

Domain Identification By Frontier Observation In Cylindrical And Spherical Reference Frame

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Abstract :

The objective of this work, which was conducted at eINOV and Laboratory of Computer Science, Telecommunications and Applications (LITA) was to carry out the determination procedures to identify a domain by looking border. We proved that if well-defined signals are sent on the known edge Γ_0 of a regular field Ω on cylindrical and spherical shapes, where prevail some phenomena modelled by partial derivative equations and that following a certain number of measurements on the same edge, knowing $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma$ (where n indicates the normal external of $\partial\Omega$), it is possible to find a method which leads to the determination of Ω . The demonstration were made in the cylindrical and spherical references frames of a domain.

Keywords: Distribution theory, Transmission theorem, Domain recognition, frontier, algorithm.

I. INTRODUCTION

The recognition of a domain consists in finding the geometric shape of an object, knowing some of its physical characteristics. This concept is close to the concept of shape optimization introduced some thirty years ago and used in mechanics for the construction of solid and light objects [1]. In general, the criteria to be optimized are, for example, strength, efficiency, or radiated energy. These involve the solution of a system of equations with partial derivatives placed on an open space Ω .

While domain recognition is defined as a way of recognizing the geometrical shape of an object according to its characteristics, we found after our research that it was not easy or even easy for telephone operators to setup methods enabling them to: manage the interventions on possible disturbances on telecommunication lines and easily locate the domain or extent of a deposit or incident. The fields of application in which one can be brought to exploit the solutions posed by this kind of problem are varied.

Telephony Operators: determining the distance between a breakpoint and the known end of a domain represented by a cable telephone line. Determining the distance between a breakpoint and the known end of a domain represented by an optical fiber transmission line.

Mining Operators and Disaster Managers: determining the extent of an oil or ore deposit determining the extent of the impact of a disaster.

To justify the reasons for our research, in this article, we discuss the position of the problem, then we will present some mathematical concepts and some models of related problems. This in order to propose solutions of domain recognition in any form, stationary, orthogonal, cylindrical and spherical.

Note: In a previous article, we have already published the results of the research that allowed us to demonstrate and prove the determination of the domain in dimension 1 (stationary) and 2 (orthogonal).

II. POSITION OF THE PROBLEM

1. Excitation of a known edge Γ_0 of a domain

We consider a regular bounded domain $\Omega \subset \mathbb{R}^n$, $c \in C^0$ with value in $\mathbb{R}^+$ and g a continuous function on $\mathbb{R}^n$. It is assumed that the phenomenon occurring in Ω is modeled by a wave equation.

Given a domain in which there is a phenomenon modeled by partial differential equations, more particularly by the equation (1):

$$\begin{cases} -\Delta u + cu = 0 & \text{on } \Omega \\ u|_{\Gamma_0} = g \\ u|_{\partial\Omega \setminus \Gamma_0} = 0 \end{cases} \quad (1)$$

We send well-determined signals to the edge Γ_0 modeled by g and measure the integral $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma$ (where n denotes the exterior normal to $\partial\Omega$).

Question: knowing $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma$, was it be possible to determine the entire domain Ω ?

Before find an answer to this problem we will firstly remind some mathematical theories.

2. Related Mathematical Theories

2.1 Theory of distribution

Let T be a linear mapping from $D(\Omega)$ to $\mathbb{R}$ (where $D(\Omega)$ is the set of functions $C^\infty(\Omega)$ with compact support on Ω). The distribution on $D(\Omega)$ is defined as any application $T: D(\Omega) \rightarrow \mathbb{R}$ linear and continuous.

We denote $\forall \varphi \in D(\Omega)$, $\langle T, \varphi \rangle = T(\varphi)$. $D'(\Omega)$ is a set of distribution on Ω . Thus for every function u of class C^1 , we

defined the distribution $T_u \in \mathcal{D}'(\Omega)$, the dual of the functional space $\mathcal{D}(\Omega)$, define by :

$$\forall \varphi \in \mathcal{D}(\Omega) \quad T_u(\varphi) = \langle T_u, \varphi \rangle = \int_{\Omega} u(x) \varphi(x) dx \text{ and}$$

$$\forall \varphi \in \mathcal{D}(\Omega) \quad \langle \frac{\partial}{\partial x_i} T_u, \varphi \rangle = - \langle T_u, \frac{\partial \varphi}{\partial x_i} \rangle, \text{ for } i = 1, \dots, n.$$

Similarly for each multi-index $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbb{N}^n$, we have :

$$\forall \varphi \in \mathcal{D}(\Omega) \quad \langle \mathcal{D}^\alpha T_u, \varphi \rangle = (-1)^{|\alpha|} \langle T_u, \mathcal{D}^\alpha \varphi \rangle.$$

2.2 Principe of maximum

Theorem 1 :

Let Ω be an open set of $\mathbb{R}^n$ of class C^1 , $f \in L^2(\Omega)$ and $u \in H^1(\Omega)$ such that:

$$\forall v \in H_0^1(\Omega), \int_{\Omega} \sum_{i,j=1}^n \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_j} dx + \int_{\Omega} a_0 u v dx = \int_{\Omega} f v dx.$$

If $a_0 \in L^2$ and $a_0 \geq 0$ on Ω then ($u \geq 0$ on Γ_0) and ($f \geq 0$ on Ω) $\rightarrow (u \geq 0$ on Ω).

III. WELL-DEFINED PROBLEM PATTERNS

1. Keeping the flows in a transmission medium

Considering a scalar quantity $u(x)$ (temperature, chemical concentration...). By associating a flow (e.g: heat, matter...).

Whatever the integration volume ω , the outward flow of $u(x)$ of this quantity, $q(x)$ is zero at equilibrium: $\int_{\partial\omega} q(x) \cdot n_x ds = 0$

Applying the Green-Riemann formula [1] of flux divergence to it, we obtain: $\int_{\omega} \operatorname{div}_x q(x) dx = 0$

Assuming good regularity for $q(x)$, we get a local version of this equation, since ω is arbitrary, $\operatorname{div}_x k(x) = 0, \forall x \in \Omega$.

Because of the independence of the integration volume of G . Demengel [2].

If one further assumes that this flow is a linear function of the gradient ∇u , and oriented in the opposite direction (flows often run opposite the gradient of a quantity), in most situations it is physically reasonable to assume that the flow is proportional to the gradient ∇u . We can then write that:

$q(x) = -k(x) \nabla_x u$, $k(x) > 0$ (Fick's law of diffusion, Fourier's law of heat conduction, Ohm's law of electricity conduction).

We obtain an equilibrium conservation law of the type $\operatorname{div}_x (-k(x) \nabla u(x)) = 0$, $k(x) > 0$, we obtain the simplest of the elliptic equations, the Laplace equation which models the stationary phenomenon of conservation of flux on a transmission medium $\operatorname{div}(-\nabla u(x)) = 0$, which leads to $-\Delta u(x) = 0$ (Laplace equation).

This notion of flux conservation provides a mathematical model for equilibrium conservation laws in linear behavior, which is applied to the modeling of stationary phenomena in several fields of engineering sciences. Among others, we can cite by R. Dautray and J. L. Lions [3]:

- In the case of particle diffusion: $u(x)$ represents the chemical concentration at position x ; k is called the diffusion coefficient; $q(x)$ represents the chemical flux field; and the Laplace equation is called Fick's law.
- In the case of fluid flow in a porous medium: $u(x)$ represents the pressure of the fluid at position x ; k is called the hydraulic

conductivity; $q(x)$ represents the flow field in the fluid and Laplace's equation is called Darcy's law.

- In the case of electron flow in a conductor: $u(x)$ is the voltage at position x ; k is called the electrical conductivity; $q(x)$ is the flow of electrical current; and Laplace's equation is called Ohm's law.
- In steady state: the temperature $T(x)$ of a medium of uniform thermal conductivity, subjected to different temperatures at its edges, verifies Laplace's equation in the form $-\Delta T(x) = 0$.
- In the static regime: the electric potential in vacuum also verifies Laplace's equation.
- In the same way, the gravitational potential between the planets, also verifies Laplace's equation.

Thus, many physical problems can be reduced to the search for a scalar field that we call potential and that we denote $u(x)$ in dimension one, satisfying the Laplace equation. The first model of problem considered here and modeled by the Laplace equation could only find a solution if it is associated with limit conditions on each section of the boundary that will ensure that the problem is well posed.

The first model of the problem considered here and modeled by the Laplace equation can only find a solution if we associate boundary conditions on each piece of the boundary that will make the problem well posed.

2. Limit conditions associated with the Laplace equation

In general, in the Laplace equation, we look for the solution in an area of space Ω . The edges of Ω noted $\partial\Omega$ impose constraints: these are called boundary conditions. We usually distinguish, three types of boundary conditions:

- The value of the flux potential on the boundary $\partial\Omega$. This is the case, for example, when we study the electric potential that prevails between different electric conductors subjected to known voltages. In this case, we speak of the Dirichlet boundary conditions.
- The value of the derivative $\frac{\partial u}{\partial n}$ normal of the flux potential is fixed. We speak in this case of the Von Neumann conditions
- We can also impose these two conditions on the boundary $\partial\Omega$. This is the Cauchy condition.

In short, the Laplace equation can only have a solution if we have specified both $u|_{\partial\Omega}$ and $\frac{\partial u}{\partial n}|_{\partial\Omega} = (\overrightarrow{\operatorname{grad} u}) \cdot \vec{n}$ along the boundary. And from Green's formulation,

$$\int_{\partial\Omega} u \frac{\partial u}{\partial n_x} ds = \int_{\partial\Omega} (u \Delta u + \nabla u \nabla u) dx = \int_{\Omega} \nabla u \nabla u dx, \text{ because } \Delta u = 0.$$

Thus according to the theorems of G. Demengel, P. Benichou [2], for any function $g, h \in C(\Omega)$, the problem defined by equation (2):

$$\begin{cases} -\Delta u = 0, & x \in \Omega \\ u(x) = g(x), & x \in \partial\Omega_1 \\ \frac{\partial u(x)}{\partial n_x} = h(x), & x \in \partial\Omega_2 \end{cases} \quad (2)$$

Equation which, provided with boundary conditions, is well posed and justifies the position of problem (1), with:

$$\begin{cases} c = 0 \\ \Omega_1 = \Gamma_0 \\ \Omega_2 = \partial\Omega \setminus \Gamma_0 \end{cases}$$

3. Wave propagation in a transmission medium (coaxial medium in free space, isotropic medium)

Given a portion of the transmission guide shown in Figure 1, Maxwell's equations [6] in local form for any point inside the guide are written:

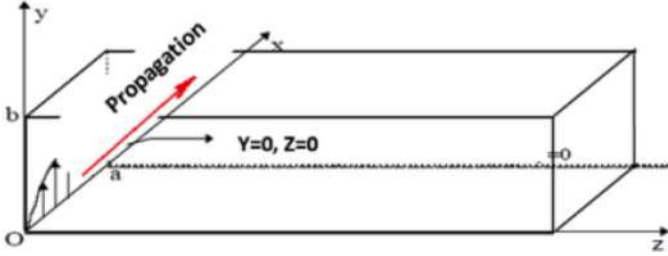


Figure 1. Transmitter unit

- Maxwell-Gauss $\text{div} \vec{E} = 0$
- Equation du flux magnétique $\text{div} \vec{B} = 0$
- Maxwell-Faraday $\text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- Maxwell-Ampère $\text{rot} \vec{E} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

The substitution of these equations allows us to obtain the wave propagation equation valid at each point inside the transmission medium.

$$\Rightarrow \Delta \vec{B} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

We are interested in the search for the solutions of the particular solutions of this equation in the form of progressive (stationary) waves, harmonic of pulsation ω , rectilinearly polarized and characterized by an electromagnetic field.

Let us consider the scalar function $s(x, t)$ defined by:

$$\frac{\partial^2 s}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 s}{\partial t^2} = 0 \text{ with } c^2 = \frac{1}{\mu_0 \epsilon_0}$$

By posing $s(x, t) = u(x)e^{-i\omega t}$, we obtain after transformation of the equation: $u''(x) - ku(x) = 0$, known as the Hemholtz with $k = \frac{\omega^2}{c^2}$. In general, for the same medium, the Dirichlet and Neumann conditions lead to different spectra [2]. And according to the theorems of G. Demengel, P. Benichou [2], for any function $g, h \in C(\Omega)$, the problem defined by equation (3):

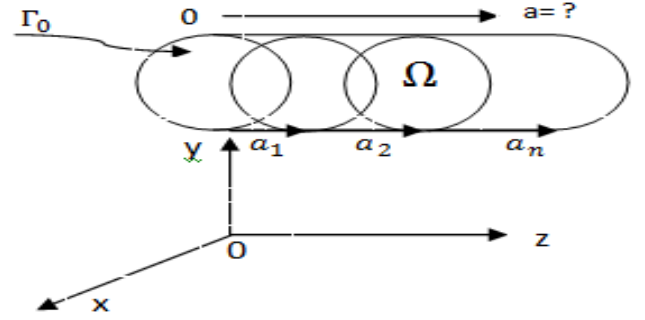
$$\begin{cases} -\Delta u(x) + cu(x) = 0, & x \in \Omega \\ u(x) = g(x), & x \in \partial\Omega_1 \\ \frac{\partial u(x)}{\partial n_x} = h(x), & x \in \partial\Omega_2 \end{cases} \quad (3)$$

And one of the boundary conditions is well posed and justifies

the position of problem (1), with $\begin{cases} c = k \\ \Omega_1 = \Gamma_0 \\ \Omega_2 = \partial\Omega \setminus \Gamma_0 \end{cases}$

Note: the problem thus posed is also valid in dimension two, three, cylindrical and spherical.

IV. DOMAIN RECOGNITION IN THE CYLINDRICAL AND SPHERICAL MARK



1. In dimension 3: cylindrical reference frame

Figure 2. Representation of Ω in dimension 3

In dimension 3, cylindrical shape, we will adopt the same considerations as in dimension 2

We consider the position of the problem defined by the following system (4):

$$\begin{cases} -\Delta u + cu = 0 & \text{on } \Omega \\ u|_{\Gamma_0} = f & \text{on } \partial\Omega \\ u|_{\partial\Omega \setminus \Gamma_0} = 0 \end{cases} \quad (4)$$

In order to determine the domain Ω of the phenomenon, it will be necessary first of all to find good function (the signal) which it will be used to excite the edge the phenomenon on the border of Ω .

Hypothesis: given function u the solution will be based on method of separation of variables of Herv DELINGETTE [11], u does not depend on the ordinate z :

$$\begin{cases} u(x, y, z) = u_1(x, y)u_3(z) \\ u(x, y, z)|_{\partial\Omega \setminus \Gamma_0} = 0 \end{cases}$$

This method of separating variables allows to obtains by application of Laplacian's formula a formulation of the problem in dimension 1, represented by the equation:

$$\begin{cases} -u_3''(z) + \lambda_1 u_3(z) = 0 & \text{on }]0, a[\\ u_3(0) = 1 \\ u_3'(0) \text{ is known} \\ u_3(a) = 0 \end{cases} \quad (5)$$

Thus the work to be done is reduced to dimension 1 and comes down to the determination of a of the interval $]0, a[$. to get there, it will be necessary to determine the values of u_1 and λ_1 .

Determination the values of u_1 and λ_1 :

To really determine the values of u_1 and λ_1 , we have to know exactly what type of signal it will be important to use to excite our phenomenon on the border of the disk $D(0, R)$, so after changing variables in the following equation:

$$\begin{cases} -\Delta u_1 + c(x, y)u_1(x, y) = \lambda_1 u_1 & \text{on } D(0, R) \\ u_1 = 0 & \text{on } C(0, R) \end{cases} \quad (6)$$

We obtain:

$$\Delta u_1 = \frac{\partial^2 u_1}{\partial^2 r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_1}{\partial \theta^2} \quad (7)$$

- In the case where have $c = 0$ then $-\Delta u_1 = \lambda_1 u_1$
- Else let's find v_1 and v_2 such that $u_1(r, \theta) = v_1(r)v_2(\theta)$ on $D(0, R)$. $v_1(R)=0$ and $v_2(\theta)$ is periodic with period of 2π .

By choosing $v_2(\theta) = \sin(\theta)$, then $u_1(r, \theta) = v_1(r)\sin(\theta)$ and we end up with the following system of equations:

$$\begin{cases} -v_1''(r)v_2(\theta) - \frac{1}{r}v_1'(r)v_2(\theta) - \frac{1}{r^2}v_1(r)v_2''(\theta) = \lambda_1 v_1(r)v_2(\theta) \\ v_1(R) = 0 \\ v_2(\theta) = \sin(\theta) \text{ is periodic of } 2\pi \end{cases}$$

It's comes that :

$$\left(v_1''(r) + \frac{1}{r}v_1'(r) + \lambda_1 v_1(r) \right) v_2(\theta) + \frac{1}{r^2}v_1(r)v_2''(\theta) = 0$$

Which implies the following result:

$$v_1''(r) + \frac{1}{r}v_1'(r) + \lambda_1 v_1(r) + \frac{1}{r^2}v_1(r) \frac{v_2''(\theta)}{v_2(\theta)} = 0$$

Therefore, given $\alpha \in \mathbb{R}$, we pose $v_2''(\theta) = \alpha v_2(\theta)$ and suppose for the continuation that $\alpha = -n^2 (n \in \mathbb{R})$. Then knowing that $v_2(\theta) = \sin(\theta)$ is 2π periodic, the above equation becomes:

$$v_1''(r) + \frac{1}{r}v_1'(r) + \lambda_1 v_1(r) + \frac{\alpha}{r^2}v_1(r) = 0, \text{ or simply:}$$

$$r^2 v_1''(r) + r v_1'(r) + (\alpha + r^2 \lambda_1) v_1(r) = 0, \text{ more precisely:}$$

$$r^2 v_1''(r) + r v_1'(r) + (\lambda_1 r^2 - n^2) v_1(r) = 0$$

By replacing r by $r\sqrt{\lambda_1}$, the new equation to be solved:

$$r^2 \lambda_1 v_1''(r\sqrt{\lambda_1}) + r v_1'(r\sqrt{\lambda_1}) + (\lambda_1^2 r^2 - n^2) v_1(r\sqrt{\lambda_1}) = 0$$

And by posing: $\rho = r\sqrt{\lambda_1}$, we finally obtain:

$$\frac{d^2 v_1}{d\rho^2} + \frac{1}{\rho} \frac{dv_1}{d\rho} + \left(1 - \frac{n^2}{\rho^2} \right) v_1 = 0 \quad (8)$$

Through equation (7), we then recognize a Bessel of differential equation whose solutions are in the form of the series:

$$v_1(\rho) = \sum_0^n a_n \rho^n, \text{ where } v_1 \text{ is an } n \text{ order function of Bessel.}$$

Thus, the signals which will be used to excite the phenomenon on the border of the domain will be the Bessel functions of first kind (Bessel J_n) with Bessel $J_n(\rho) = v_1(r\sqrt{\lambda_1})$. We find that

$v_1(r) = v_1\left(\frac{\rho}{\sqrt{\lambda_1}}\right)$ and the eigenvalues of the operator their roots, which will be calculated as follows:

$v_1(\rho) = 0$, so if ρ_0 is the root of an n order function of Bessel, then according to the above, $\rho_0 = r\sqrt{\lambda_1}$.

From where $\lambda_1 = \left(\frac{\rho_0}{\sqrt{\lambda_1}}\right)^2$ is the eigenvalue of the operator associate to a corresponding function of Bessel.

We then deduce an algorithm to determine u_1 and λ_1 :

Algorithm 1: determination of the u_i and λ_1

1. *Begin*
2. *Consider* $n \in \mathbb{N}$
3. *Determining* $v_1(r) = J_n(r)$
4. *Determining a root* ρ_0 *of* $v_1(r)$
5. *For* $\sqrt{\lambda_1} = \frac{\rho_0}{R}$, *then* $\lambda_1 = \left(\frac{\rho_0}{\sqrt{\lambda_1}}\right)^2$
6. *For* $v_1(r) = v_1\left(\frac{R}{\sqrt{\lambda_1}}\right) = \text{Bessel}J_n\left(\frac{R}{\sqrt{\lambda_1}}\right) = \text{Bessel}J_n\left(\frac{R_r}{\rho_0}\right)$, *then* $u_1(r, \theta) = v_1(r) \sin(\theta) = \text{Bessel}J_n\left(\frac{R_r}{\rho_0}\right)$
7. *End*

After the values have been determined, we can deduce the algorithm for determining the domain.

Algorithm 2: procedure to determine domain Ω

1. *Begin*
2. *Consider sub program to excite the phenomenon at the border of the domain (here disk, cylindrical) with the signal (equation (6));*
3. *It is important to note here that the phenomenon must be excited only on the disk $D(0, R)$ with a specific signal that will determine the domain;*
4. *Call algorithm 1;*
5. *Knowing* $u_1(x, y)$ *that:* $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma = -u_1'(0) \int_{\Gamma_0} u_1(x, y) d\Gamma;$
6. *The domain will be determined by solving the following equation in dimension 1:*

$$\begin{cases} -u_3''(z) + \lambda_1 u_3(z) = 0 & \text{on }]0, a[\\ u_3(0) = 1 \\ u_3'(0) \text{ is known} \\ u_3(a) = 0 \end{cases}$$
7. *Call for algorithm to calculate the domain in dimension 1.[]*
8. *End*

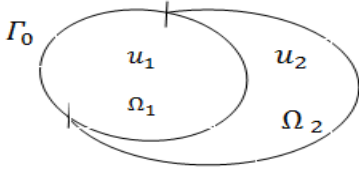
Assuming beforehand that the corresponding $u_{a_i}'(0)$ values for each node a_i of the domain representing an optical fiber transmission line. Values were measured and stored in a database, as shown in the table below.

a_1	a_2	...	a_n
$u'_{a_1}(0)$	$u'_{a_2}(0)$	...	$u'_{a_n}(0)$

Table 1: Results of the values of a_i in cylindrical form of the domain.

2. Any dimension : spherical reference frame

For a spherical shape of the domain, Γ_0 a surface of the domain represented by the following figure:

Figure 2. Representation of Ω in spherical domain

For the res we consider the following hypothesis: assuming that there exists a nonzero and positive function g_0 not defined on Γ_0 . $\forall \Omega \in E_{\Gamma_0}$, $\widetilde{g}_0: \partial\Omega \rightarrow \mathbb{R}$ defined by

$$\widetilde{g}_0(x) = \begin{cases} g_0(x) & \text{if } x \in \Gamma_0 \\ 0 & \text{if } x \in \partial\Omega \setminus \Gamma_0 \end{cases} \quad \widetilde{g}_0 \text{ admits a bearing on } \Omega \text{ belonging to } H^2(\Omega).$$

Theorem 2: given $E = \{\Omega \text{ bound domains of } \mathbb{R}^3 | \Gamma_0 \subset \partial\Omega\}$, $\forall \Omega_1, \Omega_2 \in E | \Omega_1 \subset \Omega_2$ then $\int_{\Gamma_0} \frac{\partial u_2}{\partial n} g d\Gamma \leq \int_{\Gamma_0} \frac{\partial u_1}{\partial n} g d\Gamma$.

This theorem according of Samuel OUYA [12] allows us to conclude for the case of spherical domain, the known conditions at the border of the domain imply by dichotomy knowledge of subdomains and gradually that of the target domain. So, in this context where the shape of the domain is spherical, we can also build an algorithm that could particularly be used industrials or researchers having detect a deposit of ores or oil to estimate the area of the overall extent of the deposit.

Algorithm describing the procedure to estimate the area of the extended domain Ω_i of is explain as follow:

Algorithm 3: Describing procedure to determine values of Ω_i (Figure 3)

1. *Begin*
2. *Consider* $E = \{\Omega \text{ bound domains of } \mathbb{R}^3 | \Gamma_0 \subset \partial\Omega\}$;
3. *Considering* $\begin{cases} -\Delta u_i + cu_i = 0 & \text{on } \Omega_i \\ u_i|_{\Gamma_0} = g \geq 0 \\ u_i|_{\partial\Omega \setminus \Gamma_0} = 0 \end{cases}$;
4. *If* $\Omega_{i-1} \subset \Omega_i$, *then* $\int_{\Gamma_0} \frac{\partial u_i}{\partial n} g d\Gamma \leq \int_{\Gamma_0} \frac{\partial u_{i-1}}{\partial n} g d\Gamma$
5. *Call algorithm to estimate the values of the domain in dimension 1.*
6. *Determine values of* Ω
7. *End*

Ω_1	Ω_2	...	Ω_n
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$\int_{\Gamma_u} \frac{\partial u_1}{\partial n} g d\Gamma$	$\int_{\Gamma_u} \frac{\partial u_2}{\partial n} g d\Gamma$	...	$\int_{\Gamma_u} \frac{\partial u_n}{\partial n} g d\Gamma$
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Table 2. Results of different values of Ω_i

V CONCLUSION

The study that we carried out made it possible to reach results allowing to propose algorithms to estimate and an entire domain by sending specifics signals on the border. In this article we extended the work cylindrical and spherical shape. This results are applicable to the domain of telephony, transmission over optical fiber, electricity, determination of overall extension area of oil or minerals field...

To achieve these results, we used Sobolev's theories, the formula for variation of variables, the green formula, the principle of the maximum for existence and the uniqueness of the solutions, Reisz theorem, Samuel OUYA and NIANE theorem and many other theorems. The excitation signals of the phenomena that we proposed in cylindrical case depend on the functions of Bessel, thus leading to justify that our problem was well defined and posed. In spherical case we have shown under certain conditions that the strict regression of the function $\int_{\Gamma_0} \frac{\partial u_i}{\partial n} g d\Gamma$ (where n denotes the normal outside of $\partial\Omega$, leads to the determination of an algorithm allowing to approach and to approximate a domain Ω by is dichotomy. In perspective, it will be interesting to assume that the domain Ω is governed by a non-stationary phenomenon, thus to study the determination of the domain in the eventual area.

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