

Construction of Optical ZCZ Sequence Set with the Zero-Correlation Zone 2^z Suitable for PPM-OCDMA System

Yasuaki Ohira ^{*}, Takahiro Matsumoto [†], Hideyuki Torii [‡], Yuta Ida [§]

^{*}Center for Information Infrastructure, Yamaguchi University,
1-1-1 Minamikogushi, Ube, Yamaguchi, 755-8505, Japan

[†]Graduate School of Science and Engineering, Kagoshima University,
1-21-40 Korimoto, Kagoshima, 890-0065, Japan

[‡]Department of Information Network and Communication, Kanagawa Institute of Technology,
1030 Ogino, Atsugi, Kanagawa, 243-0292, Japan

[§]Graduate School of Sciences and Technology for Innovation, Yamaguchi University,
2-16-1 Tokiwadai, Ube, Yamaguchi, 755-8611, Japan

yohira@yamaguchi-u.ac.jp, matugen@ibe.kagoshima-u.ac.jp, torii@nw.kanagawa-it.ac.jp, y.ida@yamaguchi-u.ac.jp

Abstract—In this paper, we propose a new generation method of optical zero-correlation zone (ZCZ) sequences suitable for the optical code division multiple access with pulse position modulation (PPM-OCDMA) system. The proposed optical ZCZ sequence set has multiple ZCZ in the side lobes, and the interval of those ZCZs is equal. In addition, this ZCZ size can be easily extended. Because of these properties, the proposed optical ZCZ sequence is suitable for the PPM-OCDMA system. The PPM-OCDMA system using the proposed optical ZCZ sequence is expected to realize visible light communication that prevents illumination degradation which is a weak point of the general PPM-OCDMA system and is resistant to interference caused by wall reflections.

Keywords—optical wireless communication, visible light communication (VLC), pulse-position modulation (PPM), optical zero-correlation zone (ZCZ) sequence

I. INTRODUCTION

In optical wireless communications, where multiple users communicate at the same time, multiple access interference (MAI) is a problem. The cause of the MAI is light from light sources adjacent to the desired ones. The optical code division multiple access (OCDMA) system is one of the optical wireless communication systems which can cancel the MAI. In the OCDMA system, spreading codes such as optical orthogonal code (OOC), modified pseudo orthogonal M-sequence sets (MPOMS), and generalized modified prime sequence codes (GMPSC) have been proposed [1]–[3]. In particular, the quasi-synchronous OCDMA system using the optical zero-correlation zone (ZCZ) sequence set suppress MAI by the correlation property of the sequence [4].

In this study, we focused on two problems of OCDMA. The first problem is the decrease in illumination due to the modulation method. The major modulation methods of the OCDMA system are equal weight orthogonal (EWO) modulation and pulse position modulation (PPM). Allocating one sequence to one user is the most efficient approach to

increase the number of users in the OCDMA system. Under this condition, the EWO modulation can transmit only one bit per symbol, which becomes a problem in terms of the speed of communication. On the other hand, the PPM can improve the bit rate by dividing the symbol into multiple slots. However, increasing the number of slots in the PPM-OCDMA means that the LED emission time per symbol becomes shorter, which causes a decrease in the original illumination brightness of the optical product.

The second problem is the interference by the reflected light from the wall. This problem occurs when the receiver is located near the wall in a room. The reflected light from the wall causes delay distortion. The delay distortion causes inter-symbol interference (ISI), which degrades the bit error rate performance [5].

In this paper, we propose a new PPM-OCDMA system and a new generation method of optical ZCZ sequence set suitable for the proposed system. The proposed optical ZCZ sequence set has multiple ZCZ in the side lobes, and the interval of those ZCZs is equal. The emission time per symbol can be improved by using the property of this sequence set in the new PPM-OCDMA system. In addition, it has been reported that when the optical ZCZ sequence set is used in EWO modulation, the ISI due to wall reflection can be suppressed [6]. This tolerance to ISI can be expected in the new PPM-OCDMA scheme.

II. PROPOSED PPM-OCDMA SYSTEM

In this section, we describe the conventional and proposed PPM-OCDMA system. In the PPM-OCDMA system, a spreading sequence is assigned to a corresponding slot in a symbol according to bit data, and its modulated symbol is transmitted. In this paper, the corresponding slot is called pulse position. In general, a symbol is divided into power-of-two slots. The transmitted data is correlated with the reference

sequence at the receiver, and is detected the pulse position in the symbol.

The difference between the conventional and proposed systems is the way to change the pulse position. In the conventional system, a spreading sequence with the same length as the slot length is used, and is assigned to a pulse position according to bit data. This means that the symbol has slots which are unassigned to a spreading sequence. On the other hand, in the proposed system, a spreading sequence with the same length as the symbol length is used, and the pulse position is changed by cyclic shifting so that the first position of the spreading sequence comes to the slot allocated. This means that the symbol does not have slots which are unassigned to a spreading sequence.

Figure 1 shows the timing charts of the transmit waveforms and their correlation outputs of the conventional and proposed 2-PPM-OCDMA systems in the case of the same symbol length, respectively. In both systems, one bit is transmitted per symbol, a symbol is divided into two slots, and the bit data is demodulated according to pulse position obtained by the peak value of the correlation output.

In Figure 1, a_i and b_i denote the spreading sequences used in the conventional and the proposed systems, respectively. The lengths of the a_i and b_i are $N/2$ and N , respectively, where i is an ordinal variable. In the conventional system, the spreading sequence a_i of length $N/2$ according to the slot length is used. The spreading sequence a_i is assigned to the first half of the slot in the case of bit data 1, and the spreading sequence a_i is assigned to the second half of the slot in the case of bit data 0. Note that slots which are unassigned a spreading sequence turn off the light. Therefore, in the case of the same amplitudes and increasing the number of slots for speed-up, the conventional system causes the problem that the unlit state becomes longer and the illumination degradation occurs. In addition, the sequence length becomes shorter as the number of slots increases, therefore the peak of the correlation outputs decreases.

On the other hand, in the proposed method, a spreading sequence b_i of length N according to the symbol length is used. The spreading sequence b_i is assigned without shifting in the case of bit data 1, and the spreading sequence $b_{(i+N/2) \bmod N}$ is assigned with a cyclic shift of $N/2$ in the case of bit data 0. Note that slots which are unassigned a spreading sequence do not exist. Therefore, the proposed system does not cause the problem that the unlit state becomes longer and the illumination degradation occurs. In addition, the peak of the correlation outputs is unchanged because the sequence length is unchanged.

As a result, it is expected that the proposed system improves the bit-error rate (BER) performance compared to the conventional system. In addition, it is expected that the proposed PPM-OCDMA system using an optical ZCZ sequence which has zero-correlation zones is suppressed the inter-symbol interference (ISI) caused by the reflection on the wall which is a problem in the downlink of visible light communication in a room [5].

Therefore, the proposed system requires an optical ZCZ sequence set having multiple ZCZ. In addition, the system requires that the interval of those ZCZs is equal to prevent degradation of BER performance [7].

III. DEFINITION OF SEQUENCES

A. Optical ZCZ Sequence Set

Let $\alpha_N^{j,Z}$ be a bi-phase sequence of length N , whose elements take 1 or -1 , written as

$$\begin{aligned} \alpha_N^{j,Z} &= (\alpha_{N,0}^{j,Z}, \dots, \alpha_{N,i}^{j,Z}, \dots, \alpha_{N,N-1}^{j,Z}), \\ \alpha_{N,i}^{j,Z} &\in \{1, -1\}, \end{aligned} \quad (1)$$

where Z is a non-negative integer, j is a sequence number, and i is an order variable and denotes $i \bmod N$. Similarly, let $\hat{\alpha}_N^{j,Z,d}$ be a binary sequence of length N , whose elements take 1 or 0, written as

$$\begin{aligned} \hat{\alpha}_N^{j,Z,d} &= (\hat{\alpha}_{N,0}^{j,Z,d}, \dots, \hat{\alpha}_{N,i}^{j,Z,d}, \dots, \hat{\alpha}_{N,N-1}^{j,Z,d}), \\ \hat{\alpha}_{N,i}^{j,Z,d} &\in \{1, 0\}, d \in \{1, 0\}. \end{aligned} \quad (2)$$

Let $\mathcal{A}^Z$ be a set of M pairs of the bi-phase sequence $\alpha_N^{j,Z}$ and the binary sequence $\hat{\alpha}_N^{j,Z,d}$, written as

$$\mathcal{A}^Z = \{(\alpha_N^{0,Z}, \hat{\alpha}_N^{0,Z,d}), \dots, (\alpha_N^{j,Z}, \hat{\alpha}_N^{j,Z,d}), \dots, (\alpha_N^{M-1,Z}, \hat{\alpha}_N^{M-1,Z,d})\}, \quad (3)$$

where M is the number of sequences in a sequence family and is called family size.

A periodic correlation function between the sequences $\alpha_N^{j,Z}$ and $\hat{\alpha}_N^{j',Z,d}$ at shift i' is defined by

$$\rho_{\alpha_N^{j,Z}, \hat{\alpha}_N^{j',Z,d}, i'} = \sum_{i=0}^{N-1} \alpha_{N,i}^{j,Z} \hat{\alpha}_{N,(i+i') \bmod N}^{j',Z,d}. \quad (4)$$

In this paper, the above correlation function $\rho_{\alpha_N^{j,Z}, \hat{\alpha}_N^{j',Z,d}, i'}$ is called the auto-correlation function for $j = j'$ and the cross-correlation function for $j \neq j'$. If the periodic auto- and cross-correlation functions satisfy

$$\rho_{\alpha_N^{j,Z}, \hat{\alpha}_N^{j',Z,d}, i'} = \begin{cases} w_\alpha & ; i' = 0, j = j', d = 0, \\ -w_\alpha & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq Z, \end{cases} \quad (5)$$

where $w_\alpha = \sum_{i=0}^{N-1} \hat{\alpha}_{N,i}^{j',Z,d} < N$, then the set $\mathcal{A}^Z$ is called an optical ZCZ sequence set with the ZCZ size Z .

B. Optical ZCZ Sequence Set Suitable for PPM-OCDMA System

Let $a_N^{j,Z,G}$ be a bi-phase sequence of length N , whose elements take 1 or -1 , written as

$$\begin{aligned} a_N^{j,Z,G} &= (a_{N,0}^{j,Z,G}, \dots, a_{N,i}^{j,Z,G}, \dots, a_{N,N-1}^{j,Z,G}), \\ a_{N,i}^{j,Z,G} &\in \{1, -1\}, \end{aligned} \quad (6)$$

where Z, G are non-negative integers, j is a sequence number, and i is an order variable and denotes $i \bmod N$. Similarly, let

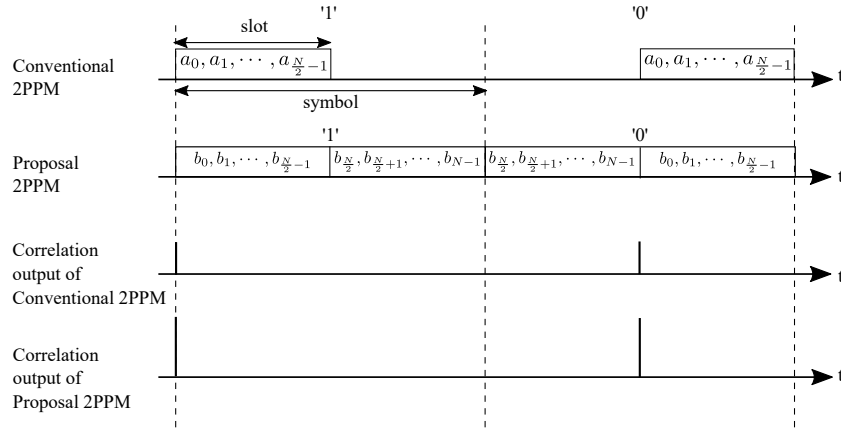


Figure 1. Timing charts of the transmit waveforms and their correlation outputs of the conventional and proposed 2-PPM-OCDMA systems.

$\hat{a}_N^{j,Z,G,d}$ be a binary sequence of length N , whose elements take 1 or 0, written as

$$\begin{aligned} \hat{a}_N^{j,Z,G,d} &= (\hat{a}_{N,0}^{j,Z,G,d}, \dots, \hat{a}_{N,i}^{j,Z,G,d}, \dots, \hat{a}_{N,N-1}^{j,Z,G,d}), \\ \hat{a}_{N,i}^{j,Z,G,d} &\in \{1, 0\}, d \in \{1, 0\}. \end{aligned} \quad (7)$$

Let $A^{Z,G}$ be a set of M pairs of the bi-phase sequence $a_N^{j,Z,G}$ and the binary sequence $\hat{a}_N^{j,Z,G,d}$, written as

$$\begin{aligned} A^{Z,G} &= \{(a_N^{0,Z,G}, \hat{a}_N^{0,Z,G,d}), \dots, (a_N^{j,Z,G}, \hat{a}_N^{j,Z,G,d}), \\ &\dots, (a_N^{M-1,Z,G}, \hat{a}_N^{M-1,Z,G,d})\}. \end{aligned} \quad (8)$$

A periodic correlation function between the sequences $a_N^{j,Z,G}$ and $\hat{a}_N^{j',Z,G,d}$ at shift i' is defined by

$$\rho_{a_N^{j,Z,G}, \hat{a}_N^{j',Z,G,d}, i'} = \sum_{i=0}^{N-1} a_{N,i}^{j,Z,G} \hat{a}_{N,(i+i') \bmod N}^{j',Z,G,d}. \quad (9)$$

In this paper, the above correlation function $\rho_{a_N^{j,Z,G}, \hat{a}_N^{j',Z,G,d}, i'}$ is called the auto-correlation function for $j = j'$ and the cross-correlation function for $j \neq j'$. If the periodic auto- and cross-correlation functions satisfy

$$\begin{aligned} &\rho_{a_N^{j,Z,G}, \hat{a}_N^{j',Z,G,d}, i'} \\ &= \begin{cases} w_a & ; i' = 0, j = j', d = 0, \\ -w_a & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq Z, \\ 0 & ; \frac{kN}{G} - Z \leq |i'| \leq \frac{kN}{G} + Z, \\ 1 \leq k < G, \end{cases} \end{aligned} \quad (10)$$

where $w_a = \sum_{i=0}^{N-1} \hat{a}_{N,i}^{j',Z,G,d} < N$, then the set $A^{Z,G}$ is called an optical ZCZ sequence set with the ZCZ size Z at G positions.

IV. GENERATION METHOD OF SEQUENCES

A. Optical ZCZ Sequence Set with $Z = 1$

A Hadamard matrix is needed for the construction of the optical ZCZ sequence set with $Z = 1$. The generation method of the optical ZCZ sequence set changes in accordance with

the type of Hadamard matrix. In this paper, the Sylvester-type Hadamard matrix is used because the generation method is simple.

Let $\mathbf{H}_{N_1}$ be the Sylvester-type Hadamard matrix of order $N_1 = 2^{n_1}$ with $n_1 \geq 2$, written as

$$\mathbf{H}_{N_1} = [h_{N_1}^0, \dots, h_{N_1}^j, \dots, h_{N_1}^{N_1-1}]^T, \quad (11)$$

$$\begin{aligned} h_{N_1}^j &= (h_{N_1,0}^j, \dots, h_{N_1,i}^j, \dots, h_{N_1,N_1-1}^j), \\ h_{N_1,i}^j &\in \{1, -1\}, \end{aligned} \quad (12)$$

where the symbol T denotes the matrix transposition, which is defined by

$$\mathbf{H}_{N_1} = \mathbf{H}_{\frac{N_1}{2}} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad (13)$$

where the operation $\otimes$ denotes the Kronecker product, and $h_{N_1}^j$ is called a Sylvester-type Hadamard sequence.

An element of the bi-phase sequence $\alpha_N^{j,1} \in \{1, -1\}$ of length $N = 2N_1$ is given by

$$\alpha_{N,i}^{j,1} = \begin{cases} h_{N_1,i \bmod N_1}^j & ; 0 \leq i < \frac{N}{2}, \\ (-1)^{i+1} h_{N_1,i \bmod N_1}^j & ; \frac{N}{2} \leq i < N, \end{cases} \quad (14)$$

where i denotes $i \bmod N$. The mean value of the bi-phase sequence $\alpha_N^{j,1}$ is given by $\sum_{i=0}^{N-1} \alpha_{N,i}^{j,1} = 0$ with $j \neq 0$ and 1. Therefore, the bi-phase sequence $\alpha_N^{j,1}$ is called a bi-phase balanced sequence. On the other hand, an element of the binary sequence $\hat{\alpha}_{N,i}^{j,1,d} \in \{1, 0\}$ of length N is given by

$$\hat{\alpha}_{N,i}^{j,1,d} = \frac{1 + (-1)^d \alpha_{N,i}^{j,1}}{2}. \quad (15)$$

The periodic correlation function between $\alpha_N^{j,1}$ and $\hat{\alpha}_N^{j',1,d}$ is given by

$$\rho_{\alpha_N^{j,1}, \hat{\alpha}_N^{j',1,d}, i'} = \begin{cases} \frac{N}{2} & ; i' = 0, j = j', d = 0, \\ -\frac{N}{2} & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; i' = \pm Z = \pm 1. \end{cases} \quad (16)$$

Therefore, the above set $\mathcal{A}^1$ which is a set of $(N/2 - 2)$ pairs of the bi-phase sequence $\alpha_N^{j,1}$ and the binary sequence $\hat{\alpha}_N^{j,1,d}$

of length $N = 2N_1$ except at $j = 0$ and 1 is an optical ZCZ sequence set with $Z = 1$ and $M = N/2 - 2$.

B. Optical ZCZ Sequence Set with ZCZ at $G = 2^g$ positions

1) *Optical ZCZ Sequence Set with $Z = 1$ at $G = 2^g$ positions:* The bi-phase sequence $a_{2N}^{j,1,2}$ of length $2N$ is generated by

$$\begin{aligned} a_{2N}^{j,1,2} = & (\alpha_{N,0}^{j-(j \bmod 2)+1,1}, \dots, \alpha_{N,\frac{N}{2}-1}^{j-(j \bmod 2)+1,1}, \\ & (-1)^{j+1} \alpha_{N,0}^{j-(j \bmod 2),1}, \dots, (-1)^{j+1} \alpha_{N,\frac{N}{2}-1}^{j-(j \bmod 2),1}, \\ & \alpha_{N,\frac{N}{2}}^{j-(j \bmod 2)+1,1}, \dots, \alpha_{N,N-1}^{j-(j \bmod 2)+1,1}, \\ & (-1)^{j+1} \alpha_{N,\frac{N}{2}}^{j-(j \bmod 2),1}, \dots, (-1)^{j+1} \alpha_{N,N-1}^{j-(j \bmod 2),1}). \end{aligned} \quad (17)$$

Eq. (17) shows that the bi-phase sequence $a_{2N}^{j,1,2}$ is generated by interleaving the optical ZCZ sequence $\alpha_N^{j,1}$ of length N . Note that also the unbalanced sequences of $\alpha_N^{j,1}$ with $j = 0$ and 1 are used.

On the other hand, elements of the binary sequence $\hat{a}_{2N}^{j,1,2,d}$ are given by

$$\hat{a}_{2N,i}^{j,1,2,d} = \frac{1 + (-1)^d a_{2N,i}^{j,1,2}}{2}. \quad (18)$$

The periodic correlation function between $a_{2N}^{j,1,2}$ and $\hat{a}_{2N}^{j',1,2,d}$, except for an unbalanced sequence with $j = 0$ is given by

$$\rho_{a_{2N}^{j,1,2}, \hat{a}_{2N}^{j',1,2,d}, i'} = \begin{cases} N & ; i' = 0, j = j', d = 0, \\ -N & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; i' = \pm 1, \\ 0 & ; N - 1 \leq |i'| \leq N + 1. \end{cases} \quad (19)$$

Therefore, the above set $A^{1,2}$ which is a set of $(N/2 - 1)$ pairs of the bi-phase sequence $a_{2N}^{j,1,2}$ and the binary sequence $\hat{a}_{2N}^{j,1,2,d}$ of length $2N$ except at $j = 0$ is an optical ZCZ sequence set with $Z = 1$ at $G = 2$ positions.

In general, the optical ZCZ sequence set $A^{1,2^g}$ with $Z = 1$ at $G = 2^g$ which is a set of $\{N/2 - 2 + (g \bmod 2)\}$ pairs of the bi-phase sequence $a_{2^g N}^{j,1,2^g}$ and the binary sequence $\hat{a}_{2^g N}^{j,1,2^g,d}$ of length $2^g N$ is generated by repeating the same g times interleaving of Eq. (17) for $\alpha_N^{j,1}$. The periodic correlation function between $a_{2^g N}^{j,1,2^g}$ and $\hat{a}_{2^g N}^{j',1,2^g,d}$ is given by

$$\begin{aligned} \rho_{a_{2^g N}^{j,1,2^g}, \hat{a}_{2^g N}^{j',1,2^g,d}, i'} = & \begin{cases} 2^{g-1} N & ; i' = 0, j = j', d = 0, \\ -2^{g-1} N & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; i' = \pm 1, \\ 0 & ; kN - 1 \leq |i'| \leq kN + 1, \\ & 1 \leq k < G = 2^g, \end{cases} \end{aligned} \quad (20)$$

where if $g \equiv 0 \pmod{2}$ then the sequence numbers $j = 0$ and 1 are excepted, else the sequence number $j = 0$ is excepted.

2) *Optical ZCZ Sequence Set with $Z = 2^z$ at $G = 2^g$ positions:* The bi-phase sequence $a_{2^{g+1}N}^{j,2,2^g}$ of length $2^{g+1}N$ and ZCZ $Z = 2$ at $G = 2^g$ positions is generated by

$$\begin{aligned} a_{2^{g+1}N}^{j,2,2^g} = & (a_{2^g N,0}^{j-(j \bmod 2)+1,1,2^g}, (-1)^{j+1} a_{2^g N,0}^{j-(j \bmod 2),1,2^g}, \\ & a_{2^g N,1}^{j-(j \bmod 2)+1,1,2^g}, (-1)^{j+1} a_{2^g N,1}^{j-(j \bmod 2),1,2^g}, \\ & \dots, a_{2^g N,2^g N-1}^{j-(j \bmod 2)+1,1,2^g}, (-1)^{j+1} a_{2^g N,2^g N-1}^{j-(j \bmod 2),1,2^g}). \end{aligned} \quad (21)$$

On the other hand, elements of the binary sequence $\hat{a}_{2^{g+1}N}^{j,2,2^g}$ are obtained by Eq. (18).

The periodic correlation function between $a_{2^{g+1}N}^{j,2,2^g}$ and $\hat{a}_{2^{g+1}N}^{j',2,2^g,d}$, except at $j = 0$ and 1 when $g + 1 \equiv 0 \pmod{2}$ is given by

$$\begin{aligned} \rho_{a_{2^{g+1}N}^{j,2,2^g}, \hat{a}_{2^{g+1}N}^{j',2,2^g,d}, i'} = & \begin{cases} 2^g N & ; i' = 0, j = j', d = 0, \\ -2^g N & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq 2, \\ 0 & ; kN - 2 \leq |i'| \leq kN + 2, \\ & 1 \leq k < G = 2^g. \end{cases} \end{aligned} \quad (22)$$

Therefore, the above set $A^{2,2^g}$ which is a set of $[N/2 - 2 + \{(g + 1) \bmod 2\}]$ pairs of the bi-phase sequence $a_{2^{g+1}N}^{j,2,2^g}$ and the binary sequence $\hat{a}_{2^{g+1}N}^{j,2,2^g,d}$ of length $2^{g+1}N$ is an optical ZCZ sequence set with $Z = 2$ at $G = 2^g$ positions.

In general, the optical ZCZ sequence set $A^{2^z,2^g}$ with $Z = 2^z$ at $G = 2^g$ which is a set of $[N/2 - 2 + \{(g + z) \bmod 2\}]$ pairs of the bi-phase sequence $a_{2^{g+z}N}^{j,2^z,2^g}$ and the binary sequence $\hat{a}_{2^{g+z}N}^{j,2^z,2^g,d}$ of length $2^{g+z}N$ is generated by repeating the z times interleaving of Eq. (21) for $a_{2^g N}^{j,1,2^g}$. The periodic correlation function between $a_{2^{g+z}N}^{j,2^z,2^g}$ and $\hat{a}_{2^{g+z}N}^{j',2^z,2^g,d}$ is given by

$$\begin{aligned} \rho_{a_{2^{g+z}N}^{j,2^z,2^g}, \hat{a}_{2^{g+z}N}^{j',2^z,2^g,d}, i'} = & \begin{cases} 2^{g+z-1} N & ; i' = 0, j = j', d = 0, \\ -2^{g+z-1} N & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq 2^z, \\ 0 & ; kN - 2^z \leq |i'| \leq kN + 2^z, \\ & 1 \leq k < G = 2^g, \end{cases} \end{aligned} \quad (23)$$

where if $g + z \equiv 0 \pmod{2}$ then the sequence numbers $j = 0$ and 1 are excepted, else the sequence number $j = 0$ is excepted.

Therefore, we can obtain the optical ZCZ sequence set $A^{2^z,2^g}$ with ZCZ $Z = 2^g$ at $G = 2^g$ positions and $M = N/2 - 2 + \{(g + z) \bmod 2\}$ by first g times interleaving of Eq. (17) for $\alpha_N^{j,1}$, and then z times interleaving of Eq. (21).

V. EXAMPLE

Let $\alpha_8^{j,1}$ be a bi-phase sequence in an optical ZCZ sequence set $\mathcal{A}^1$ with $N = 8$ and $Z = 1$. From Eq. (14), the bi-phase sequence $\alpha_8^{j,1}$ is written as

$$\begin{aligned}\alpha_8^{0,1} &= (+, +, +, +, -, +, -, +), \\ \alpha_8^{1,1} &= (+, -, +, -, -, -, -, -), \\ \alpha_8^{2,1} &= (+, +, -, -, -, +, +, -), \\ \alpha_8^{3,1} &= (+, -, -, +, -, -, +, +).\end{aligned}$$

where + and - denote 1 and -1, respectively. Let $a_{16}^{j,1,2}$ be a bi-phase sequence in an optical ZCZ sequence set $A^{1,2}$ with $N = 16$ and $Z = 1$ at $G = 2$ positions. From Eq. (17), the bi-phase sequence $a_{16}^{j,1,2}$ is written as

$$\begin{aligned} a_{16}^{0,1,2} &= (+, -, +, -, -, -, -, -, -, -, -, -, -, -, -, -), \\ a_{16}^{1,1,2} &= (+, -, +, -, +, +, +, +, -, -, -, -, -, -, -), \\ a_{16}^{2,1,2} &= (+, -, -, +, -, -, +, +, -, -, +, +, +, -, -, +), \\ a_{16}^{3,1,2} &= (+, -, -, +, +, +, -, -, -, -, +, +, -, +, +, -). \end{aligned}$$

Let $a_{32}^{j,2,2}$ be a bi-phase sequence in an optical ZCZ sequence set $A^{j,2,2}$ with $N = 32$ and $Z = 2$ at $G = 2$ positions. From Eq. (21), the bi-phase sequence $a_{32}^{j,2,2}$ is written as

$$\begin{aligned} a_{32}^{0,2,2} &= (+, -, -, +, +, -, -, +, +, +, +, +, +, +, +, \\ &\quad -, +, -, +, -, +, -, +, -, -, +, +, -, -, +, -), \\ a_{32}^{1,2,2} &= (+, +, -, -, +, +, -, -, +, -, +, -, +, -, +, -, \\ &\quad -, -, -, -, -, -, -, -, +, +, -, -, +, +, -), \\ a_{32}^{2,2,2} &= (+, -, -, +, -, +, +, -, +, +, +, +, -, -, -, -, \\ &\quad -, +, -, +, +, -, +, -, -, -, +, +, +, +, -, -), \\ a_{32}^{3,2,2} &= (+, +, -, -, -, -, +, +, +, -, +, -, -, +, -, + \\ &\quad -, -, -, -, +, +, +, +, -, +, +, -, +, -, -, +). \end{aligned}$$

The mean value of the bi-phase sequence $a_{32}^{j,2,2}$ is given by $\sum_{i=0}^{31} a_{32,i}^{j,2,2} = 0$ with $j \neq 0$ and 1. From Eq. (18), the binary sequence number $\hat{a}_{32}^{j,2,2,d}$ of $d = 0$ and 1, and the sequence number $j = 2$ and 3 are written as

$$\begin{aligned} \hat{a}_{32}^{2,2,2,0} &= (+, 0, 0, +, 0, +, +, 0, +, +, +, +, 0, 0, 0, 0, \\ &\quad 0, +, 0, +, +, 0, +, 0, 0, 0, +, +, +, +, 0, 0), \\ \hat{a}_{32}^{3,2,2,0} &= (+, +, 0, 0, 0, 0, +, +, +, 0, +, 0, 0, +, 0, + \\ &\quad 0, 0, 0, 0, +, +, +, +, 0, +, +, 0, +, 0, 0, +), \\ \hat{a}_{32}^{2,2,2,1} &= (0, +, +, 0, +, 0, 0, +, 0, 0, 0, 0, +, +, +, +, \\ &\quad +, 0, +, 0, 0, +, 0, +, +, +, 0, 0, 0, 0, +, +), \\ \hat{a}_{32}^{3,2,2,1} &= (0, 0, +, +, +, +, 0, 0, 0, +, 0, +, +, 0, +, 0 \\ &\quad +, +, +, +, 0, 0, 0, 0, +, 0, 0, +, 0, +, +, 0). \end{aligned}$$

$A^{2,2} = \{(a_{32}^{2,2,2}, \hat{a}_{32}^{2,2,2,d}), (a_{32}^{3,2,2}, \hat{a}_{32}^{3,2,2,d})\}$ is an optical ZCZ sequence set with $N = 32$ and $Z = 2$ at $G = 2$ positions.

Figures 2 and 3 show the periodic auto-correlation values of $\hat{a}_{32}^{2,2,2}$ and $\hat{a}_{32}^{2,2,0}$, and the periodic cross-correlation values of $\hat{a}_{32}^{2,2,2}$ and $\hat{a}_{32}^{2,2,0}$, respectively. From figures, we can see that the ZCZ with $Z = 2$ appears on the left of right sides of shift $i' = 0$ and 16.

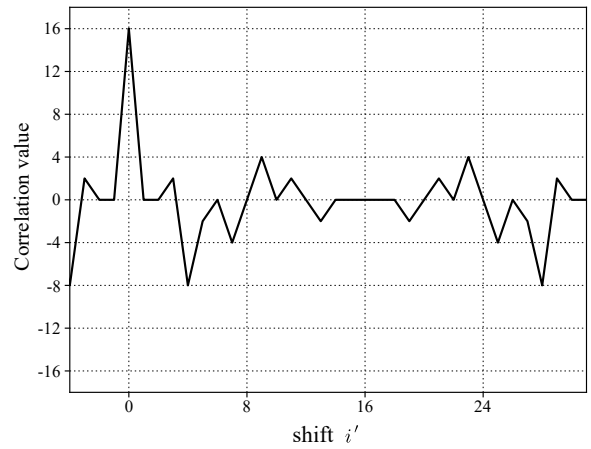


Figure 2. A periodic auto-correlation value of the optical ZCZ sequence set $A^{2,2}$ with $N = 32$, $Z = 2$ at $G = 2$ positions.

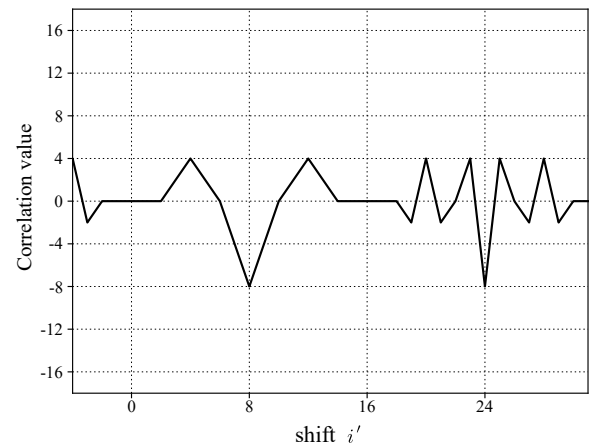


Figure 3. A periodic cross-correlation value of the optical ZCZ sequence set $A^{2,2}$ with $N = 32$, $Z = 2$ at $G = 2$ positions.

VI. CONCLUSION

In this paper, we have proposed a new PPM-OCDMA system that prevents the degradation of the emission time per symbol and suppresses the interference caused by reflected light, and have proposed a new generation method of optical ZCZ sequences suitable for the proposed system. By the proposed generation method, we can obtain an optical ZCZ sequence set having multiple ZCZ in the side lobes at power-of-two positions and equal intervals. In addition, this ZCZ size can be easily extended. The proposed PPM-OCDMA system using the new optical ZCZ sequence is expected to realize visible light communication that prevents illumination degradation and is resistant to interference caused by wall reflections.

Future research is needed to evaluate the performance of the proposed system by simulations and experiments assuming the actual indoor environment.

ACKNOWLEDGMENT

This work was supported by the Japan Society for the Promotion of Science (JSPS), Grant-in-Aid for Scientific Research

(C) (18K04145) and (C) (21K04062).

REFERENCES

- [1] F. R. K. Chung, J. A. Salehi, and V. K. Wei, "Optical orthogonal codes: design, analysis and applications," *IEEE Trans. on Information Theory*, vol. 35, issue 3, pp. 595–604, 1989.
- [2] Y. Kozawa and H. Habuchi, "Theoretical analysis of atmospheric optical DS/SS with On-Off Orthogonal M-sequence Pairs," *Proc. of the 6th International Conference on Information, Communications and Signal Processing*, pp. 1–5, 2007.
- [3] T.K. Matsushima, T. Nagao, N. Ochiai, and Y. Teramachi, "Generalization of modified prime sequence codes and its properties," *IEICE Trans. Fundamentals (Japanese Edition)*, vol. J91-A, no. 5, pp. 559–573, 2008.
- [4] T. Matsumoto and S. Matsufuji, "Optical ZCZ code generators using Sylvester-type Hadamard matrix," *Int. J. of Communications*, vol. 4, issue 1, pp. 22–29, 2010.
- [5] T. Matsumoto, M. Kurashita, Y. Ohira, H. Torii, Y. Ida, and S. Matsufuji, "Influence of ISI on synchronous OCDMA system using an optical ZCZ sequence set on a diffuse channel," *Proc. of the 8th International Workshop on Signal Design and Its Applications in Communications*, pp. 159–163, 2017.
- [6] D. Ozaki, T. Matsumoto, H. Torii, Y. Ida, and S. Matsufuji, "Study on BER performance of synchronous OCDMA system using optical ZCZ sequence with diffused link using rake reception," *Proc. of the 2021 RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing*, pp. 186–189, 2021.
- [7] K. Okawa, T. Matsumoto, H. Torii, Y. Ida, and S. Matsufuji, "Transmission experiment of optical ZCZ-CDMA system using 2- or 4-PPM," *Proc. of the 2019 International Workshop on Electronics, Information and Communication*, pp. 41–44, 2019.

Yasuaki Ohira received his B.Eng. degree in information science and engineering from Yamaguchi University, Japan, in 2013, and his Ph.D. degree in engineering from Yamaguchi University, Japan, in 2019. Since 2013, he has been a Technical Staff at Yamaguchi University, Japan. His research interests include optical CDMA systems and their applications. Dr. Ohira is a member of IEICE.

Takahiro Matsumoto received his B.Eng. and M.Eng. degrees in information and computer science from Kagoshima University, Japan, in 1996 and 1998, respectively, and his Ph.D. degree in engineering from Yamaguchi University, Japan, in 2007. He was a research associate from 1998 to 2007, an Assistant Professor from 2007 to 2012, an Associate Professor from 2012 to 2021 at Yamaguchi University, Japan. Since 2021, he has been a Professor at Kagoshima University. From 2010 to 2011, he was a visiting researcher at the University of Melbourne, Australia. His current research interests include spread spectrum systems and their applications. Prof. Matsumoto is a member of IEEE and IEICE.

Hideyuki Torii received his B.Eng., M.Eng., and Ph.D. degrees from the University of Tsukuba, Tsukuba, Japan, in 1995, 1997, and 2000, respectively. In 2000, he joined the Department of Network Engineering, Kanagawa Institute of Technology as a research associate. He is currently a Professor in the Department of Information Network and Communication at the same university. His research interests include spreading sequences, CDMA systems, and mobile communication systems. Prof. Torii is a member of IEEE and IEICE.

Yuta Ida received his B.E., M.E., and Ph.D. degrees from Hiroshima City University, Japan in 2008, 2010, and 2013, respectively. From 2011 to 2013, he was a special research student at Chiba University, Japan. From 2013 to 2021, he was an Assistant Professor at Yamaguchi University, Japan. Since 2021, he has been an Associate Professor at Yamaguchi University. His research interests are OFDM, MIMO, and cooperative communications for wireless communications. Prof. Ida is a member of IEEE and IEICE.