

Physics-Informed Neural Networks for solving Blood Flows

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Abstract—The Physics-informed Neural Networks Deep Learning (PINN) framework has been introduced with the primary objective of advancing the field of blood flow simulations. PINN Deep Learning involves data-driven training for flow prediction and can incorporate the understanding of physical laws described by partial differential equations (PDEs). This paper employs the PINN Method for simulating blood flows. Multiple test cases will be computed and compared with other numerical and experimental results to validate the approach. The results demonstrate that the PINN method functions as expected, and validation against experimental and other researchers' results ensures the generation of meaningful output data and the prudent selection of parameters.

Keywords—Physics-informed Neural Networks, Deep Learning, Cardiovascular System, 1D Blood Model.

I. INTRODUCTION

The cardiovascular system, also known as the circulatory system, is composed of the heart, blood vessels (arteries, veins, capillaries), and blood. Representing it mathematically and analyzing its dynamic mechanisms is still a challenging research. Traditionally, the computation of cardiovascular system flow fields involved the solution of the three-dimensional Navier-Stokes equations and solved it numerically. However, this is time consuming and wastes a lot of computational resources.

To simplify this problem, certain assumptions can be made, reducing it to a one-dimensional problem., thus, the axisymmetric Navier-Stokes equations [1][2]. Axisymmetric equations may have a single point where the radius is zero, making numerical solutions singularity. Huang [3] applied the TVD Scheme to solve cardiovascular systems, simplifying the dynamics of cardiovascular flow fields into a one-dimensional model. This model is employed to study and comprehend the characteristics of cardiovascular flow dynamics and pressure propagation. It simplifies blood vessels into one-dimensional tube flow problems, allowing for the calculation of flow rates, blood vessel cross-sectional areas, and lengths to further understand and predict pressure distribution within blood vessels. . Consequently, scholars have started exploring the feasibility of solving the one-dimensional Navier-Stokes equations [4][5].

In recent years, the development of Artificial Intelligence (AI) has been one of the hottest topics. Physics-informed Neural Networks (PINN), proposed by scholar M. Raissi[6], are a type of neural network that uses physical equations as computational constraints for solving partial differential equations[6]. They belong to the supervised learning category. This network utilizes data-driven training, learning through loss functions described by partial differential equations. During the training process, PINN benefits from the constraints imposed by physical equations, enabling it to learn fundamental physical laws such as mass and momentum conservation. In recent years, PINN has gradually become a focal point in machine learning and computational fluid dynamics, achieving significant progress in both theory and applications [7][8][9].

To enhance the performance of computation of cardiovascular system flow fields, the systems can be treated as 1D tube like structures and PINN method is adopted for simulations. Further, we solve several preambles to demonstrate obtaining the results accuracy and efficiency.

II. METHODS

2.1 One-dimensional formulation

Blood flow through arteries treats as 1D tube flow is governed by the Navier-Stokes equations in 1D-cylinder coordinate systems:

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\alpha \frac{q^2}{A} \right) + \frac{A}{\rho} \frac{\partial p}{\partial x} + \frac{f}{\rho} = 0 \quad (2)$$

$$p - p_{ext} - \beta(\sqrt{A} - \sqrt{A_0}) = 0 \quad (3)$$

where $A(x, t)$ represents the cross-sectional area of the blood vessel, which is a function of spatial position x and time t . $q(x, t)$ denotes the volumetric flow rate within the blood vessel, while $p(x, t)$ represents the average pressure of the fluid within the blood vessel. The fluid density, denoted as ρ , is assumed to have a default value of 1050 kg/m^3 . Additionally, the frictional force, $f(x, t)$, experienced per unit length within the blood vessel, can be expressed as $f = \beta\pi\mu u$, where $\mu = 0.0045$

Pa/s and u represents the fluid velocity. For the Poiseuille flow, the fluid, β takes a value of 8 and $\alpha=1$ for the uniform velocity distribution assuming. The pressure $p(x, t)$ in the above equations can be described using the following equation:

$$p(x, t) = p_{ext}(x, t) + \Psi(x, t) \quad (4)$$

where $p_{ext}(x, t)$ is the external pressure acting on the fluid and $\Psi(x, t)$ can be represented as the follows:

$$\Psi(x, t) = K(x)\phi(A(x, t), A_0(x)) + p_0 \quad (5)$$

In the above equation, $K(x)$ is a positive function that depends on the Young's modulus $E(x)$ and the thickness $h(x)$ of the blood vessel wall. P_0 represents the reference pressure at the blood vessel cross-sectional area A_0 .

$$\phi(x, t) = \left(\frac{A(x, t)}{A_0(x)} \right)^m - \left(\frac{A(x, t)}{A_0(x)} \right)^n \quad (6)$$

where the values of m and n can be determined experimentally. For arteries, the values are $(m, n) = (0.5, 0)$, and for veins, the values are $(m, n) = (10, -1.5)$ [4].

2.2 Physics-informed Neural Networks

Consider a general form of partial differential equation as an example, assuming that the function $u = u(t, x)$ and satisfies a partial differential equation in the following form:

$$u_t + N[u; \lambda] = 0, \quad x \in \Omega, \quad t \in [0, T] \quad (7)$$

where $N[u; \lambda]$ represents a parameterized non-linear operator that operates on u with parameter λ . Ω denotes the Euclidean geometric space, and T is the terminal time. Given the initial state, boundary conditions, and physical parameter λ for $u(t, x)$, solving the partial differential equation allows us to predict the values of $u(t, x)$ at any spatiotemporal point. Numerical methods such as finite differences or finite elements can be employed for solving. First, let's define:

$$f : u_t + N[u; \lambda] \quad (8)$$

Subsequently, a neural network can be constructed to approximate the solution to the partial differential equation. Therefore, the root mean square loss function is defined as following:

$$MSE = MSE_u + MSE_f \quad (9)$$

$$MSE_u = \frac{1}{N} \sum_{i=1}^{N_u} |u(t_u^i, x_u^i) - u^i|^2 \quad (10)$$

$$MSE_f = \frac{1}{N} \sum_{i=1}^{N_f} |f(t_f^i, x_f^i)|^2 \quad (11)$$

the data-driven of the loss function, acquired through the initial and boundary condition in the training data, represents the physics model-driven portion of the loss function, which is trained in the system's machine learning process to approximate the neural network function. The network must not only minimize the error with known training data but also satisfy the constraints of the partial differential equation as much as possible. Therefore, the network structure and loss function of PINN need to be customized according to the form of the partial differential equation, while also maintaining the model's scalability.

The architecture of the PINN in this study is shown in Figure 1. After inputting temporal and spatial data, it initially employs a fully connected neural network to approximate the function, subsequently obtaining residuals for the partial differential equation and initial/boundary value constraints. These residuals are then incorporated as regularization terms in the loss function. Finally, optimization algorithms like gradient descent are used to obtain the neural network's connection weights and the physical parameters of the partial differential equation.

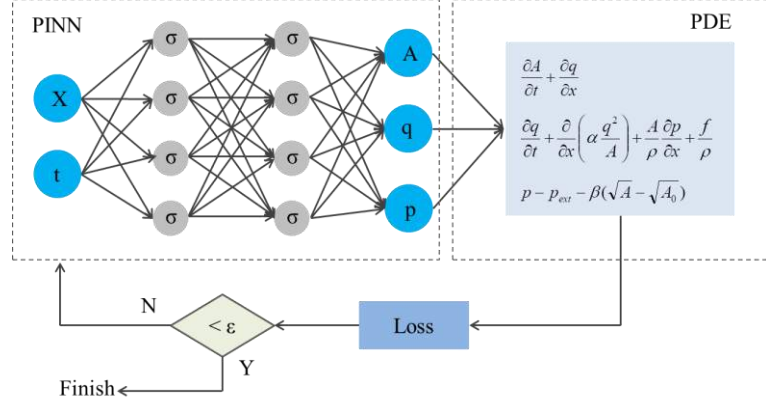


Figure 1. A schematic representation of the proposed PINN.

III. RESULTS

An arterial tree with branches :

The main objective of this test is to evaluate the capability and efficiency of the developed numerical method in computing bifurcated tube calculations. The geometry of a bifurcation blood flows shown in figure 2.

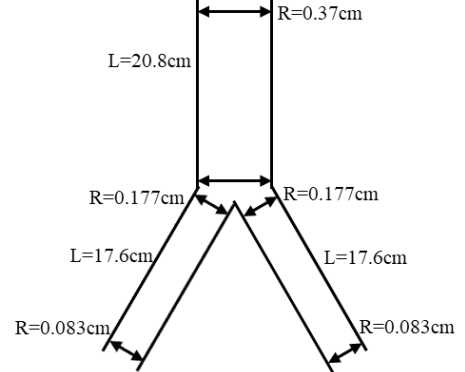


Figure 2. Geometry of a bifurcation blood flows

The input velocity profile was obtained from [1] by curve fit as shown in figure. The main vessel was composed with uniform slender cylinder and has two branch vessel connected to the end. The branch vessel has a linearly tapered cylinder with varying radius as follows:

$$r_0(z) = R_0 \exp(\log(R_0/R_u)(z/L)) \quad (15)$$

where $r_0(z)$ radius of a vessel in a cylinder coordinate system. The elastic property of vessel is calculated from Young's modulus:

$$Eh/r_0 = k_1 \exp(k_2 r_0) + k_3 \quad (16)$$

Where h is vessel wall thickness, r_0 is vessel radius and the constant $k_1 = 2 \times 10^7 \text{ g.cm}^{-1}.\text{s}^{-2}$, $k_2 = -22.53 \text{ cm}^{-1}$ and $k_3 = 8.65 \times 10^5 \text{ g.cm}^{-1}.\text{s}^{-2}$ are adopted in simulation.

The inflow at the inlet of common carotid artery (CCA) is given by the a periodic pulse in [11] as shown in figure 3.. Time period $T = 0.917\text{s}$ is the period of one pulse cycle and the kinematic viscosity is $\nu = 0.046 \text{ cm}^2.\text{s}^{-1}$. The bifurcation occurs at a point where the outflow from the CCA is balanced with the inflow from the ICA and the external carotid artery (ECA). The geometry of the CCA, ICA and ECA branches is summarized in Table 1.

Table 1 Geometry parameter of CCA, ICA and ECA

	r_u	r_d	L
CCA	0.370 cm	0.370 cm	20.8 cm
ICA	0.177 cm	0.083 cm	17.6 cm
ECA	0.177 cm	0.083 cm	17.7 cm

The problem is addressed by utilizing neural networks with seven hidden layers, each consisting of 100 neurons, and employing a hyperbolic tangent activation function. The pressure waveform at various locations in a bifurcation vessel, obtained through the application of the PINN method to solve the 1-D equations, is illustrated in Figure 4. Specifically, the plots are at a location 6 cm from the common carotid artery (CCA), internal carotid artery (ICA), and external carotid artery (ECA). These waveforms closely resemble the corresponding ones calculated by solving the Navier-Stokes equations, considering compatible geometrical and mechanical properties, as well as identical inflow and outflow boundary conditions [12]. A noticeable increase in pressure gradient discrepancy is observed, attributed to the tapered shape of the vessel.

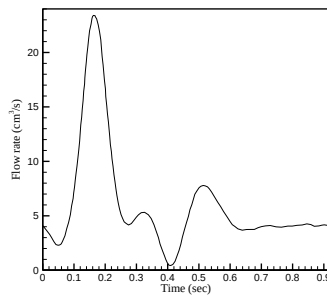
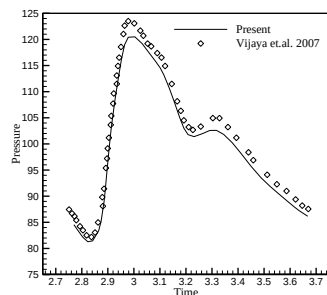
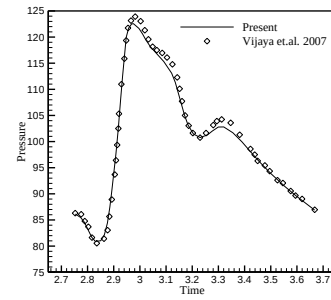


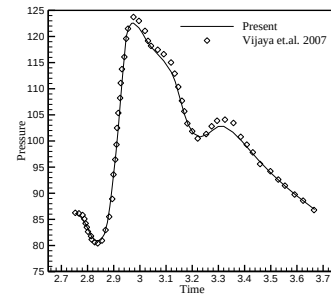
Figure 3. Inflow rate as a function of time.



(a) Common carotid artery



(b) External carotid artery



(c) Internal carotid artery

Figure 4. Pressure waveform with time.

IV. CONCLUSIONS

In this study, the PINN method is employed to simulate one-dimensional blood flows. Simulations are conducted using the hyperbolic mathematical model of one-dimensional Navier-Stokes equations in cylindrical coordinate systems. Several test cases are employed to validate the chosen approach. The proposed PINN method's results are compared to those in [12] for the common carotid artery bifurcation. The results demonstrate that the PINN method functions as expected, and validation against experimental and other researchers' results ensures the generation of meaningful output data and the judicious selection of parameters. The validation of larger arterial networks with multiple bifurcation levels has not been conducted yet, but is scheduled for future research. Demonstrations indicate that considering bifurcation pressure drops can significantly enhance accuracy. Consequently, achieving comparable accuracy in blood flow solutions for 1D models using the PINN method compared to traditional numerical methods for 2D or 3D models is feasible.

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