calculation of the solution of the linear system shown in Equation (1) through matrix multiplication, as outlined in Equation (4). Previous works are in line with these claims [11], [17].

IV. CONCLUSIONS

The eigenvalue decomposition, which is the most computationally demanding portion of a MMIMO receiver algorithm, can be implemented in fixed-point arithmetic with wordlengths of 7 and 10 bits for eigenvectors and eigenvalues, respectively, with minimal degradation of the Bit-Error Rate achieved in a double-precision floating-point implementation. This allows the reduction of the cost of massive MIMO base stations, which is a key impediment to their widespread deployment. In future work, the wordlength of the fixed-point implementation will be assessed in the presence of signal contamination from adjacent networks.

REFERENCES

- [1] C. Mollén, "High-End Performance with Low-End Hardware. Analysis of Massive MIMO Base Station Transceivers," Ph.D. dissertation, Linköping University, Linköping, Sweden, 2018.
- [2] C. Mollén, J. Choi, E. G. Larsson, and J. Robert W. Heath, "Uplink Performance of Wideband Massive MIMO with One-Bit ADCs," *IEEE Transactions on Wireless Communications*, vol. 16, no. 1, pp. 87–100, January 2017.
- [3] —, "Achievable Uplink Rates for Massive MIMO with Coarse Quantization," in *Proceedings of the 2017 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2017)*. New Orleans, Louisiana: IEEE, March 2017, pp. 6488–6492.
- [4] G. H. Golub and C. F. van Loan, *Matrix Computations*, 3rd ed. The Johns Hopkins University Press, 1996.
- [5] B. Parhami, *Computer Arithmetic: Algorithms and Hardware Designs*. Oxford University Press, 2000.
- [6] M. Ercegovac and T. Lang, *Digital Arithmetic*, Morgan Kaufmann, 2003.
- [7] A. Chockalingam and B. S. Rajan, *Large MIMO Systems*. Cambridge University Press, 2014.
- [8] M. K. Ozdemir and H. Arslan, "Channel Estimation for Wireless OFDM systems," *IEEE Communications Surveys and Tutorials*, vol. 9, no. 2, pp. 18–48, July 2007.
- [9] Z. Zhang, M. McGuire, and M. Sima, "Iterative Channel Estimation for Large Scale MIMO with Highly Quantized Measurements in 5G," in *The 28th European Signal Processing Conference (EUSIPCO 2020)*. Amsterdam, The Netherlands: European Association for Signal Processing (EURASIP), January 2021, pp. 1643–1647.
- [10] *IEEE Standard for Floating-Point Arithmetic. IEEE STD 754-2019*, IEEE Computer Society, July 2019.
- [11] M. Tian, M. McGuire, and M. Sima, "FPGA Arithmetic for Eigenvalue Decomposition with Reduced Precision in Massive MIMO Systems," *IEEE Access*, November 2020, submitted.
- [12] J. M. Mendel, *Lessons in Estimation Theory for Signal Processing, Communications, and Control*, 2nd ed. Prentice-Hall, 1995.
- [13] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. Wiley-Interscience, 2006.
- [14] J. Francis, "The QR transformation – Part 1," *The Computer Journal*, vol. 4, no. 3, pp. 265–272, October 1961.
- [15] —, "The QR transformation – Part 2," *The Computer Journal*, vol. 4, no. 4, pp. 332–345, January 1962.
- [16] V. N. Kublanovskaya, "On some algorithms for the solution of the complete eigenvalue problem," *USSR Comput. Math. and Math. Physics*, no. 3, pp. 637–657, March 1961.
- [17] G. Xu, Y. Li, J. Yuan, R. Monroe, S. Rajagopal, S. Ramakrishna, Y. H. Nam, J.-Y. Seol, J. Kim, M. M. U. Gul, A. Aziz, and J. Zhang, "Full Dimension MIMO (FD-MIMO): Demonstrating Commercial Feasibility," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 8, pp. 1876–1886, August 2017.



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