

Domain Recognition By Border Observation In Dimension 1 & 2

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Abstract :

The objective of this work, which was conducted at eINOV and Laboratory of Computer Science, Telecommunications and Applications (LITA) was to carry out the determination procedures to identify a domain by looking border. We proved that if well-defined signals are sent on the known edge Γ_0 of a regular field Ω of $\mathbb{R}^n$ ($n=1, 2$), where prevail some phenomena modeled by partial derivative equations and that following a certain number of measurements on the same edge, knowing $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma$ (where n indicates the normal external of $\partial\Omega$), it is possible to find a method which leads to the determination of Ω . The demonstration were made in the stationary case with dimension $n=1, 2$ and arbitrary form.

Keywords: Distribution theory, Transmission theory, Domain recognition, border.

I. INTRODUCTION

The recognition of a domain consists in finding the geometric shape of an object, knowing some of its physical characteristics. This concept is close to the concept of shape optimization introduced some thirty years ago and used in mechanics for the construction of solid and light objects [1]. In general, the criteria to be optimized are, for example, strength, efficiency, or radiated energy. These involve the solution of a system of equations with partial derivatives placed on an open space Ω .

While domain recognition is defined as a way of recognizing the geometrical shape of an object according to its characteristics, we found after our research that it was not easy or even easy for telephone operators to setup methods enabling them to: manage the interventions on possible disturbances on telecommunication lines and easily locate the domain or extent of a deposit or incident. The fields of application in which one can be brought to exploit the solutions posed by this kind of problem are varied.

Telephony Operators: determining the distance between a breakpoint and the known end of a domain represented by a cable telephone line. Determining the distance between a breakpoint and the known end of a domain represented by an optical fiber transmission line.

Mining Operators and Disaster Managers: determining the extent of an oil or ore deposit determining the extent of the impact of a disaster.

To justify the reasons for our research, in this article, we discuss the position of the problem, then we will present some mathematical concepts and some models of related problems. This in order to propose solutions of domain identification in dimension $n = 1, 2$.

II. POSITION OF THE PROBLEM

1. Excitation of a known edge Γ_0 of a domain

We consider a regular bounded domain $\Omega \subset \mathbb{R}^n$ ($n = 1, 2$), $c \in C^0(\mathbb{R}^n$ ($n=1,2$)) with value in $\mathbb{R}^+$ and g a continuous function on $\mathbb{R}^n$ ($n = 1, 2$). It is assumed that the phenomenon occurring in Ω is modelled by a wave equation.

Given a domain in which there is a phenomenon modeled by partial differential equations, more particularly by the equation (1):

$$\begin{cases} -\Delta u + cu = 0 & \text{on } \Omega \\ u|_{\Gamma_0} = g \\ u|_{\partial\Omega \setminus \Gamma_0} = 0 \end{cases} \quad (1)$$

We send well-determined signals to the edge Γ_0 modeled by g and measure the integral $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma$ (where n denotes the exterior normal to $\partial\Omega$).

Question: knowing $\int_{\Gamma_0} \frac{\partial u}{\partial n} g d\Gamma$, was it be possible to determine the entire domain Ω ?

Before find an answer to this problem we will firstly remind some mathematical theories.

2. Related Mathematical Theories

2.1 Theory of distribution

Let T be a linear mapping from $D(\Omega)$ to $\mathbb{R}$ (where $D(\Omega)$ is the set of functions $C^\infty(\Omega)$ with compact support on Ω). The distribution on $D(\Omega)$ is defined as any application $T: D(\Omega) \rightarrow \mathbb{R}$ linear and continuous.

We denote $\forall \varphi \in D(\Omega)$, $\langle T, \varphi \rangle = T(\varphi)$. $\mathcal{D}'(\Omega)$ is a set of distribution on Ω . Thus for every function u of class C^1 , we defined the distribution $T_u \in \mathcal{D}'(\Omega)$, the dual of the functional space $\mathcal{D}(\Omega)$, define by :

$$\forall \varphi \in \mathcal{D}(\Omega) \quad T_u(\varphi) = \langle T_u, \varphi \rangle = \int_{\Omega} u(x) \varphi(x) dx \text{ and}$$

$$\forall \varphi \in \mathcal{D}(\Omega) \quad \left\langle \frac{\partial}{\partial x_i} T_u, \varphi \right\rangle = - \langle T_u, \frac{\partial \varphi}{\partial x_i} \rangle, \text{ for } i = 1, \dots, n.$$

Similarly for each multi-index $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbb{N}^n$, we have :

$$\forall \varphi \in \mathcal{D}(\Omega) \quad \langle \mathcal{D}^\alpha T_u, \varphi \rangle = (-1)^{|\alpha|} \langle T_u, \mathcal{D}^\alpha \varphi \rangle.$$

2.2 Principe of maximum

Theorem 1 :

Let Ω be an open set of $\mathbb{R}^n$ of class C^1 , $f \in L^2(\Omega)$ and $u \in H^1(\Omega)$ such that:

$$\forall v \in H_0^1(\Omega), \int_{\Omega} \sum_{i,j=1}^n \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_j} dx + \int_{\Omega} a_0 uv dx = \int_{\Omega} f v dx.$$

If $a_0 \in L^2$ and $a_0 \geq 0$ on Ω then ($u \geq 0$ on Γ_0) and ($f \geq 0$ on Ω) $\rightarrow (u \geq 0$ on Ω).

III. WELL-DEFINED PROBLEM PATTERNS

1. Keeping the flows in a transmission medium

Considering a scalar quantity $u(x)$ (temperature, chemical concentration...). By associating a flow (e.g: heat, matter...).

Whatever the integration volume ω , the outward flow of $u(x)$ of this quantity, $q(x)$ is zero at equilibrium: $\int_{\partial\omega} q(x) \cdot n_x ds = 0$

Applying the Green-Riemann formula [1] of flux divergence to it, we obtain: $\int_{\omega} \operatorname{div}_x q(x) dx = 0$

Assuming good regularity for $q(x)$, we get a local version of this equation, since ω is arbitrary, $\operatorname{div}_x k(x) = 0, \forall x \in \Omega$.

Because of the independence of the integration volume of G . Demengel [2].

If one further assumes that this flow is a linear function of the gradient ∇u , and oriented in the opposite direction (flows often run opposite the gradient of a quantity), in most situations it is physically reasonable to assume that the flow is proportional to the gradient ∇u . We can then write that:

$q(x) = -k(x) \nabla_x u$, $k(x) > 0$ (Fick's law of diffusion, Fourier's law of heat conduction, Ohm's law of electricity conduction).

We obtain an equilibrium conservation law of the type $\operatorname{div}_x (-k(x) \nabla u(x)) = 0$, $k(x) > 0$, we obtain the simplest of the elliptic equations, the Laplace equation which models the stationary phenomenon of conservation of flux on a transmission medium $\operatorname{div}(-\nabla u(x)) = 0$, which leads to $-\Delta u(x) = 0$ (Laplace equation).

This notion of flux conservation provides a mathematical model for equilibrium conservation laws in linear behavior, which is applied to the modeling of stationary phenomena in several fields of engineering sciences. Among others, we can cite by R. Dautray and J. L. Lions [3]:

- In the case of particle diffusion: $u(x)$ represents the chemical concentration at position x ; k is called the diffusion coefficient; $q(x)$ represents the chemical flux field; and the Laplace equation is called Fick's law.
- In the case of fluid flow in a porous medium: $u(x)$ represents the pressure of the fluid at position x ; k is called the hydraulic conductivity; $q(x)$ represents the flow field in the fluid and Laplace's equation is called Darcy's law.

- In the case of electron flow in a conductor: $u(x)$ is the voltage at position x ; k is called the electrical conductivity; $q(x)$ is the flow of electrical current; and Laplace's equation is called Ohm's law.
- In steady state: the temperature $T(x)$ of a medium of uniform thermal conductivity, subjected to different temperatures at its edges, verifies Laplace's equation in the form $-\Delta T(x) = 0$.
- In the static regime: the electric potential in vacuum also verifies Laplace's equation.
- In the same way, the gravitational potential between the planets, also verifies Laplace's equation.

Thus, many physical problems can be reduced to the search for a scalar field that we call potential and that we denote $u(x)$ in dimension one, satisfying the Laplace equation. The first model of problem considered here and modeled by the Laplace equation could only find a solution if it is associated with limit conditions on each section of the boundary that will ensure that the problem is well posed.

The first model of the problem considered here and modeled by the Laplace equation can only find a solution if we associate boundary conditions on each piece of the boundary that will make the problem well posed.

2. Limit conditions associated with the Laplace equation

In general, in the Laplace equation, we look for the solution in an area of space Ω . The edges of Ω noted $\partial\Omega$ impose constraints: these are called boundary conditions. We usually distinguish, three types of boundary conditions:

- The value of the flux potential on the boundary $\partial\Omega$. This is the case, for example, when we study the electric potential that prevails between different electric conductors subjected to known voltages. In this case, we speak of the Dirichlet boundary conditions.
- The value of the derivative $\frac{\partial u}{\partial n}$ normal of the flux potential is fixed. We speak in this case of the Von Neumann conditions
- We can also impose these two conditions on the boundary $\partial\Omega$. This is the Cauchy condition.

In short, the Laplace equation can only have a solution if we have specified both $u|_{\partial\Omega}$ and $\frac{\partial u}{\partial n}|_{\partial\Omega} = (\overrightarrow{\operatorname{grad} u}) \cdot \vec{n}$ along the boundary. And from Green's formulation,

$$\int_{\partial\Omega} u \frac{\partial u}{\partial n_x} ds = \int_{\partial\Omega} (u \Delta u + \nabla u \nabla u) dx = \int_{\Omega} \nabla u \nabla u dx, \text{ because } \Delta u = 0.$$

Thus according to the theorems of G. Demengel, P. Benichou [2], for any function $g, h \in C(\Omega)$, the problem defined by equation (2):

$$\begin{cases} -\Delta u = 0, & x \in \Omega \\ u(x) = g(x), & x \in \partial\Omega_1 \\ \frac{\partial u(x)}{\partial n_x} = h(x), & x \in \partial\Omega_2 \end{cases} \quad (2)$$

Equation which, provided with boundary conditions, is well posed and justifies the position of problem (1), with:

$$\begin{cases} c = 0 \\ \Omega_1 = \Gamma_0 \\ \Omega_2 = \partial\Omega \setminus \Gamma_0 \end{cases}$$

3. Wave propagation in a transmission medium (coaxial medium in free space, isotropic medium)

Given a portion of the transmission guide shown in Figure 1, Maxwell's equations [6] in local form for any point inside the guide are written:

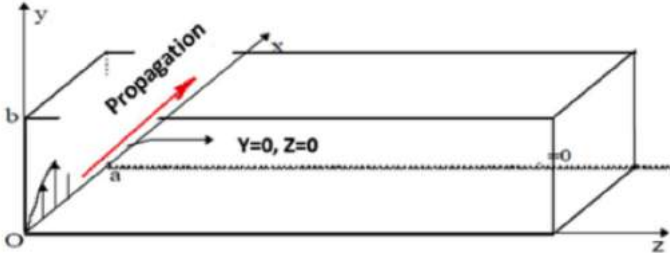


Figure 1. Transmitter unit

- ▶ *Maxwell-Gauss* $\text{div} \vec{E} = 0$
- ▶ *Equation du flux magnétique* $\text{div} \vec{B} = 0$
- ▶ *Maxwell-Faraday* $\text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- ▶ *Maxwell-Ampère* $\text{rot} \vec{E} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

The substitution of these equations allows us to obtain the wave propagation equation valid at each point inside the transmission medium.

$$\Rightarrow \Delta \bar{B} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{B}}{\partial t^2} = 0$$

We are interested in the search for the solutions of the particular solutions of this equation in the form of progressive (stationary) waves, harmonic of pulsation ω , rectilinearly polarized and characterized by an electromagnetic field.

Let us consider the scalar function $s(x,t)$ defined by:

$$\frac{\partial^2 s}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 s}{\partial t^2} = 0 \text{ with } c^2 = \frac{1}{\mu_0 \epsilon_0}$$

By posing $s(x,t) = u(x)e^{-i\omega t}$, we obtain after transformation of the equation: $u''(x) - ku(x) = 0$, known as the Hemholtz with $k = \frac{\omega^2}{c^2}$. In general, for the same medium, the Dirichlet and Neumann conditions lead to different spectra [2]. And according to the theorems of G. Demengel, P. Benichou [2], for any function $g, h \in C(\Omega)$, the problem defined by equation (3):

$$\begin{cases} -\Delta u(x) + cu(x) = 0, & x \in \Omega \\ u(x) = g(x), & x \in \partial\Omega_1 \\ \frac{\partial u(x)}{\partial n_x} = h(x), & x \in \partial\Omega_2 \end{cases} \quad (3)$$

And one of the boundary conditions is well posed and justifies

the position of problem (1), with $\begin{cases} c = k \\ \Omega_1 = \Gamma_0 \\ \Omega_2 = \partial\Omega \setminus \Gamma_0 \end{cases}$

Note: the problem thus posed is also valid in dimension two, three, cylindrical and spherical.

IV. DOMAIN RECOGNITION IN THE LINEAR AND ORTHOGONAL REFERENCE FRAME

1. In dimension 1 : stationary shape

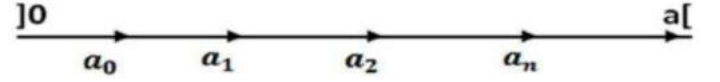


Figure 2. Representation of Ω in dimension 1

Given figure 2, if $\Omega =]0, a[$ then then determining Ω would be like determining a knowing that the 0 end is known.

It is assumed that the phenomenon is modeled by equation (4)

below

$$\begin{cases} -u''(x) + c(x)u(x) = 0 & (1) \text{ pour } 0 < x < a \\ u(0) = 1 \text{ and } u(a) = 0 \end{cases} \quad (4)$$

with $a \in \mathbb{R}$ et $c \in C^0]0, a[$

We also assume that $c(x) \geq 0, \forall x \in \mathbb{R}$

It is further supposed that $u'(0)$ is given, then how do we find a $(0 < x < a)$ so that the problem is well posed?

Theorem 2: $\forall c, a$ positive function $\in C^0(\mathbb{R}^n (n = 1, 2, 3))$ with values in $\mathbb{R}^+$, so the function $g: a \rightarrow g(a) = u'(0)$ de $\mathbb{R}_*^+$ with values in $] -\infty, 0[$ is injective and strictly decreasing.

With this theorem, we show that a is uniquely determined.

Algorithm 1: determination of the a_i values of the domain Ω (Figure 2.)

1. Start
2. Consider a breakpoint
3. Successively browse database containing different pre-registered nodes (a_i) to know where a is located.
4. If $a > a_i$ then $g(a) < g(a_i)$
5. Considering (a_1) (the first node of the known domain)
6. If $g(a) > g(a_1)$ then $a_1 < a$ else $a > a_1$
7. Endif
8. Look for $i_0 \in \{1, \dots, n\}$ such that
9. If $g(a_{i_0+1}) < g(a) < g(a_{i_0})$ then $a_{i_0+1} < a < a_{i_0}$
10. EndiF
11. EndiF
12. End

a_1	a_2	...	a_n
$u'_{a_1}(0)$	$u'_{a_2}(0)$	...	$u'_{a_n}(0)$

Table 1: Results of the values of a_i in dimension 1

2. In dimension 2 : orthogonal reference frame

Let O be a known nonempty bounded open of $\mathbb{R}^2$ and of Lebesgue measure denoted $\text{mes}(O)$. Γ_0 be a nonempty part of the adherence $\bar{O}$ of the open O . $E_{\Gamma_0}(O) = \{\Omega \subset O, \Omega \text{ bounded domain of class } C^2 \text{ with } \Gamma_0 \subset \partial\Omega\}$

It is assumed that there exists a nonzero and positive function g_0 defined on Γ_0 .

$\forall \Omega \in E_{\Gamma_0}(O)$, $\tilde{g}_0: \partial\Omega \rightarrow \mathbb{R}$ defined by

$$\tilde{g}_0(x) = \begin{cases} g_0(x) & \text{si } x \in \Gamma_0 \\ 0 & \text{si } x \in \partial\Omega \setminus \Gamma_0 \end{cases} \quad \tilde{g}_0 \text{ admits a bearing on } \Omega$$

belonging to $H^2(\Omega)$.

For $\Omega \in E_{\Gamma_0}(O)$, we say that u is a solution of the equation below from the problem posed.

$$\begin{cases} -\Delta u(x) + q(x)u(x) = 0 & \text{in } \Omega \\ u = g_0 & \text{on } \Gamma_0 \\ u = 0 & \text{on } \partial\Omega \setminus \Gamma_0 \end{cases} \quad (5)$$

We define on $E_{\Gamma_0}(O)$ the application δ :

$\delta: E_{\Gamma_0}(O) \rightarrow \mathbb{R}$, such that $\forall \Omega$ we associate $\delta(\Omega) = \int_{\Gamma_0} \frac{\partial u}{\partial n} g_0 d\Gamma$

The fundamental problem in this part is to answer the following question:

Let $\Omega \in E_{\Gamma_0}(O)$, does knowledge of $\delta(\Omega)$ as stated in the problem posed allow us to determine Ω ?

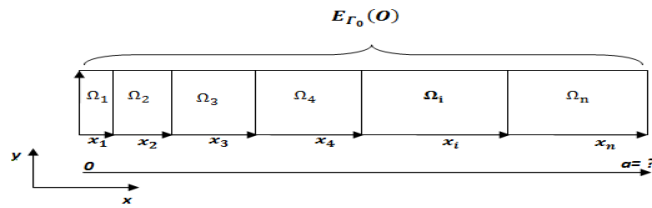


Figure 3. consideration of the domain Ω in dimension 2

Theorem 3: OUYA and NIANE basic theorem [5] [6]

Let $\Omega_1, \Omega_2 \in E_{\Gamma_0}(O)$ such that $\Omega_1 \subset \Omega_2$ then $\delta(\Omega_1) - \delta(\Omega_2) > 0$

The proof of this fundamental result relies on the following preliminary lemmas:

Lemma 1: Let $\Omega_1, \Omega_2 \in E_{\Gamma_0}(O)$ such that $\Omega_1 \subset \Omega_2$ then $\partial\Omega_1 \cap \Omega_2 \neq \emptyset$ [9].

Lemma 2: Let Ω an open a bounded open of class C^2 and Γ_1 an open non-empty part of its edge. If $u \in C(\Omega \cup \Gamma_1)$ verified $\begin{cases} \Delta u = 0 & \text{in } D'(\Omega) \\ u = 0 & \text{on } \Gamma_1 \end{cases}$

Lemma 3: Let Ω an open a bounded open of class C^2 . If $u \in H_0^1(\Omega)$ et $-\Delta u \in C(\bar{\Omega})$ then $\forall \alpha \in]0, 1[$, $u \in C^{1,\alpha}(\bar{\Omega})$.

Lemma 4: Let $\Omega \in E_{\Gamma_0}(O)$ and u solution of (1) then $u \in C^1(\Omega \cup (\partial\Omega \setminus \Gamma_0))$ [9].

Lemma 5: Let Ω an open a bounded open of class C^2 of edge $\partial\Omega$ et $q \in L^\infty(\Omega)$. Let B an open ball of Ω such that $B \cap \partial\Omega \neq \emptyset$, if $u \in H^2(\Omega)$ and verify $\begin{cases} -\Delta u + qu = 0 & \text{sur } \Omega \\ \frac{\partial u}{\partial n} = 0 & \text{sur } B \cap \partial\Omega \end{cases}$ then $u=0$ in Ω , where hence the proof of theorem 3.

This theorem is the fundamental result on which is based the method we propose here to recognize the domain Ω sought in dimension 2 stationary case, knowing the value of $\delta(\Omega)$. This

theorem generalizes to the determination of the domain for any form.

Algorithm 2: Describing procedure to determine values of Ω_i (Figure 3)

1. Start
2. Considering
3. $E_{\Gamma_0}(O) = \{\Omega \subset O, \Omega \text{ bounded domain of class } C^2 \text{ with } \Gamma_0 \subset \partial\Omega\}$;
4. Considering $\begin{cases} -\Delta u_i + cu_i = 0 & \text{on } \Omega_i \\ u_i|_{\Gamma_0} = g \geq 0 \\ u_i|_{\partial\Omega \setminus \Gamma_0} = 0 \end{cases}$;
5. Following the iteration on i ;
6. If for any $\delta: E_{\Gamma_0}(O) \rightarrow \mathbb{R}$, the value of $\delta(\Omega_i) = \int_{\Gamma_0} \frac{\partial u_i}{\partial n} g_0 d\Gamma$ exist then application of the algorithm developed in dimension 1.
7. Determination of Ω
8. End

$\Omega_1 \text{ ou } x_1$	$\Omega_2 \text{ ou } x_2$	...	$\Omega_n \text{ ou } x_n$
$\delta(\Omega_1)$	$\delta(\Omega_2)$	...	$\delta(\Omega_n)$

Table 2. Results of different values of Ω_i

V CONCLUSION

The study that we carried out made it possible to reach results allowing to propose algorithms to estimate and an entire domain by sending specifics signals on the border. In this article we extended the work cylindrical and spherical shape. This results are applicable to the domain of telephony, transmission over optical fiber, electricity, determination of overall extension area of oil or minerals field...

To achieve these results, we used Sobolev's theories, the formula for variation of variables, the green formula, the principle of the maximum for existence and the uniqueness of the solutions, Reisz theorem, Samuel OUYA and NIANE theorem and many other theorems. The excitation signals of the phenomena that we proposed in dimension 1 and 2, thus leading to justify that our problem was well defined and posed. In stationary and orthogonal references frames we have shown under certain conditions that the signal sent outside of $\partial\Omega$, leads to the determination of an algorithm allowing to approach and to approximate a domain Ω . In perspective, it will be interesting to assume that the domain Ω is governed by a non-stationary phenomenon, thus to study the determination of the domain in the eventual area.

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