

# A Reliable Routing Method for Remote Entanglement Distribution under Limited Resources

Tianzhu Hu\*, Xiaofeng Jiang\*\*, Tianze Zhu\*\*, Xin Sun\*\*, Haomin Chen\*\*, and Jian Yang\*\*

\*Institute of Advanced Technology, University of Science and Technology of China, Hefei, China

\*\*the Department of Automation, University of Science and Technology of China, Hefei, China

htzustc@mail.ustc.edu.cn, jxf@ustc.edu.cn, tz\_19@mail.ustc.edu.cn, sx21010097@mail.ustc.edu.cn, chen0215@mail.ustc.edu.cn, jianyang@ustc.edu.cn

**Abstract**—Generating and distributing entangled pairs between arbitrary nodes is essential to fully realize the network’s capabilities, with the challenges of limited qubit resources, severe decoherence and stochastic physical mechanism. In this paper, we model the service process of quantum repeater nodes based on the concept of queuing theory to help characterize their availability. Further, we propose a link-disjoint multi-path routing algorithm, with repeaters’ availability, nodes’ qubit capacity, entangled links’ fidelity and classical delay taken into consideration. The performance of our scheme has been evaluated with simulated environment and compared with other existing routing schemes.

**Keywords**—Quantum Networks, Entanglement Distribution, Queuing Theory, Routing Algorithm, Quantum Repeaters

## I. INTRODUCTION

Quantum communication technology has unveiled the possibility of numerous ground-breaking applications, including highly secure communication, distributed quantum computing, and remote quantum clock synchronization [1]. The realization of these capabilities relies on the utilization of quantum entanglements and their unique features. Entanglement is a phenomenon in quantum mechanics where two or more particles become correlated as a indivisible entity, regardless of how distant they are physically separated. Entangled qubits are well suited for tasks that require coordination, since any alteration made to one particle’s state will be instantly reflected in the state of other particles. Also, the laws of quantum mechanics ensure that entanglements are inherently private and hence resilient to eavesdropping from a third party. Generally, the word “entanglements” in this article refers to the Bell states  $|\Phi^\pm\rangle = \frac{\sqrt{2}}{2}(|00\rangle \pm |11\rangle)$  and  $|\Psi^\pm\rangle = \frac{\sqrt{2}}{2}(|01\rangle \pm |10\rangle)$ . So far, short-lived entanglement distribution between adjacent nodes has been achieved on the ground by sending photons over standard optical fibre [2], or from free-space based on a satellite [3]. Long-lived, remote entanglement between two quantum nodes are generated step-by-step: link-level heralded entanglements are established between adjacent nodes first, followed by entanglement swapping operations performed at intermediate nodes known as quantum repeater nodes. Additionally, entanglement purification allows for higher fidelity with consumption of entangled pairs of lower fidelity, as shown in Fig.1.

Quantum memory plays a significant role in the entanglement distribution process, with the challenge of balancing

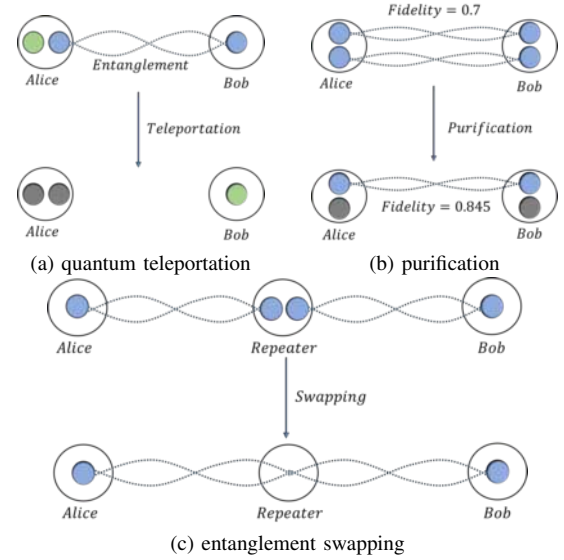


Fig. 1: Common applications of entanglements

between capability, lifetime, and store-and-retrieve fidelity. Limited resources may lead to severe contention under the multi-request scenario, calling for an effective routing design. At specific moment, it’s possible that most nodes lack enough resources for a newly arrived request. Furthermore, the stochastic physical mechanisms underlying the entanglement distribution will undoubtedly lead to a decrease in the entanglement generation success rate.

Recently, huge efforts have been devoted to the development of efficient entanglement routing protocols, aiming to obtain higher throughput with finite qubit resources [4]. Reference [5] focused on capacity allocation on the edges of a quantum network for multiple requests of entanglement generation. Shouqian Shi *et al.* discussed a four-phase routing realization based on the “advance generation” model [6]. Jian Li *et al.* proposed a method to find the optimal purification decisions along the routing path with minimum entangled pair cost [7]. Yiming Zeng *et al.* studied the problem to simultaneously maximize the number of quantum-user pairs and their expected throughput [8]. Laszlo Gyongyosi *et al.* introduced the service rate of quantum repeater nodes based on G/G/1 priority queuing model and existing entangled pairs of different levels

[9].

Almost all papers mentioned above introduced phases in the entanglement distribution process, and primarily focused on link status. Therefore, these studies will fail to avoid the most congested nodes under continuous network environment, leading to unacceptable response time for individual requests and unbalanced resource utilization of the entire network. Compared to them, we explore a solution for reliable entanglement distribution under limited node resources. In this paper, we model the service process of quantum repeater nodes which support link-level entanglement establishment, multi-round purification and entanglement swapping, and derive their service availability based on the concept of queuing theory. Taking the specific queuing process of each repeater node into account, the routing algorithm can choose nodes with less waiting time to mitigate the impact of limited resources, which will help to provide a bandwidth guarantee for every request. A link-disjoint multi-path routing algorithm is proposed accordingly in the prevention of potential failure or memory shortage, with repeaters' availability, links' fidelity and classical delay taken into consideration. We use NetSquid simulation platform [10] to analyze the performance of our scheme, including end-to-end entanglement fidelity and throughput of requests.

## II. NETWORK MODEL

In this section, we will first clarify the key components of a typical quantum network, provide individual descriptions of their essential capabilities separately, and then supplement additional assumptions.

### A. Network Components

**End nodes.** Analogous to end users in classical networks, end nodes in quantum networks communicate and run quantum applications. Each end node is equipped with a quantum processor which has a certain number of memory qubits and necessary hardware to perform basic quantum gates on the qubits, enabling computation, storage, measurement and other functions. We assume that all memory qubits within an end node can be manipulated in parallel, allowing the end node to initiate multiple entanglement attempts with its neighbour simultaneously with the help of multiplexing.

**Repeaters.** Since the quantum channel built by optical fibre is inherently imperfect, the success rate of entanglement establishments decays exponentially with the channel length. Through entanglement swapping, quantum repeaters serve as intermediate nodes to support entanglement distribution between remote nodes. Here we assume that repeaters are also capable of link-level entanglement purification to improve entanglement fidelity, which means all necessary purification is performed before the swapping process. Additionally, we assume each quantum node has the full functionality of both an end node and a repeater. Within a specific entanglement distribution scenario, these two concepts are employed to differentiate the roles of quantum nodes along the path.

**Quantum channels.** As discussed above, quantum channels connect adjacent nodes through optical fibres. Under the con-

dition that both nodes have sufficient qubit storage, multiple entanglement attempts can be made at the same time under our assumptions.

### B. Network topology

For a quantum network  $G = (V, E, C)$  of  $V$  quantum nodes and  $E$  edges,  $C_i$  is defined as the number of memory qubits able to be manipulated in a node  $v_i$ . We assume the fidelity of entanglements generated on each edge to follow a normal distribution  $N(\mu, \sigma^2)$ .

## III. AVAILABILITY OF QUANTUM REPEATER

With the assumption that each qubit is able to build entangled pairs with other nodes independently, an arbitrary quantum repeater node  $v_i \in V$  with qubit capacity  $C_i$  is regarded as a  $G/G/C_i$  queuing system and works in an FCFS manner.

Our entanglement distribution scheme utilizes the “on-demand generation” model, in which the entangled pairs are distributed on demand, along the selected path(s) to save qubit resources and avoid decoherence in quantum memories. A remote entanglement generation request will experience path selection and resource allocation phase in order, finally be converted to requests on the source node, the destination node and several repeater nodes. Different from customers in a general queuing system, the service process of a request  $r$  on node  $v_i$  is described from two dimensions: resource demand, and service time. The qubit resource demand  $n_r$  includes basic bandwidth requirements for link-level entanglement establishment with both the previous-hop node  $v_f$  and the next-hop node  $v_l$ , and potential extra qubits consumed in entanglement purification to meet fidelity requirements. The service time can be divided into three stages in order according to the service process, as shown in Fig.2:

**Entanglement establishment:** Due to the existence of quantum link layer protocol [11] providing robust link layer entanglement service, the average service time of this stage is calculated by  $\max\{t_{fe}, t_{le}\}$ , where  $t_{fe}, t_{le}$  denote the time consumed with  $v_f$  and  $v_l$  separately.

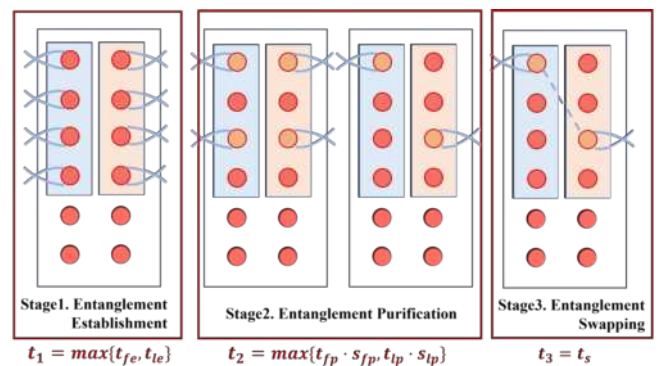


Fig. 2: Service process of a request in repeater  $v_i$ . Here  $n_r = 8$ ,  $s_{fp} = s_{lp} = 2$ .

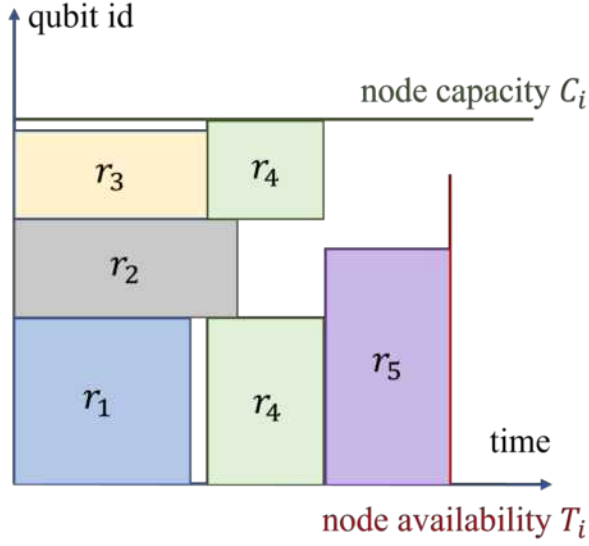


Fig. 3: Example of availability estimation in repeater  $v_i$ . Here,  $R = \{r_1, r_2, r_3, r_4, r_5\}$ , each request is represented as a block, whose length and width denote the resource demand and service time separately.

**Entanglement purification:** The time purification stage required depends on single-round purification time  $t_{fp}, t_{lp}$  in the node, and the number of rounds  $s_{fp}, s_{lp}$  with  $v_f$  and  $v_l$ .

**Entanglement swapping:** At last, all entanglement swapping processes take constant time  $t_s$  in node  $v_i$ .

Combining the time spent on different phases above together, request  $r$  will occupy  $n_r$  qubits for a period of

$$t_r = \max\{t_{fe}, t_{le}\} + \max\{t_{fp}s_{fp}, t_{lp}s_{lp} + t_s\}$$

Since both the resource demand and the service time vary from specific request  $r$ , the service rate is inaccurate in representing a repeater node's service capability. Instead, we use the whole waiting time of current requests in a node to characterize its availability. To estimate the queuing time of a newly arrived request  $u$  when the sorted list of tasks in the node  $v_i$  is  $R = \{1, \dots, r, \dots, u-1\}$ , a recursive algorithm is utilized. The departure time of requests being serviced currently can be sequentially predicted using their estimated service time, revealing the next task to be completed and the number of qubits it will release. Afterwards, the moment when the next task to be serviced is able to enter the system is determined according to its resource demand, and then its departure time is confirmed by service time. This process is repeated until the completion time of all tasks is obtained, the latest time among which is the availability attribute  $T_i$  of node  $v_i$ . Fig.3 illustrates how this algorithm works.

#### IV. ENTANGLEMENT ROUTING ALGORITHM

Introducing multi-path methods in the entanglement distribution routing process can reduce the storage capacity require-

ments for individual quantum repeater nodes. Also, it allows for a more efficient utilization of resources from different nodes to handle probabilistic failures that may occur on some certain paths, improving the overall robustness. In this paper, we adopt a link-disjoint multi-path algorithm Q-PSBA(Path Selection Based on Availability), to avoid resource contention on a single link caused by different paths of the same request. It is achieved by an extended multi-round Dijkstra algorithm, where the edges associated with the optimal path found in the previous round are deleted between rounds. An example of link-disjoint path exploration is shown in Fig.4.

Compared to the original Dijkstra algorithm, we introduced innovative routing metrics with nodes' resource availability and links' entanglement quality along the path included. To maximally loosen the requirement for qubit coherence time, an end-to-end entanglement generation will be started when all nodes on the path are prepared. Therefore, resource availability is depicted by the minimum node qubit capacity  $C_i$  and the maximum node availability  $T_i$  among the repeater nodes of a path. The entanglement quality is represented by both the product  $\prod_{i=1}^n F_i$  of fidelity, and the sum  $\sum_{i=1}^n D_i$  of classical communication delay on each edge of the path. The composite metric  $K$  is defined as:

$$K = k_1 T_i / C_i - k_2 \log \prod_{i=1}^n F_i + k_3 \sum_{i=1}^n D_i$$

, where  $k_1, k_2, k_3 \geq 0$  are parameters to adjust the routing strategy.

#### V. PERFORMANCE EVALUATION

In this section, we evaluated the performance of the simulation outcomes from multiple perspectives compared to another link-status-based Dijkstra routing algorithm Q-CAST [6] using metrics of request throughput and average fidelity. The simulation platform is based on NetSquid, a discrete event simulator for quantum networks. In addition to routing algorithms, several protocols were deployed on each node to simulate qubit resource management and quantum operations.

**Network parameters:** We constructed a  $10 \times 10$  grid network that contains 100 quantum nodes, and set the number of quantum memory positions in each node to 100(i.e. ignore the influence of node qubit capacity  $C_i$ ). The entanglement establishment between two neighbor nodes is regarded as a robust service with a 100% success rate, and an average completion time of 10ms. The success probability of imperfect entanglement swapping operations is set to 0.9 in all nodes.

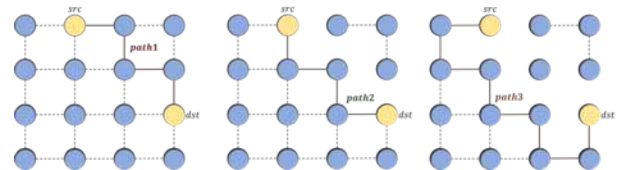


Fig. 4: A three-round routing example on a  $4 \times 4$  grid network, with network topology updated between rounds.

The average operation time of entanglement purification and swapping is set to 2ms and 10ms, separately. Without loss of generality, we also set some random parameters in the network. First, we randomly select an initial fidelity from a Gaussian distribution  $N = (0.8, 0.001)$  for each edge and assume the fidelity of multiple entangled pairs generated along the same edge to be identical. Then we defined a random request generator, which simulates a request flow following a Poisson distribution with a mean inter-arrival time of 50ms. Each request in the flow has random, non-adjacent source and destination nodes. Except for entanglement establishment and entanglement swapping, necessary entanglement purification on a single link is engaged based on the end-to-end fidelity calculation model in [12] to ensure that the final entanglement's fidelity is above 0.5.

**Performance with end-to-end distance:** For the multi-path routing algorithm Q-PSBA, the S-D pair distance is defined as the Manhattan distance between the source node and the destination node. Under the experimental setup described above, 100 requests were generated randomly and finished with paths selected by Q-PSBA and Q-CAST. All available qubits are utilized in nodes on each path, and the fidelity of every successfully constructed target entanglement is measured. Then we group the requests by their S-D pair distance to calculate the average performance, as shown in Fig.5. The results imply that Q-PSBA has an advantage of about 5 entangled pairs in terms of throughput compared to Q-CAST, while the fidelity performance is similar.

**Stability with time:** To study the sustainability of our method, we randomly generate 25 rounds of requests with S-D pair distance of 5 hops, with 4 requests in each round. The performance of requests with different sequences (i.e. arrival time) is shown in Fig.6. We found that the difference in average fidelity and throughput between requests with different sequences is within 0.04 and 3, thus the stability with time indicated.

## VI. CONCLUSION

In this paper, we have proposed a reliable routing protocol for entanglement distribution in quantum networks to deal with limited resources and existing multiple requests. We have modelled the service process of quantum repeaters in both qubit usage and service time dimensions, which helps to characterize the node's service availability. A flexible multi-path routing scheme has been developed, with qubit capacity, service availability, link fidelity and classical channel delay considered. Our algorithm has been evaluated on a grid network of small scale and proved to provide higher throughput and better fidelity for each request with time stability. To strengthen our theory, more experiments based on other topology network structures remain to be conducted in both simulation and physical environments.

To our best knowledge, we have proposed the first multi-request routing scheme without the setting of processing time windows. In future work, it would be interesting to include historical request information in the repeater node's service

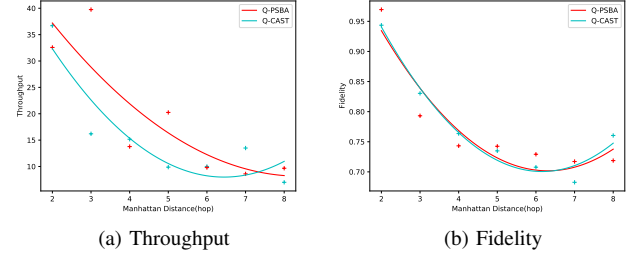


Fig. 5: Performance with different S-D pair distance

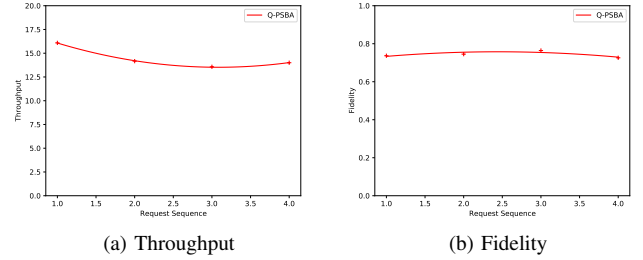


Fig. 6: Performance with different request sequence

model, and an optimal resource allocation algorithm to meet the bandwidth requirements of requests is desired.

## ACKNOWLEDGMENT

The corresponding author Xiaofeng Jiang would like to acknowledge the financial support of National Natural Science Foundations of China (Grant No. 62173315, 62101525, 62021001), Youth Innovation Promotion Association CAS (Grant No. 2020450), Strategic Priority Research Program of CAS (Grant No. XDC07020200), Fundamental Research Funds for the Central Universities, China Environment for Network Innovations (CENI).

## REFERENCES

- [1] S. Wehner, D. Elkouss, and R. Hanson, "Quantum internet: A vision for the road ahead," *Science*, vol. 362, no. 6412, 2018. doi:10.1126/science.aam9288.
- [2] J. F. Dynes *et al.*, "Efficient entanglement distribution over 200 kilometers," *Optics Express*, vol. 17, no. 14, p. 11440, 2009. doi:10.1364/oe.17.011440.
- [3] J. Yin *et al.*, "Satellite-based entanglement distribution over 1200 kilometers," *Science*, vol. 356, no. 6343, pp. 1140–1144, 2017. doi:10.1126/science.aan3211.
- [4] F. Dupuy, C. Goursaud, and F. Guillemin, "A survey of quantum entanglement routing protocols—challenges for wide-area networks," *Advanced Quantum Technologies*, vol. 6, no. 5, 2023. doi:10.1002/qute.202200180.
- [5] C. Li, T. Li, Y.-X. Liu, and P. Cappellaro, "Effective routing design for remote entanglement generation on Quantum Networks," *npj Quantum Information*, vol. 7, no. 1, 2021. doi:10.1038/s41534-020-00344-4.
- [6] S. Shi and C. Qian, "Concurrent entanglement routing for Quantum Networks," *Proceedings of the Annual conference of the ACM Special Interest Group on Data Communication on the applications, technologies, architectures, and protocols for computer communication*, 2020. doi:10.1145/3387514.3405853.

- [7] J. Li *et al.*, “Fidelity-guaranteed entanglement routing in Quantum Networks,” *IEEE Transactions on Communications*, vol. 70, no. 10, pp. 6748–6763, 2022. doi:10.1109/tcomm.2022.3200115.
- [8] Y. Zeng, J. Zhang, J. Liu, Z. Liu, and Y. Yang, “Multi-entanglement routing design over Quantum Networks,” *IEEE INFOCOM 2022 - IEEE Conference on Computer Communications*, 2022. doi:10.1109/infocom48880.2022.9796810
- [9] L. Gyongyosi and S. Imre, “Routing space exploration for scalable routing in the Quantum internet,” *Scientific Reports*, vol. 10, no. 1, 2020. doi:10.1038/s41598-020-68354-y
- [10] T. Coopmans *et al.*, “NetSquid, a network simulator for quantum information using discrete events,” *Communications Physics*, vol. 4, no. 1, 2021. doi:10.1038/s42005-021-00647-8
- [11] A. Dahlberg *et al.*, “A link layer protocol for Quantum Networks,” *Proceedings of the ACM Special Interest Group on Data Communication*, 2019. doi:10.1145/3341302.3342070.
- [12] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, “Quantum repeaters: The role of imperfect local operations in quantum communication,” *Physical Review Letters*, vol. 81, no. 26, pp. 5932–5935, 1998. doi:10.1103/physrevlett.81.5932