

On Concatenated Coding Scheme for High-Speed Ethernet

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Abstract— A concatenated forward error correction (FEC) scheme, which takes a binary code and a Reed-Solomon code as its inner and outer code, respectively, has been adopted as one of the candidate schemes in the next-generation high-speed Ethernet standard (IEEE 802.3df). Unlike the other concatenated scheme used before, to possess a low latency feature, the interleaving depth in the new scheme is very small, such as 2 or 4. In this case, the bit error ratio (BER) performance evaluation at low BER needs mass computer simulation, and the BER analysis proposed for large interleaving depth is no longer suitable. We propose a new BER analysis for a concatenated coding scheme with a small interleaving depth in this work. From the simulation results, the analysis is very accurate at a large range of BER. We also show that, contrary to intuition, a short inner code outperforms a long inner code in concatenated decoding performance, even though the long code performs better without being concatenated with the outer code.

Keywords—forward error correction, concatenated coding scheme, Reed-Solomon codes, soft-decision decoding, performance analysis

I. INTRODUCTION

Next-generation high-speed Ethernet (IEEE 802.3df) needs to support 200Gbps optical fiber (see [1]). Therefore, a Forward Error Correction (FEC) scheme, capable of providing more coding gain than the current standard (IEEE 802.3bs), should be adopted. In IEEE 802.3bs, the (544, 514) Reed-Solomon (RS) code with interleaving depth 2 is used as the FEC scheme. In the next-generation standard, concatenating an RS and short binary codes is a potential method. This scheme is shown in Figure 1.

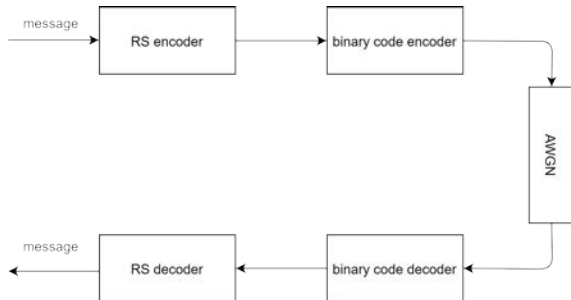


Fig. 1. A concatenated coding scheme as the candidate for high-speed Ethernet.

In the above concatenated coding scheme, the inner code is decoded by a soft-decision decoding algorithm, and a

hard-decision decoding algorithm decodes the outer code. This concatenated coding scheme can reuse the RS decoder while providing additional coding gain. Furthermore, as soft-decision decoding (e.g., Chase II algorithm [2]) for a short binary code can be done in parallel, a low latency decoder is obtained. For a FEC scheme to be used in Ethernet, both the BER and uncorrected codeword ratio (UCR) are critical. The UCR could be derived from the BER, which implies that we only need to analyze the BER. Computer simulations can get the BER performance. However, we need mass computer simulations or even FPGA implementation for extremely low BER performance, such as $\text{BER}=10^{-15}$. Hence, obtaining the BER performance at a low BER range without mass computer simulations is a challenging problem.

The BER performance of the concatenated coding scheme with large interleaving depth has been studied in [3], [4]. In the concatenated coding scheme, the interleaving depth is large enough for the symbols in an RS codeword to be independent. However, when the depth is small, this is not necessary to hold. Hence, a new analysis method must be derived to evaluate the performance at very low BERs.

This paper proposes a method for analyzing the performance of the above coding scheme. We show that, contrary to intuition, a short code outperforms a long code in concatenated decoding performance, even though the long code performs better without being concatenated with the outer code.

II. ANALYSIS OF CONCATENATED CODING SCHEME

This section proposes a method for evaluating the BER performance of the concatenated coding scheme in Figure 1.

Let (N_1, K_1) represent the code parameter of the inner code where N_1 is the length of the code, and K_1 is the dimension of it. Similarly, let the code parameter of the outer code be (N_2, K_2) . $\mathbb{F}_{2^m}$ denotes the finite field in which RS symbols appear. Hence, there are mN_2 bits in a codeword of the outer code. D indicates the interleave depth of the outer code.

An RS encoder first encodes the message. A set of D interleaved RS codewords is called an RS block. An RS block is illustrated in Figure 2, where each row is an RS codeword. $c_{i,j}$ represents the j -th symbol of the i -th RS codeword. Without loss of generality, we assume that the codes presented in this paper are systematic. Hence, $c_{i,j}$ are the message symbols for $j \leq K_2$ and the parity symbols for $K_2 < j \leq N_2$.

c_{1,N_2}	...	...	$c_{1,4}$	$c_{1,3}$	$c_{1,2}$	$c_{1,1}$
c_{2,N_2}	...	...	$c_{2,4}$	$c_{2,3}$	$c_{2,2}$	$c_{2,1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_{D,N_2}	...	...	$c_{D,4}$	$c_{D,3}$	$c_{D,2}$	$c_{D,1}$

Fig. 2. The structure of the RS block.

Next, the RS symbols are fed into a binary inner encoder in a column-by-column fashion, that is, in the sequence as $c_{1,1}, c_{2,1}, \dots, c_{D,1}, c_{1,2}, c_{2,2}, \dots, c_{D,N_2}$. We assume that the first bit of $c_{1,1}$ in the first RS block is encoded as the first bit of a binary linear block code. Then the binary codewords generated by the inner encoder are sent into the channel. The received vectors are first decoded in the receiver by a soft decision method to obtain the binary codewords. Next, the information parts of these inner codewords are fed into RS decoders. Finally, the message can be recovered.

We assume that K_1 is much larger than m since m is usually around 10 in practice, and binary codes of length about 10 are rarely used in the concatenated coding scheme. Therefore, several RS symbols are contained in a single binary codeword of the inner code. Due to the channel noise and the inner decoder, the received symbols of those RS symbols included in a binary codeword at the receiver are no longer independent. The key to analyzing the performance of the concatenated coding scheme is finding the independent structure in the received symbols corresponding to the transmitted RS symbols.

Let B_i represent the sequence of the input bits fed to the RS decoder, which corresponds to the i -th received RS block S_i , $i = 1, 2, \dots$. B_i can be seen as a $D \times mN_2$ matrix in which each entry is a binary random variable or a $D \times N_2$ matrix in which each entry is a random variable (symbol) from $\mathbb{F}_{2^m}$. B_i can also be seen as a random variable. Hence, $\{B_i, i = 1, 2, \dots\}$ is a random process. Two symbols are dependent if any of their bits are from the same codeword of the inner code. Similarly, if any bits from B_i and B_j are from the same codeword of the inner code, they are dependent. In general, if B_i and B_j are dependent, and B_j and B_ℓ are dependent, then B_i and B_ℓ are dependent. Let $l = \text{LCM}(K_1, DmN_2)/DmN_2$, where LCM is the least common multiplier. We have the following result.

Lemma 1: B_i and B_{i+l} are independent for all $i > 0$.

Proof: First, we consider the case with $i = 1$. There are DmN_2 bits in B_1 such that $B_1, B_2, \dots, B_l$ have exactly $lDmN_2/K_1$ binary codewords of the inner code. Hence, B_l is not dependent to B_{1+l} and B_1 is also not dependent to B_{1+l} . The same conclusion also holds for $i > 1$ for any positive integer i . We then conclude that B_i and B_{i+l} must be independent. $\square$

Note that the set of random variables $\{B_{j+l+1}, B_{j+l+2}, \dots, B_{j+l+j}\}$ can be treated as a single random variable for nonnegative integer j . We denote this

single random variable by $\hat{B}_j$.

Lemma 2: $\hat{B}_j$ are independent and identically distributed (i.i.d.) random variables for all $j \geq 0$.

Proof: Lemma 1 implies that $\hat{B}_j$ are independent for all j . In addition, it is straightforward to see that B_i and B_{i+l} are from the same structure in both binary codes and RS symbols. Thus $\hat{B}_j$ are identically random variables. $\square$

The bit error number of the concatenated coding scheme can be seen as a function of $\hat{B}_j$, and the desired BER is the expectation of this function divided by DmN_2 . Lemma 2 tells us that we can focus only on $\hat{B}_0$, i.e., $\{B_1, B_2, \dots, B_l\}$.

Next, we focus on a special kind of binary sub-blocks in $B_1, B_2, \dots, B_l$, which are independent of each other. Consider a sub-block of length $\text{LCM}(Dm, K_1)$ bits, which starts from the common starting point of a binary codeword and an RS symbol. We name such a sub-block as A -block and $|A| = \text{LCM}(Dm, K_1)$. It is easy to see that A contains exactly $|A|/K_1$ binary codewords. This implies that A is independent of other data in B_i . Furthermore, A includes $|A|/Dm$ symbols from each row of B_i .

We write

$$DmN_2 = \rho|A| + \mu,$$

where ρ is the quotient and μ is the residue. Let N_A be the minimum number among the maximum numbers of A -blocks each B_i can have, where $1 \leq i \leq l$. That is

$$N_A = \min_{1 \leq i \leq l} \{\text{the maximum number of } A\text{-blocks inside } B_i\}.$$

Lemma 3: $N_A = \rho$ or $N_A = \rho - 1$.

Proof: An A -block starts from the common starting point of an RS symbol and a binary codeword. Denote the first position in B_i such that the starting point of an RS symbol coincides with a binary codeword by δ_i . If any $\delta_i > \mu$ for $1 \leq i \leq l$, then there exists no enough room to contain ρ A -blocks in B_i . In this case, we only can have $\rho - 1$ A -blocks in B_i . Hence, $N_A = \rho$ or $N_A = \rho - 1$. $\square$

If we remove N_A A -blocks in B_i , there are μ or $|A| + \mu$ bits in B_i . We call these residue bits by T_i -block. $T_1, T_2, \dots, T_l$ are different in both RS symbols and binary codewords structure, and they are not independent. One should note that T_i is not successive in B_i in general, and it influences exactly

$$N_2 - N_A \frac{|A|}{Dm}$$

symbols in each row in B_i . According to the above discussion, the bit error number can be seen as a function of lN_A A -block and $T_1, T_2, \dots, T_l$.

Note that A includes $|A|/Dm$ symbols from each row of B_i . We define the probability that v symbol errors occur in a row of B_i that includes in an A -block by $P\{E_A = v\}$. Furthermore, we denote the average bit error number when v symbol errors occur in a row by $N(E_A = v)$. The same notations for T_i are $P\{E_{T_i} = v\}$ and $N(E_{T_i} = v)$. Let $P\{E_{T'} = v\}$ and $N(E_{T'} = v)$ represent the average of

$P\{E_{T_i} = v\}$ and $N(E_{T_i} = v)$ for $i = 1, 2, \dots, l$, respectively. Then the BER can be estimated as

$$P_b \approx \frac{1}{mN_2} \sum_{\substack{v_i, v \\ t_2 < v + \sum_{i=1}^{N_A} v_i \leq N_2}} \prod_{i=1}^{N_A} P\{E_A = v_i\} P\{E_{T'} = v\} \left(\sum_{i=1}^{N_A} N(E_A = v_i) + N(E_{T'} = v) \right),$$

where t_2 is the error correction capability of the RS outer code, $0 \leq v_i \leq |A|/Dm$, and $0 \leq v \leq N_2 - N_A \frac{|A|}{Dm}$. Note that we ignore the probability of undetectable errors since it is proven to be negligibly small compared to P_b . More discussion can be found in [4].

Now we discuss how to obtain $P\{E_A = v\}$, $N(E_A = v)$, $P\{E_{T'} = v\}$ and $N(E_{T'} = v)$ by simulation. We consecutively send $|A|/K_1$ inner codewords through the AWGN channel and decode the received vectors. Then one can count the symbol and bit errors of an A -block. Thus $P\{E_A = v\}$ and $N(E_A = v)$ can be finally obtained. The simulations of T_i can be done with those of an A -block. Since T_i is the same as a part of or a compound of an A -block, we can also investigate the statistics of T_i . Hence, if we count the symbol errors and bit errors in all T_i , $P\{E_{T'} = v\}$ and $N(E_{T'} = v)$ can be computed.

III. SIMULATION RESULT AND ANALYSIS

In this section, we provide several examples and analyses. Consider the concatenated coding scheme given in Section II, which has a (544, 514) RS code as the outer code. The inner codes considered are (128, 119), (144, 136), and (360, 340) binary codes, where (128, 119) is the code specified by OIF (see [5] for details), and (144, 136) code and (360, 340) code are BCH codes. The interleaving depth is set to 2, 4, or 8. The inner codes are decoded by a generalized Chase II algorithm, where 6 least reliable positions are selected and thus 64 test patterns needed to be decoded (see [2] for more details), and a maximum-likelihood (ML) decoding algorithm (A^* algorithm, see [6] for details). A bounded distance decoder decodes the outer codes up to its error-correction capability (e.g., the Berlekamp-Massey algorithm).

Figure 3 presents the BER performance of concatenated schemes obtained by the proposed method and computer simulations. We choose the generalized Chase II algorithm for decoding the inner codes. The reason is that the Chase II algorithm has good parallel architecture such that it can be efficiently implemented in hardware and that its performance is good enough. From this figure, the results demonstrate that the proposed method for evaluating the BER performance is very accurate. The inner codes BCH(144, 136) and BCH(360, 340) have the same code rate. It is well-known that the performance of BCH(360, 340) code is superior to BCH(144, 136) when using the same decoding algorithm. Nevertheless, contrary to intuition, the concatenated scheme with short inner code outperforms that of using long inner code. This is because the interleaving depth is small, so if an inner received vector is

wrong, it influences more RS symbols than a short code. On the other hand, if the code length is too small, a low-rate code should be selected to ensure the error correction capability. Hence, a medium-length code is preferred, which balances the code rate and performance.

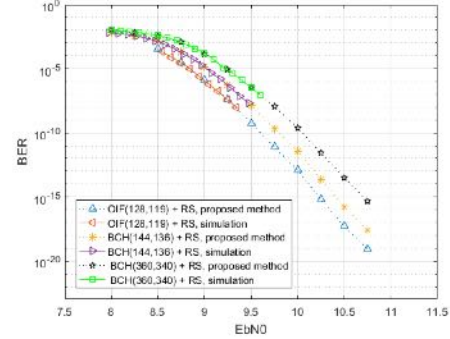


Fig. 3. BER performance of concatenated schemes with Chase decoding. The interleaving depth is set to 2.

Figure 4 compares the BER performance of the concatenated scheme with OIF(128,119) as the inner code when the generalized Chase II and the ML algorithm are implemented. The interleaving depth is set to 2. The result shows that, at $\text{BER}=10^{-15}$, there is only a 0.55dB loss in the BER performance between the generalized Chase II and the ML algorithm. This implies that the generalized Chase II algorithm is good enough while holding the parallel feature.

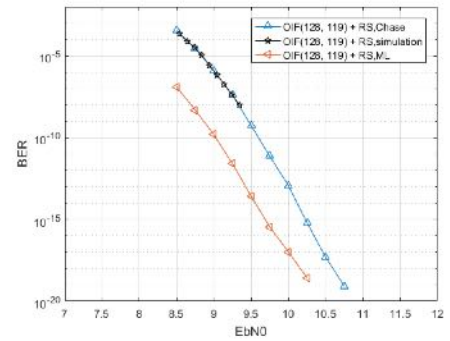


Fig. 4. BER performance of the concatenated scheme with OIF(128,119) as the inner code. The interleaving depth is set to 2.

Figure 5 compares the BER performance for various interleaving depths. At $\text{BER}=10^{-15}$, the scheme with interleaving depth 8 can offer an extra 0.6dB coding gain than that with depth 2. Moreover, an additional 1dB coding gain can be obtained when the desired BER is 10^{-20} . This implies that the small interleaving depth limits the performance of the concatenated scheme. Nevertheless, a large interleaving depth results in high latency.

IV. CONCLUSION

In this work, we consider the concatenated coding scheme for high-speed Ethernet. In this case, the interleaving depth

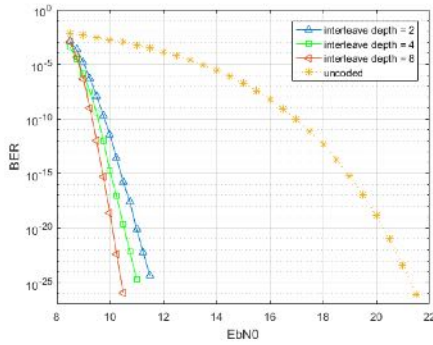
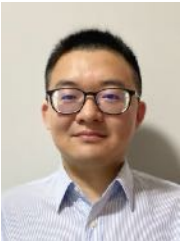


Fig. 5. BER performance of concatenated schemes with Chase decoding.

in the system is usually small. For very low BERs, the BER performance is challenging to obtain by computer simulations. We propose a method to evaluate the BER performance without mass computer simulations. This significantly reduces the simulation time. This result also reveals that the proposed method is very accurate. As expected, the larger the interleaving depth is, the better the BER performance is. However, with the same code rate, the scheme with a short code as the inner code performs better than that with a long code as the inner code, which is surprising. The future work is to extend the proposed method to other channels.

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