

A Game Theoretic Framework for Voting Behaviors and Candidate Strategies Using Monte Carlo Methods

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Abstract—This paper introduces a simulation framework to study voter-candidate dynamics and interactions in multi-issue elections. By combining voter decision-making models with candidate update strategies, we explore various interaction scenarios, including independent updates, tit-for-tat responses, and zero-determinant (ZD) strategies—a novel application in this context. Using Monte Carlo simulations, we evaluate the impact of these strategies on key metrics such as vote counts, positional stability, and extremism under varying hyper-parameter configurations. Our results reveal counterintuitive equilibrium states, demonstrating how certain strategies foster extremism or moderation, providing fresh insights into dynamic voting mechanisms using artificial intelligence.

Index Terms—Game Theory, Monte Carlo, Reinforcement Learning, Elections

I. INTRODUCTION

Political elections can be designed as a competitive, multi-agent game in which candidates compete for voter support by strategically positioning themselves in a policy space that addresses the continuous presentation of voters with respect to the weighted balance of their issues, whether it be extreme or moderate, focused or wide-ranging. This dynamic interplay between candidates and voters has long been studied in game theory, although often in terms of plurality rule elections [1], in which voting strategies can include voting for a less preferred candidate due to a most preferred candidate unlikely to win [2]. In this work, we focus on a stable two-candidate games and propose a dynamic, multi-agent, adaptive game-theoretic model for candidate behavior, extending elements of Hotelling’s Spatial Competition Model [3] with additional considerations for voter preference, regularization to retain base support, and competitive interactions via cooperation and defection.

Some traditional models assume static voter preferences and, in the best case, independent candidate updates, simplifying assumptions so as to create static models of past achievements [6], ignoring the dynamics of an election. However, real-world elections often involve candidates responding not only to voters, but also to their competitors.

A. Goals

Our research aims to explore critical questions in the context of modern two-candidate elections.

- What are the optimal strategies for candidates when considering the opposition’s tactics?
- Are (coercive) zero-determinant strategies effective in elections?
- When is it advantageous for candidates to appeal to voters who lean against them at the risk of alienating their base?

B. Dynamic, Multi-Agent Adaptive Game Framework

The proposed model represents an iterated dynamic game in which two candidates compete to maximize voter support while responding to each other’s position. Each candidate’s objective is influenced by the distance between their position and voter preferences, reflecting ideological alignment or satisfaction while maintaining core support by introducing a regularization term. Strategic interactions are also taken into account, giving candidates the ability to maintain competitiveness or differentiate themselves.

In this game, candidates act as *adaptive agents*, iteratively adjusting their positions based on feedback from voters and their competitor. Voters, on the other hand, dynamically assess candidates based on proximity in the ideological space, weighted by individual preferences. The iterative nature of the game introduces non-linear dynamics, leading to a variety of possible outcomes, including convergence, divergence, or oscillatory behavior.

C. Connection to Hotelling’s Spatial Competition Model

Hotelling’s Spatial Competition Model is a foundational framework for studying competition in an ideological or geographic space. In its classical form, the model predicts that competing agents (e.g., businesses or candidates) will converge toward the median position to capture the largest share of the population. However, this median voter theorem fails to account for real-world complexities, such as:

- The risk of alienating a candidate’s base when moving toward the center.
- Non-uniform voter distributions with varying levels of influence.
- Voter extremism leading to a tipping point
- Strategic interactions between competitors, where differentiation is often preferred over direct convergence.

Our model extends Hotelling’s framework by introducing a regularization term that penalizes candidates for moving too

far from their base voters. Additionally, the model incorporates a competitive interaction term, allowing candidates to respond directly to their opponent's movements. These extensions create a richer and more realistic representation of political strategy.

D. Nash Equilibrium and Stability in the Model

In game theory, a Nash equilibrium occurs when no player can improve their utility by unilaterally changing their strategy, given the strategy of the other player. In the context of this model, Nash equilibrium represents a stable configuration of candidate positions where (1) each candidate maximizes their voter support given the position of their competitor, and (2) neither candidate has an incentive to further adjust their position.

E. Key Contributions of the Model

The primary contributions of this work are as follows:

- We extend Hotelling's Spatial Competition Model by incorporating regularization for base retention, penalizing candidates for alienating their extreme supporters.
- We introduce a competitive interaction term that explicitly models how candidates respond to one another's positioning.
- We analyze the dynamics of candidate positioning in terms of Nash equilibrium, stability, and potential non-equilibrium behaviors, such as oscillations or divergence, thus providing a robust mathematical foundation for simulating and understanding political strategy in a dynamic, multi-agent environment.

Another benefit of our model is that while sequential voting aggregates information better as compared to simultaneous voting, sequential voting also leads to significant inequities, with later voters being better off than early voters [4]. In our model, as candidates update their policies, voters are equally as well off before or after choosing, with the candidate accounting for previous voters whilst still having a dynamic presentation of issues. Some authors [5] also remark that in terms of large electorates, because the probability of being pivotal goes to zero, the expected utility difference between any two candidates also goes to zero. However, in our model, since the candidates regularly update their position upon gaining perspective voters, this lack of utility does not exist.

II. LITERATURE REVIEW

Krueger & Acevedo [8] provide a perspective on the game-theoretic approach to voting, emphasizing its implications for understanding voter behavior, arguing that the restrictive assumptions of traditional rational choice theory can blend well with the more flexible theories of expressive voting, creating a duality of a more nuanced strategic nature in elections, particularly in contexts where preferences may not align with the majority in multi-party elections.

Building on this, Meir [9] introduces a behavioral game-theoretic model that addresses strategic voting under uncertainty, an advancement in social choice theory wherein voters adapt their based on the perceived behavior of others.

In the realm of multi-agent systems, Novikov [10] presents a comprehensive model of strategic decision-making that integrates game theory with social psychology and multi-agent systems, emphasizing the interconnectedness of agents' decisions. While this model is very comprehensive from a psychological point of view, accounting for such features as self-reflection and other internal states as well as external states such as social influence, it presupposes baseline self-awareness, uses complex and nearly unknowable modeling assumptions that are difficult to generalize to an autonomous system or other logic-based choice applications, subsequently muting extension to computational models, and the voters choice on how to act on their beliefs. We note that our model, as discussed below, does not suffer from these disadvantages.

We further note research by Madani and Lund [11] that uses Monte Carlo methods in game-theoretic frameworks for multicriteria decision making under uncertainty, similar to our own application of identifying hyperparameters. Their findings suggest that traditional decision-making methods often assume cooperation among decision-makers, while game theory allows for the modeling of self-optimizing behaviors. This distinction is vital for analyzing real-world scenarios where cooperation may not be feasible, thus providing a more realistic depiction of decision-making processes. We take this insight and apply it to cooperative and defector games as discussed in our results.

III. MATHEMATICAL MODEL

Our model incorporates three key components: utility functions for candidates, gradient-based updates for movement, and regularization to prevent candidates from alienating their base. We also consider scenarios in which the candidate's utility responds as an interaction to competition.

A. Decision-Making

Voters select candidates based on their proximity to voter preferences on multi-issue vectors. Let $v \in \mathbb{R}^d$ represent a voter's position on d issues and $c_1, c_2 \in \mathbb{R}^d$ represent the positions of two candidates. A voter chooses candidate c_i based on a decision function f :

$$f(v, c_1, c_2) = \begin{cases} c_1 & \text{if } d(v, c_1) < d(v, c_2), \\ c_2 & \text{otherwise,} \end{cases}$$

where $d(v, c_i)$ is a distance metric. Various f functions simulate different voter behaviors, as seen in Table 1. Candidate update strategies are described in Table 2 and the scenarios as to how candidates respond to their opponent are listed in Table 3.

B. Experiment Setup

The simulation iterates over:

- **Voter Functions:** $\{f_1, f_2, \dots, f_k\}$.
- **Proactive Candidate Functions:** $\{c_{p1}, c_{p2}, \dots, c_{pm}\}$.
- **Reactive Candidate Functions:** $\{c_{r1}, c_{r2}, \dots, c_{rn}\}$.
- **Hyperparameters:** α, β, γ .

To assess the performance and behavior of candidates under various scenarios, we created a model wherein co

TABLE I
DESCRIPTIONS OF VOTER DECISION-MAKING METHODS.

Voter Decision Method	Description
Most Extreme Position	The voter weighs their extreme issue differences more heavily.
Geometric Average	The voter considers the geometric mean of distances across issues compared to the candidates'.
Variance-Based Choice	The voter favors candidates whose variance is similar to theirs.
Closest Average	The voter compares the average of their position with the average of the positions of each candidate.
Harmonic Average	The voter uses the harmonic mean of their issues and compares it to the harmonic means of the candidates, emphasizing smaller differences.
Least Extreme Position	The voter chooses based on the issue they feel least strongly about.
Strongest Three Average	The voter calculates the average position of the three issues they care most strongly about and chooses the closest candidate.
Least Extreme Three Average	The voter calculates the average position of the three issues they care least strongly about.
Weighted Complex Choice	The voter combines multiple decision methods (e.g., most extreme, closest average) with assigned weights for each.
Cluster-Based Influence	The voter incorporates the positions of their social group or community and chooses the candidate closest to the average cluster position.
Weighted Extreme and Moderate	This voter decision-making function evaluates candidates based on a combination of the voter's most extreme and least extreme preferences for issues, weighted by an extreme weight.

TABLE II
DESCRIPTIONS OF CANDIDATE UPDATE METHODS.

Candidate Update Method	Description
Average Update	Uses the average between the candidate and the voter.
Geometric Update	Uses the geometric mean for smoother adjustments.
Harmonic Update	Uses the harmonic mean for balancing extreme outliers in voter positions.
Log Adjustment Update	Uses logarithmic scaling to adjust positions based on voter preferences.
Fixed Step Update	Moves candidate positions by a fixed step size in the direction of voter preferences.
Reinforcement Learning Update	Adjusts positions based on a reward signal, mimicking learning from voter feedback.
Decaying Update	Adds resistance to large shifts in position over time, simulating inertia.
Consensus Update	Accounts for the preferences of multiple voters by averaging their positions.
Cluster Update	Allows candidates to adapt based on groups of voters with different weights (e.g., demographics or regions).
Perturbation Update	Introduces random noise to simulate external shocks or unexpected events affecting positions.

TABLE III
DESCRIPTIONS OF CANDIDATE STRATEGIES IN DYNAMIC ELECTIONS.

Strategy	Description
No Interaction	Each candidate updates their positions independently, solely in response to voter preferences.
Tit-for-Tat	Candidate 2 mimics Candidate 1's previous move, fostering a reactive dynamic that mirrors actions taken by the opponent.
Generous Tit-for-Tat	An extension of the Tit-for-Tat strategy, where Candidate 2 occasionally forgives deviations by Candidate 1, introducing a probabilistic element of leniency.
Zero-Determinant (ZD)	Candidate 2 strategically enforces a linear payoff relationship, manipulating the balance of utility between themselves and Candidate 1.
Retaliatory ZD	A variation of the ZD strategy where Candidate 2 penalizes extreme or aggressive moves by Candidate 1, encouraging moderation in the competition.
Mixed Strategy	Candidate 2 dynamically switches between Tit-for-Tat and Zero-Determinant strategies, depending on the behavior of Candidate 1, adapting to maximize strategic advantage.

(conservative) was 0-50 and *li* (liberal) was 50-100. Moderate positions were considered to be 40-60, leaning right and left, respectively. We further accounted for gradually more extreme voter positions as the numbers tended towards the tails, with the voter distribution modeled on the current conservative/moderate/liberal breakdown in the United States.

The analysis includes several metrics to assess candidate behavior and the effectiveness of the strategy. Standard deviation captures the variability in a candidate's positions across all issues, reflecting how consistently they adhere to a particular stance. Complementing this, the average movement and median movement provide measures of central tendency, summarizing a candidate's typical positions. The range of movement highlights the breadth of their positional shifts, while waviness, defined as the variance in successive position changes, captures oscillatory behavior or erratic adjustments.

To assess moderation and stability, the study evaluates how often candidates remained in predefined ranges. Moderate

positions are defined as falling between 45 and 55, while balanced positions cover the broader range of 40 to 60. Conversely, extreme positions are those below 30 or above 70. The percentage of time spent in these categories reveals the alignment of candidate strategies with centrist or polarizing approaches.

Dynamic trends are analyzed through peak and valley counts, which track significant increases and decreases in positions over time, and directional trends, which indicate whether movements predominantly ascend, descend, or remain stable. The study also includes cluster analysis to identify whether candidate positions gravitate toward specific ranges, shedding light on the strategic appeal to distinct voter blocks. Finally, vote counts per candidate quantify their success in gaining voter support throughout the simulation.

This comprehensive set of metrics offers a nuanced view of candidate strategies and their impact on voter alignment and overall campaign dynamics.

C. Candidate Utility Function

The utility of a candidate C is defined as:

$$U_C = \sum_{i=1}^n W_i \cdot f(d(C, V_i)) - \lambda \cdot \sum_{j \in V_b} g(d(C, V_j)) - \gamma \cdot d(C, C_{-i}), \quad (1)$$

where:

- W_i : Weight of voter i 's influence.
- $f(d(C, V_i))$: Satisfaction function for voter i , based on the distance $d(C, V_i)$ between candidate C and voter V_i .
- $g(d(C, V_j))$: Penalty function for the distance between candidate C and their base voter V_j , where V_b is the set of base voters.
- λ : Regularization parameter controlling the trade-off between appealing to moderates and retaining base voters.
- γ : Interaction parameter that penalizes candidates for being "too distant" from their competitor C_{-i} .
- $d(C, V_i)$: Euclidean distance between candidate C and voter V_i , given by:

$$d(C, V_i) = \sqrt{(x_C - x_{V_i})^2 + (y_C - y_{V_i})^2}. \quad (2)$$

The utility function in Equation 1 has three components:

- 1) The first term accounts for voter satisfaction, weighted by each voter's influence.
- 2) The second term introduces a regularization to prevent the candidate from moving too far from their base.
- 3) The third term models the interaction between candidates by penalizing excessive distance from the competitor.

D. Gradient of the Utility Function

To determine how candidates adjust their positions, we calculate the gradient of the utility function U_C with respect to the candidate's position C . The gradient is expressed as:

$$\begin{aligned} \nabla U_C = & \sum_{i=1}^n W_i \cdot \nabla f(d(C, V_i)) \\ & - \lambda \cdot \sum_{j \in V_b} \nabla g(d(C, V_j)) \\ & - \gamma \cdot \nabla d(C, C_{-i}) \end{aligned} \quad (3)$$

where:

$$\nabla d(C, V_i) = \frac{C - V_i}{\|C - V_i\|}, \quad \nabla d(C, C_{-i}) = \frac{C - C_{-i}}{\|C - C_{-i}\|}. \quad (4)$$

Gradient Terms:

1. Voter Feedback:

$$\nabla f(d(C, V_i)) = f'(d(C, V_i)) \cdot \nabla d(C, V_i), \quad (5)$$

where $f'(d(C, V_i))$ is the derivative of the satisfaction function.

2. Base Retention:

$$\nabla g(d(C, V_j)) = g'(d(C, V_j)) \cdot \nabla d(C, V_j), \quad (6)$$

where $g'(d(C, V_j))$ is the derivative of the penalty function.

3. Competitor Interaction:

$$\nabla d(C, C_{-i}) = \frac{C - C_{-i}}{\|C - C_{-i}\|}. \quad (7)$$

E. Candidate Movement Formula

Candidates update their positions iteratively based on the gradient of their utility function. The movement formula is:

$$C_{t+1} = C_t + \alpha \cdot \nabla U_C + \beta \cdot \nabla U_{C_{-i}}, \quad (8)$$

where:

- α : Learning rate for voter feedback, controlling how strongly candidates adapt to voter preferences.
- β : Learning rate for competitor interaction, determining how candidates respond to their competitor's position.
- ∇U_C : Gradient of the candidate's own utility function.
- $\nabla U_{C_{-i}}$: Gradient of the competitor's utility function with respect to their position C_{-i} .

IV. EXPERIMENTAL RESULTS

Here we summarize key experimental results from Figure 1 and Figure 2, which highlight counter-intuitive outcomes in scenarios without candidate responsiveness. Figure 1 illustrates voter behavior and candidate strategies across different models with 10,000 voters. The top left shows minimal variance when voters chose their most extreme positions and candidates updated geometrically. Reinforcement learning (top right) reduced variance further. In contrast, the bottom left, where voters chose their least extreme positions, and candidates used an average update, led to greater variability but did not converge on moderation. The bottom right reveals a candidate crossing the moderate line (50) to capture votes when voters selected based on their least extreme positions and candidates used a log update.

Figure 2, with 5,000 voters, shows significant waviness when voters chose their least extreme positions and candidates updated based on average voter distance.

These simplified models, while not accounting for penalties or regularization, demonstrate that extremism can stabilize without incentivizing moderation. Symmetry in candidate extremism is not guaranteed, tipping points of extremism may limit voter appeal, and strategies balancing base appeasement with moderate positions can broaden voter support. Comprehensive model statistics follow, covering various hyperparameters and metric outputs.

A. Results

We report the results of the full test below:

- 1) **Strategy distributions:** Cluster distribution highlights how a candidate's positions align with predefined categories over time, offering insights into their appeal to centrists, extremists, or a mix of voters. Our findings show that Zero-Determinant (ZD) strategies consistently drive extremism. By enforcing a strict utility relationship between candidates, ZD strategies demand significant positional adjustments to maintain the desired payoff

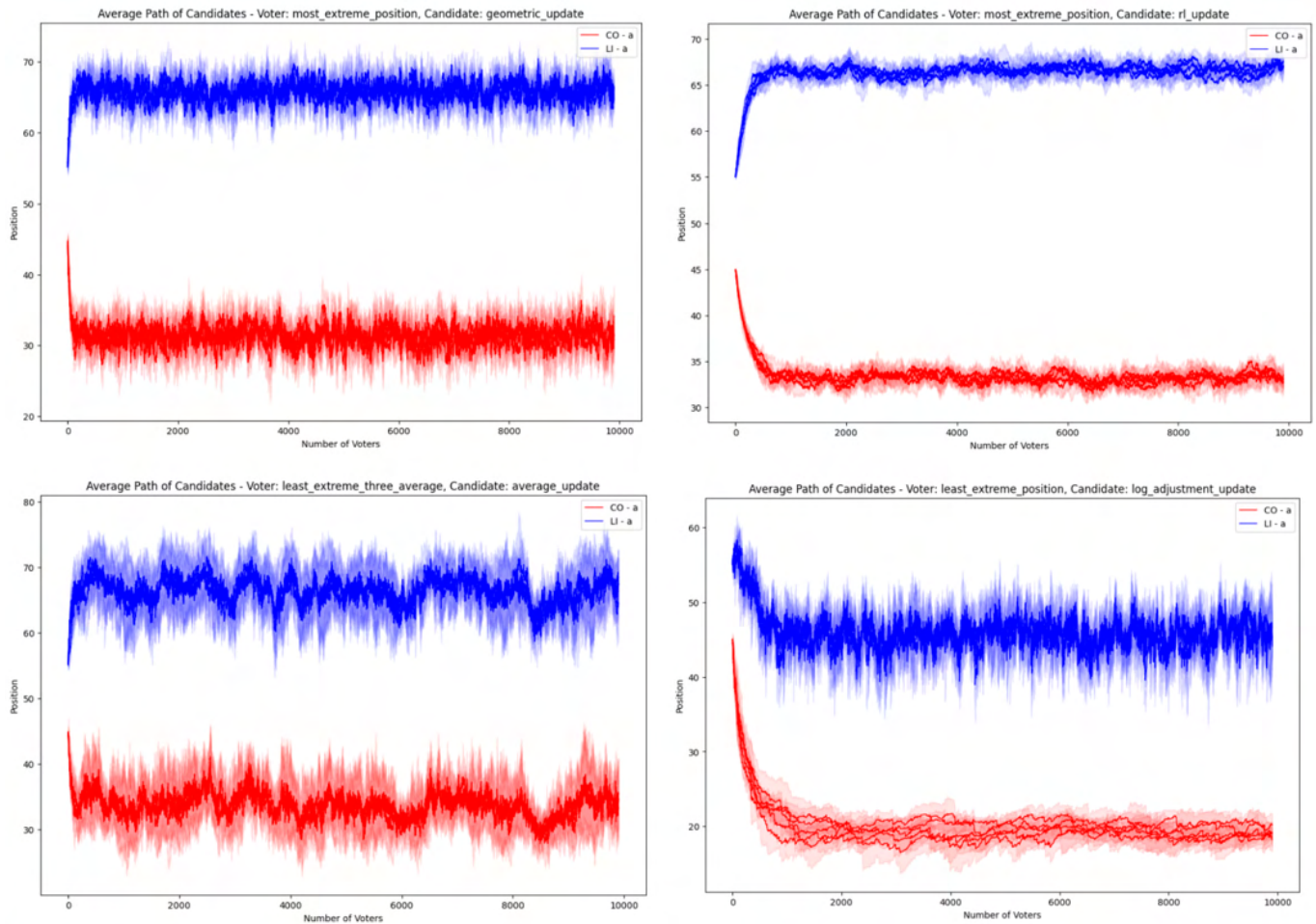


Figure 1. Sample paths for no interaction scenarios

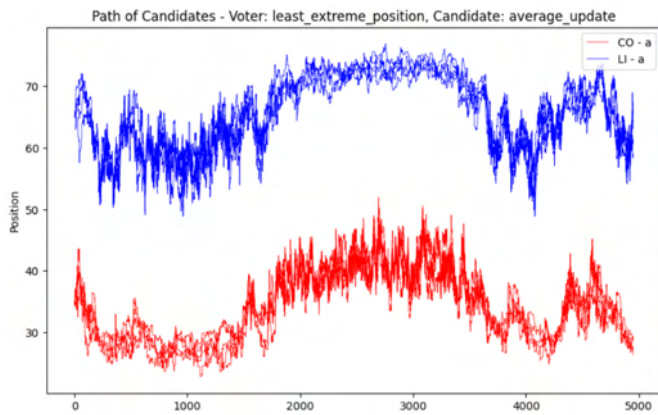


Figure 2. Sample path for no interaction scenario with voter least extreme position

balance. This often pushes candidates away from moderate stances and toward extremes, prioritizing strategic gains over balanced or centrist policies. In contrast, No Interaction strategies resulted in a broad spread across all voter blocks, characterized by low kurtosis. Tit-for-Tat and Mixed Strategies maintained a concentrated focus on

the balanced center, while Generous Tit-for-Tat captured the moderate position with a higher kurtosis than a Gaussian distribution, reflecting its ability to balance responsiveness and moderation effectively.

- 2) **Reinforcement Learning is an optimal candidate strategy:** As can be seen in Figure 2 & Table IV, as a candidate strategy, reinforcement learning consistently produced the highest median score across all scenarios, indicating its robust performance in aligning candidate positions with voter preferences. Additionally, it exhibited the highest standard deviation in most cases, highlighting its capacity for significant adaptability and responsiveness to varying voter dynamics. The only exception was under the weighted complex choice, where it was the lowest standard deviation.
- 3) **Waviness of Candidates:** As shown in Table VI, ZD strategies exhibited the highest waviness due to their strict utility relationships, causing abrupt, erratic position shifts. In contrast, Tit-for-Tat and Mixed Strategies showed the lowest waviness, relying on calculated, measured adjustments, with Mixed Strategies tempering extreme reactions. Generous Tit-for-Tat demonstrated a broad range of positions with moderate waviness, as

Alpha	Avg of li_median	Avg of li_moderate %	Avg of co_moderate %
0.1	67.83	25.18	4.96
0.3	67.85	25.15	22.27

Beta	Avg of li_median	Avg of li_moderate %	Avg of co_moderate %
0.1	69.98	6.84	7.51
0.3	65.71	43.49	19.72

Gamma	Avg of li_median	Avg of li_moderate %	Avg of co_moderate %
0.1	67.85	25.17	10.25
0.3	67.84	25.16	16.98

Figure 3. Greater responsiveness to voters makes exceptional gains in getting the vote

its forgiveness mechanism enabled gradual shifts across the ideological spectrum, avoiding erratic swings while maintaining adaptability.

- 4) **Moderate percentage gain:** Table VII shows that the strongest approach to appeal to moderates was when voters chose candidates based on the geometric average, least extreme position, and cluster-based influence, in this order. Geometric average minimizes the impact of extreme positions, favoring balanced candidates; least extreme position explicitly rewards moderation by prioritizing centrist issues; and cluster-based influence aligns candidates with broader voter trends, reflecting the preferences of the moderate majority. These methods collectively emphasize balance and centrality, making them effective for attracting moderates.
- 5) **Gaining the moderate vote:** Table VIII shows that Generous Tit-for-Tat and No Interaction strategies have the highest peak and valley counts. Generous Tit-for-Tat's forgiveness mechanism encourages smoother but broader movements, while No Interaction allows unrestricted oscillations driven by voter preferences. Median numbers to gain the moderate vote are strongest for Generous Tit-for-Tat, giving the candidate the ability to maintain their base while broadening their appeal.
- 6) **Alpha, beta, and gamma settings:** Figure 3 shows how alpha, beta, and gamma influence candidate alignment with moderate and balanced voters. Beta has the strongest impact, with a higher value (0.3) significantly increasing candidate-voter alignment, indicating a better alignment with centrist voters due to stronger responsiveness to voter preferences. In contrast, changes in alpha and gamma have minimal effects, as they primarily influence competitor-driven adjustments and utility strategies rather than direct voter alignment.

B. Summary

Our study reveals that candidate strategies significantly influence election dynamics. ZD strategies foster extremism through aggressive adjustments, while reinforcement learning consistently achieves the highest voter alignment by adapting effectively to preferences. Generous Tit-for-Tat excels in gaining moderate votes by balancing forgiveness with broad but gradual position shifts. Finally, voter behaviors like geometric

averaging and least extreme positioning strongly favor centrists, emphasizing the importance of moderation in electoral outcomes.

V. DISCUSSION

Our model has broader implications beyond political science, extending to any multi-agent dynamic system with layered interactions. Notably, it aligns well with image processing frameworks, where agents explore images and respond to the strength of matrix eigenvalues across multiple inferential levels. This dynamic enables agents to process different aspects, such as smoothness or detail, based on the principles outlined above. By analyzing the choices in reaction and strategy management, our study reveals which models exhibit rigidity or fluidity, variation, or bias. This framework also connects to the concept of temperature in modern AI systems, emphasizing its relevance to adaptable decision-making.

our model has potential applications in video game design, cryptocurrency security [13], and network theory [12], particularly in scenarios requiring critical node detection. It offers a more dynamic and scalable approach compared to existing methods.

VI. CONCLUSION AND FUTURE WORK

This work introduces a comprehensive multi-agent dynamic voting model, highly applicable to spatial systems and interactive environments where agents, such as reinforcement learning entities, iteratively update and evaluate multiple contributing factors. By leveraging this layered framework, we capture the duality of agent interactions, as illustrated by the strategic positioning of candidates within our model.

The model's robustness lies in its versatility and the breadth of evaluation metrics it employs, providing deep insights into candidate strategies and voter behavior. These metrics highlight that strategic choices are not static; adaptability and responsiveness to environmental and competitive factors significantly influence outcomes. For example, our findings demonstrate that even seemingly minor assumptions, such as voters favoring candidates based on their least extreme positions, can profoundly affect candidate volatility and foster strong, loyal followings.

Moreover, this study emphasizes the importance of nuanced strategic approaches in elections. Simulated scenarios reveal how dynamically adapting candidate strategies can shape election outcomes. For instance, we hypothesize that in the 2024 U.S. presidential election, the ability to appeal to moderate voters on select issues while maintaining a strong base played a pivotal role in the sweeping mandate achieved by the winning party.

Future research will focus on integrating real-world voter preference data to further validate and refine the model. Aligning voter preference distributions with empirical data will enhance our understanding of the interplay between voter behavior and candidate strategies. Additionally, we plan to expand the model's applications beyond electoral systems to other domains, such as image generation in convolutional neural networks. Here, agents would analyze and adapt based

TABLE IV
COMPARISON OF UPDATE STRATEGIES ACROSS METRICS

Update Strategy	Geometric Average	Least Extreme Position	Most Extreme Position	Strongest Three Average	Weighted Complex Choice
Average Update	3.665	3.850	3.780	3.806	3.891
RL Update	4.403	3.895	4.018	4.200	3.684
Decaying Update	3.833	3.841	3.772	3.900	3.558
Fixed Step Update	3.800	3.822	3.779	3.916	3.844
Geometric Update	3.800	3.872	3.885	3.914	3.942
Harmonic Update	3.910	3.702	3.899	3.934	3.158
Log Adjustment Update	3.817	3.857	4.009	3.923	3.859
Perturbation Update	3.900	3.797	3.889	3.902	3.833

TABLE V
REINFORCEMENT LEARNING PRODUCED THE HIGHEST STANDARD DEVIATION AMONG CANDIDATE STRATEGIES

Update Strategy	Generous Tit-for-Tat	Mixed Strategy	No Interaction	Tit-for-Tat	Zero-Determinant
Average Update	35.125	35.125	35.125	35.125	35.125
RL Update	38.899	38.899	38.899	38.899	38.899
Decaying Update	35.125	35.125	35.125	35.125	35.125
Fixed Step Update	35.120	35.120	35.120	35.120	35.120
Geometric Update	35.119	35.119	35.119	35.119	35.119
Harmonic Update	35.119	35.119	35.119	35.119	35.119
Log Adjustment Update	35.120	35.120	35.120	35.120	35.120
Perturbation Update	35.121	35.121	35.121	35.121	35.121

TABLE VI
SCENARIO STRATEGIES AS MEASURED BY STANDARD DEVIATION AND WAVINESS

Strategy	Average Standard Deviation	Average Waviness
Generous Tit-for-Tat	13.4766	0.4116
Mixed Strategy	0.4034	0.0002
No Interaction	3.5324	0.4496
Tit-for-Tat	0.4034	0.0002
Zero-Determinant	1.6755	0.9523

TABLE VII
VOTER STRATEGIES AGAINST GAINING THE MODERATE VOTE

Strategy	Average Moderate Percentage
Cluster-Based Influence	6.964
Geometric Average	7.133
Harmonic Average	6.764
Least Extreme Position	7.072
Least Extreme Three Average	6.851
Most Extreme Position	6.794
Strongest Three Average	6.509
Variance-Based Choice	6.623
Weighted Complex Choice	6.767
Weighted Extreme and Moderate	6.897

on different image features, reflecting the strategies outlined in this study.

In conclusion, this work provides a solid foundation for studying dynamic voting systems, showcasing both its theoretical depth and practical relevance to elections and broader multi-agent systems. We aim to continue refining and expanding this framework through interdisciplinary collaborations and empirical validation.

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TABLE VIII
MEDIAN, PEAKS, AND VALLEYS FOR DIFFERENT STRATEGIES

Strategy	Avg of li_median	Avg of li_peak_count	Avg of li_valley_count
Generous Tit-for-Tat	52.659	233.64	265.99
No Interaction	74.97	61.77	56.12
Zero-Determinant	100	33	4
Tit-for-Tat	61.09	2	2
Mixed Strategy	61.09	2	2

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