

Drone Signal TDOA Estimation Algorithm Based on an Improved Wavelet Threshold Denoising Technique

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abstract—In the passive detection and positioning technology of UAV based on the principle of time difference of arrival (TDOA), the existing time-difference estimation method which relies on signal cross-correlation puts forward high requirements for waveform correlation of received signals. Noise interference in the received signal will lead to a large error in time delay estimation, which will seriously affect the positioning accuracy. To solve this problem, an improved wavelet threshold estimation method for UAV signal arrival time difference is proposed in this paper. In order to solve the problem of signal correlation and noise in traditional TDOA estimation method, wavelet denoising technology is used to pre-process the received signal and improve the accuracy of TDOA estimation. In addition, a new soft threshold function is introduced, which is improved on the traditional threshold function, reduces the oscillation of the hard threshold function and the constant error of the soft threshold function, while maintaining the local characteristics of the signal. The effectiveness of the proposed method is verified by simulation and measured data. The results show that the improved wavelet threshold denoising method has better denoising performance and significantly improves the stability and accuracy of TDOA estimation.

Keywords—Wavelet threshold denoising; Arrival time difference TDOA estimate; Threshold optimization; Signal processing.

I. INTRODUCTION

Currently, civil drones have found widespread applications in areas such as aerial photography, disaster detection, and parcel delivery. With the increasing frequency of drone operations, security concerns in sensitive regions are also on the rise. Drones are classified as "low, slow, and small" objects, which makes them challenging to detect and locate via radar systems. As a result, distributed passive localization techniques are typically employed for drone detection. Among these methods, Time Difference of Arrival (TDOA)-based positioning algorithms are commonly used[1]. This approach calculates the position of a drone by utilizing the time differences at which the received signals reach various locations[2]. The estimation of the time difference between the signal's arrival at each base station is a critical step, directly influencing the accuracy of the target signal's localization.

At present, the most widely adopted method for time difference estimation is based on cross-correlation between two received signals[3]. This technique determines the time delay by calculating the peak of the cross-correlation function of the two signals. However, the accuracy of this method is significantly affected by the correlation between the signals and the presence of noise in the received signals, leading to substantial errors in time delay estimation, and consequently, localization[4]. In the field of signal denoising, wavelet denoising techniques have garnered considerable attention, with a focus on optimizing the wavelet thresholding parameters[5].

In reference [6], a comprehensive review and comparison of wavelet threshold denoising methods are presented. The study examines various threshold selection strategies and their effects on signal denoising performance, providing both theoretical insights and experimental data to support future research. In reference [7], an adaptive wavelet threshold denoising method is proposed, where thresholds are dynamically selected based on signal characteristics. The authors validate the method's effectiveness under low signal-to-noise ratio (SNR) conditions through simulation experiments and compare it with traditional threshold methods. The results demonstrate that this approach significantly outperforms conventional methods in preserving signal details.

In reference [8], a time difference estimation algorithm based on variational mode decomposition and wavelet threshold denoising is introduced. Simulation analysis verifies the feasibility of this algorithm in improving TDOA accuracy by denoising signals. In reference [9], an adaptive threshold selection mechanism is proposed to enhance wavelet threshold denoising performance. By incorporating local SNR, the authors dynamically adjust thresholds for different signal regions, thus improving denoising effectiveness. The method's validity is confirmed through multiple sets of experimental data.

In reference [10], an improved wavelet threshold algorithm for denoising time-series signals is presented. By integrating multi-scale analysis with wavelet coefficient reconstruction, the authors design a novel threshold strategy. Experimental

results show that this method outperforms traditional approaches in both denoising effectiveness and signal reconstruction accuracy.

The downlink transmission signal of a drone typically exhibits a certain period and bandwidth. During transmission, however, the received signal displays non-stationary characteristics due to factors such as propagation path and environmental noise. The use of wavelet threshold denoising methods for preprocessing the received drone signal can significantly influence the denoising results, depending on the selection of parameters. This paper proposes an improved drone signal TDOA estimation algorithm based on wavelet threshold denoising theory. Specifically, it improves the threshold function, threshold selection method, and wavelet decomposition level. Time delay estimation is then performed using a generalized cross-correlation algorithm. The effectiveness of the proposed method is verified through a set of simulation signals and experimental data.

II. PASSIVE TIME DIFFERENCE OF ARRIVAL (TDOA) LOCALIZATION PRINCIPLE

Passive Time Difference of Arrival (TDOA) localization estimates the position of a mobile station by measuring the time difference between the arrival of a signal emitted by a target source at two receiving base stations. This time difference is then used to calculate the distance difference between the target and the stations, leading to the formation of a hyperbola with the two base stations as its foci.

To determine the position of the target source, theoretically, only two sets of TDOA values are required in a two-dimensional scenario. These two sets of TDOA data yield two hyperbolas, and their intersection corresponds to the target's position. In a three-dimensional scenario, however, at least four receiving base stations are necessary to accurately localize the target.

The localization principle in a two-dimensional plane is illustrated in Figure 1, where three base stations (BS1, BS2, and BS3, with BS1 as the primary station) are deployed in different locations to simultaneously receive signals emitted by the target source S. The TDOA estimation algorithm can generally be classified into two approaches: one approach directly calculates the Time of Arrival (TOA) from the target source to the base stations and estimates the TDOA by taking the difference, while the other approach uses a cross-correlation algorithm, where signals received at different base stations are cross-correlated, and the TDOA is estimated based on the peak of the cross-correlation function.

For non-cooperative target sources, Time of Arrival (TOA) values cannot be directly obtained between the target node and the reference node. Therefore, in passive Time Difference of Arrival (TDOA) localization of non-cooperative targets, cross-correlation algorithms are typically employed to estimate the TDOA. This method does not require synchronization between the target node and the base stations, but only necessitates time synchronization between the base stations.

In practical scenarios, the accuracy of time synchronization between base stations, the AD sampling rate of the receivers, and the noise interference during signal propagation all have a significant impact on the final TDOA estimation. Given that the speed of electromagnetic wave propagation in air is approximately 3×10^8 m/s, even a small error in distance (e.g., a few meters) can lead to a substantial positioning error, potentially on the order of hundreds of meters. As a result, accurately determining the target source's position is challenging in real-world applications. Noise interference is particularly problematic, as the effective signal is contaminated by various noise sources during transmission. In extreme cases, the noise may completely obscure the signal, leading to significant deviations in TDOA estimation between base stations. Therefore, denoising the received signals and mitigating the impact of propagation noise on TDOA estimation is crucial for improving the accuracy of passive TDOA localization.

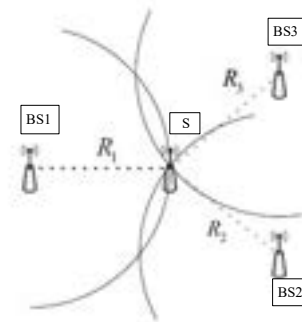


Fig 1 Two dimensional TDOA positioning principle

III. WAVELET DENOISING PRINCIPLE

A. Wavelet Decomposition Principle

Wavelet transforms are utilized to analyze signal details in both the time and frequency domains. They serve to extract the characteristic waveforms of the signal while also detecting anomalous transient information embedded within the normal signal. The wavelet analysis window has a fixed size and various shapes, and by selecting an appropriate wavelet basis, any function can be decomposed. As illustrated in the figure below, this process demonstrates the decomposition of a signal. The left diagram shows the signal passing through high-pass and low-pass filters, resulting in the filtered signals H_1 and L_1 , which represent the first level of decomposition. The signal is then passed through both high-pass and low-pass filters at the L_1 level to obtain H_2 and L_2 , which represent the second level of decomposition.

Following this pattern, the output of the low-pass filter is processed in the same manner at each step, resulting in successive levels of decomposition. After obtaining the L and H signals at each level, wavelet transforms are applied to compute the corresponding wavelet decomposition coefficients. Once the wavelet basis function and the number of decomposition levels are selected, multi-scale wavelet decomposition of the signal can be performed. The signal X is

convolved with the low-pass filter to obtain the low-frequency coefficients cA_1 , and with the high-pass filter to obtain the detail coefficients cD_1 , continuing this process for each subsequent decomposition level.

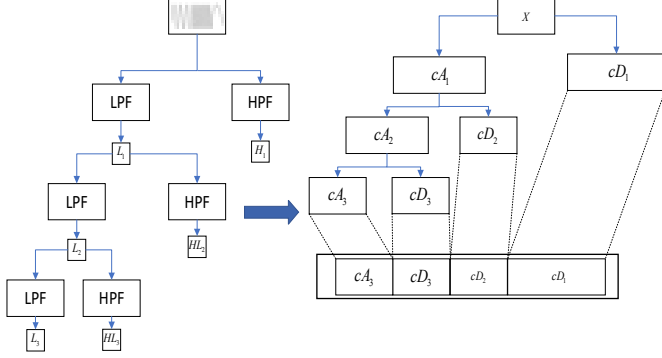


Fig 2 Wavelet threshold denoising process

B. Wavelet Threshold Denoising Algorithm

The denoising process for a noisy signal using wavelet transform can be summarized in the following three steps:

Wavelet Decomposition: Select an appropriate wavelet basis and the number of decomposition levels M . Then, apply M -level wavelet decomposition to the noisy signal, obtaining the wavelet coefficients at each level, with noise primarily located in the high-frequency components.

Thresholding: Apply a thresholding criterion to the high-frequency coefficients of each decomposition level. A typical approach is to set a threshold w for the high-frequency coefficients of each level: coefficients smaller than the threshold are set to zero, while those greater than the threshold are retained.

Reconstruction: Reconstruct the original signal by combining the low-frequency coefficients from the final level of wavelet decomposition with the high-frequency coefficients, which have been quantized (thresholded), from all the decomposition levels.

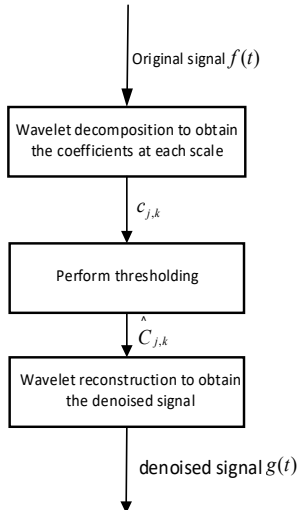


Fig 3 Wavelet threshold denoising process

IV. TRADITIONAL THRESHOLD FUNCTIONS AND IMPROVED SOFT THRESHOLD FUNCTION

A. Traditional Threshold Functions

There are two traditional wavelet threshold functions: the hard thresholding function and the soft thresholding function.

Hard Thresholding Function

$$C_{j,k} = \begin{cases} c_{j,k}, & |c_{j,k}| \geq w; \\ 0, & |c_{j,k}| < w; \end{cases} \quad (1)$$

The hard thresholding algorithm sets wavelet coefficients with absolute values below the threshold to zero, while preserving all coefficients above the threshold unchanged. This method is effective in preserving local features of the signal, such as edges and spikes. However, the hard thresholding function is discontinuous at the threshold $\pm w$, which introduces significant variance and causes oscillations during signal reconstruction..

Soft Thresholding Function

$$C_{j,k} = \begin{cases} \text{sgn}(c_{j,k})(|c_{j,k}| - w), & |c_{j,k}| \geq w; \\ 0, & |c_{j,k}| < w; \end{cases} \quad (2)$$

In the equation: $c_{j,k}$ represents the wavelet coefficient, $C_{j,k}$ is the quantized wavelet coefficient, sign is the sign function, and w is the threshold.

The soft thresholding algorithm is continuous at the threshold $\pm w$, providing better smoothing of the wavelet coefficients in the reconstructed signal, which leads to improved denoising performance. However, for coefficients corresponding to $|c_{j,k}| > w$, the same operation $c_{j,k} = c_{j,k} - w$ is applied to all, introducing errors in the wavelet coefficients. Consequently, the reconstructed signal also contains errors, causing the loss of abrupt changes in the original signal. This results in a lower signal-to-noise ratio (SNR) and a higher mean square error (MSE) in the reconstructed signal.

B. Improved Threshold Function

In addition to the widely used hard and soft thresholding functions, another type of thresholding function known as the non-negative dead-zone thresholding function also exists:

$$C_{j,k} = \begin{cases} c_{j,k} - \frac{w^2}{c_{j,k}}, & |c_{j,k}| \geq w \\ 0, & |c_{j,k}| < w \end{cases} \quad (3)$$

The non-negative dead-zone thresholding function considers that as the wavelet coefficients of the useful signal increase, the attenuation of the noise signal also decreases. This characteristic ensures the function's continuity at the

threshold $\pm w$, providing a balanced compromise between the soft and hard thresholding methods. However, the non-negative dead-zone thresholding function does not account for the exponential decay of the wavelet transform magnitudes of the noise signal. Like the soft and hard thresholding functions, it performs a zeroing operation within a specified interval $|c_{j,k}| < w$, which results in the loss of some useful signal information. Based on this limitation, this paper proposes an improved thresholding function, defined as follows:

$$C_{j,k} = \begin{cases} \text{sgn}(c_{j,k}) \left(|c_{j,k}| - \frac{w^2}{2|c_{j,k}|} e^{2(w-|c_{j,k}|)} \right), & |c_{j,k}| \geq w \\ \text{sgn}(c_{j,k}) \left(\frac{w(e^{k|c_{j,k}|} - e^p)}{2(e^w - e^p)} \right), & |c_{j,k}| < w \end{cases} \quad (4)$$

$p \in (0, w)$.

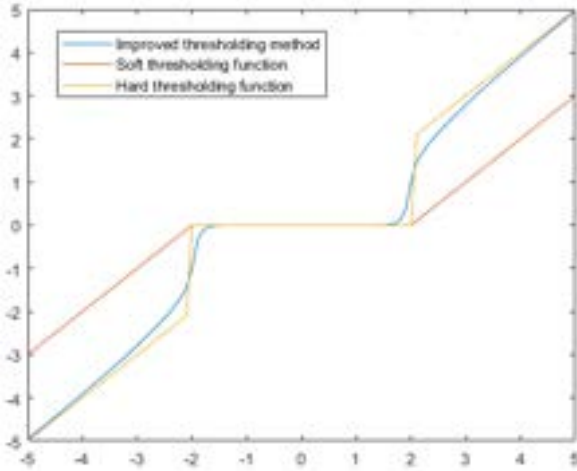


Fig 4 the comparison curves between the improved threshold function and the soft and hard thresholding functions

It can be observed that at point $|c_{j,k}| = w$, the improved threshold function overcomes the discontinuity at point $\pm w$ in the hard thresholding function, and avoids oscillations during signal reconstruction. In segment $|c_{j,k}| \geq w$, the improved threshold function more closely aligns with the hard thresholding function and does not exhibit the constant error observed in the soft thresholding function in this region.

C. TDOA Estimation Algorithm for Drone Signals Based on Improved Wavelet Threshold Denoising

Based on the aforementioned improved wavelet threshold denoising algorithm, both simulated and real DRONE signal data are first processed using this technique. Subsequently, the generalized cross-correlation method is applied for time delay estimation. The overall algorithmic process is illustrated in Figure 5:

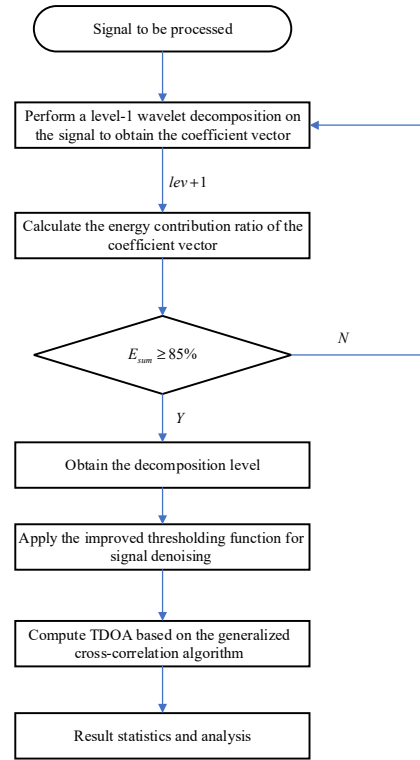


Fig 5 The flow of TDOA estimation algorithm based on improved wavelet threshold denoising

V. SIMULATION AND EXPERIMENTAL ANALYSIS

To validate the effectiveness of the proposed method, we first conducted experiments using simulated signals. In the MATLAB simulation environment, a single-tone sinusoidal signal was generated as the input source. To simulate the non-stationary characteristics of actual drone signals, the signal frequency was varied within the range of 5 to 5.5 Hz. The simulation parameters are detailed in Table 1.

Figure 6 shows the initial noisy signal used in the simulation, which includes additive Gaussian noise to emulate real-world conditions.

TABLE I Simulation Parameters for Noisy Signal

Parameter	Value
Sampling rate (Hz)	1000
Sampling time (s)	1
Signal frequency (Hz)	5~5.5
SNR range (dB)	10:5:30
Noise type	Gaussian white noise

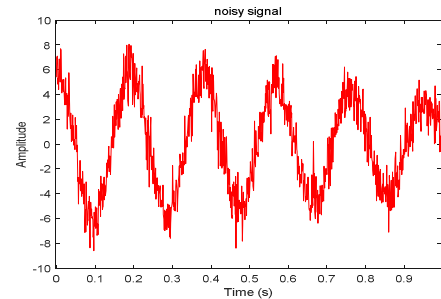


Fig 6 Time domain waveform of simulated signal with noise (SNR=20 dB)

According to the proposed improved wavelet threshold denoising algorithm, the optimal wavelet decomposition level for noisy simulation signals is first determined. Taking a noisy signal with an SNR of 20 dB as an example, we calculate the cumulative energy values of the coefficient vectors at each decomposition level until the cumulative energy exceeds 85%. The wavelet decomposition level at this point is selected as the final level.

For the simulated signals used in this study, the optimal decomposition level was determined to be 5 levels. Thus, when performing wavelet threshold denoising on noisy signals with different SNRs, a decomposition level of 5 was chosen before subsequent denoising.

Table 2 lists the parameters used for the wavelet threshold denoising algorithm. The simulation results in Figure 7 illustrate the denoising performance at different SNR levels. Table 3 presents statistical results of the denoised signal quality, including the Signal-to-Noise Ratio (SNR), Mean Squared Error (MSE), and Normalized Cross-Correlation (NCC) metrics.

TABLE II Parameters of wavelet denoising

Parameter Name	Parameter Value	Remarks
Threshold Selection	sqtolog	$w = \sigma\sqrt{2 \ln(n)}$
Threshold Function	Improved soft threshold function	
Denoising Wavelet	'db4'	Filter coefficient of 4
Decomposition Levels	5	Adaptive selection to 5 levels

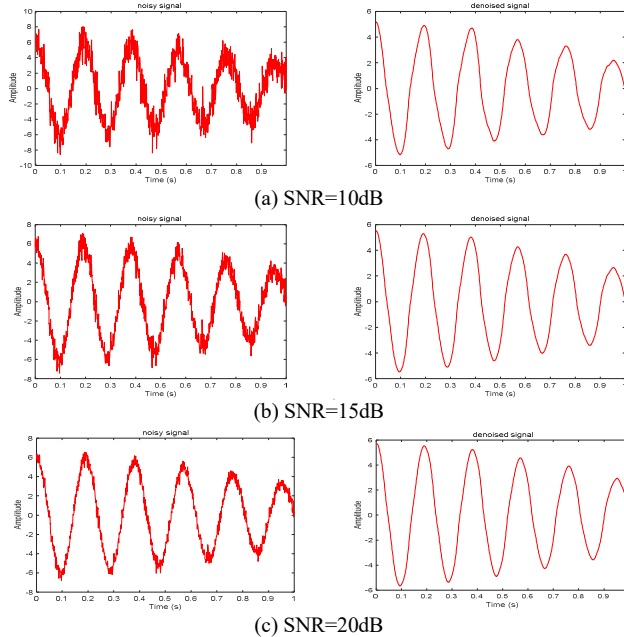


Fig 7 Denoising results of noisy signal under different SNR

TABLE III Evaluation of filtering effect of noisy signal

Initial SNR(dB)	SNR after filtering(dB)	MSE	NCC
10	13.2791	0.59665	0.9919
15	17.6279	0.2192	0.99668
20	22.0074	0.079963	0.99872

It can be seen from Figure 7 and Table 3 that the improved wavelet threshold method can achieve better denoising effect under different SNR.

Finally, the effectiveness of the proposed algorithm is validated in a real drone positioning scenario. The distribution of the base stations and the target drone is illustrated in Figure 8. The system sampling rate of the base station is set at 61.44MHz, and four groups of target source signals are synchronously collected. Each group of signals consists of the in-phase and quadrature components of 16384 data points. Figure 9 demonstrates the time domain waveform diagram (I signal) of one group of signals processed by the algorithm.



Fig 8 Location diagram of base station and drone

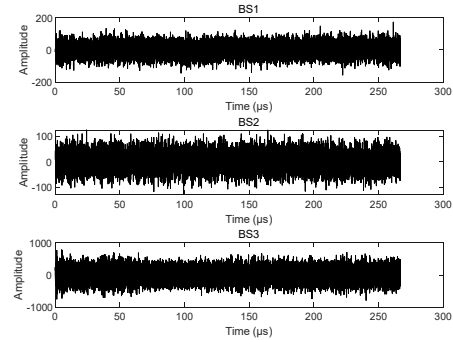


Fig 9 A group of drone measured signals after denoising

Table 4 shows the drone position calculated using the original received signal and the drone position calculated using the improved wavelet threshold denoising TDOA estimation algorithm, and the calculated error with the real drone position. This paper uses calculation to estimate the Euclidean distance between the drone position and the real drone as the error value. The calculation method is as follows, (x_r, y_r) is the real position of the drone.

$$error_d = \sqrt{(x - x_r)^2 + (y - y_r)^2} \quad (5)$$

TABLE IV Drone position estimation results before and after denoising

Group	Error Before denoising/m	Error After denoising/m
1	90.9725	12.3607
2	86.3278	10.5183
3	78.0147	18.2843
4	92.3369	14.2074

As can be seen from Table 4, there is a large deviation between the drone position obtained from the undenoised signal by TDOA positioning algorithm and the real position. The drone position accuracy obtained from the signal processed by the improved threshold wavelet denoising algorithm has been greatly improved and has good estimation stability.

VI. CONCLUSION

In this paper, an improved TDOA estimation method for wavelet threshold denoising is proposed, which improves the traditional threshold function and overcomes the problem of oscillation of hard threshold function and constant error of soft threshold function. An adaptive method is used to determine the wavelet decomposition series of noisy signals by analyzing the energy proportion of wavelet coefficients. Finally, the effectiveness of the proposed method is verified by simulation.

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