

Radar Pulse repetition interval estimation based on Blind Source Separation

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Abstract— To address the challenge of radar signal aliasing in complex electromagnetic environments, this study proposes a PRI estimation method based on signal separation preprocessing. Mixed signals from a one-dimensional uniform antenna array are separated using blind source separation, isolating single radar signal pulse flows. This paper proposes an innovative waveform shaping method based on the floating threshold of Hilbert envelope, which is then applied to obtain rectangular pulses, enabling precise estimation of pulse arrival and end times. This leads to accurate PRI calculation. Simulation experiments validate the method's effectiveness, providing a solid foundation for PRI modulation recognition, radar mode analysis, and threat assessment.

Keywords— radar signal aliasing; pulse repetition interval; signal separation preprocessing; waveform shaping

I. INTRODUCTION

Pulse Repetition Interval (PRI) represents the time interval between two adjacent pulses of the same radar, which reflects the radar's working mode, use and even threat level, and is an important characteristic parameter of radar radiation source[1]. Multiple modulation styles of PRI are used in new radars. The estimation of PRI parameters is very important for radar radiation source identification. Under complex electromagnetic environment conditions, it is difficult to estimate PRI parameters due to the following two factors:

(1) PRI of various modulation styles is often used in new radars, and the PRI parameter changes are complicated[2];

(2) Due to the large number of radiation sources and high pulse density, different radar signals are aliasing each other, causing false signal interference to PRI parameter estimation[3-4].

At present, for the fixed, staggered PRI modulation style, it is usually possible to estimate the Arrival Time (TOA) of each pulse in the radar pulse train during deinterlacing. TOA statistical histogram method, linear stochastic dynamic model method, Hough transform method, plane transform method, fast Fourier transform method and so on are used to distinguish. However, for PRI modulation styles such as jitter, slip, sinusoidal modulation and resident and switch, deinterleaving can only be roughly identified as "complex" type, and can not make a specific judgment. The existing methods for complex PRI modulation recognition basically estimate the TOA of each

pulse front in the pulse train, then obtain its differential sequence (called PRI sequence), and then use the different features of the PRI sequence to realize the modulation recognition of the complex PRI system. The existing recognition methods can be roughly divided into two categories: PRI sequence differential sign feature recognition method[5-7] and PRI sequence autocorrelation function shape feature recognition method[8].

In view of the importance of PRI parameters, it is urgent to develop effective methods to overcome the influence of signal aliasing on their accurate estimation in complex electromagnetic environments. In this paper, a technical idea based on blind signal separation is proposed. Radar signals are separated by blind source separation method, and then PRI parameters are estimated for single radar signal, which lays the foundation for PRI modulation recognition, radar operation mode and threat level determination.

II. MATHEMATICAL MODEL OF TYPICAL RADAR PULSE REPETITION INTERVAL AND RADAR SIGNAL BLIND SOURCE SEPARATION

A. Mathematical model of typical radar pulse repetition interval

Due to the different working mode, application, principle or parameter requirement of radar, there are many modulation styles of radar pulse repetition interval. Radar using different PRI modulation modes can also achieve the purpose of resolving range blur and velocity blur and anti-jamming. There are many PRI modulation modes, and several typical PRI modulation modes are introduced as follows[9-10].

1) The PRI is fixed:

PRI fixed refers to a group of pulse signals in which the interval size of adjacent pulses and the repetition rate of pulses remain stable and unchanged. Radar PRI fixed modulation mode is more commonly used, such as the 1960s to 1980s long-range early warning radar, air defense and anti-missile radar, sea search radar, conventional moving target display (MTI) and pulse Doppler (PD) radar. Radar using PRI fixed modulation usually leads to the error of parameter measurement, weak multi-target processing ability and poor anti-jamming ability.

PRI In fixed modulation mode, the value expression of PRI is as follows:

$$PRI_i \equiv PRI_0 \quad i = 1, 2, 3, \dots \quad (1)$$

Where PRI_0 is a constant value that does not change with time, and N is the total number of pulses. Under normal circumstances, when the maximum change value of the PRI parameter does not exceed 1% of the central value of the PRI, the PRI modulation mode can be approximated as PRI fixed modulation. Figure 1 shows the schematic diagram of PRI fixed modulation mode.

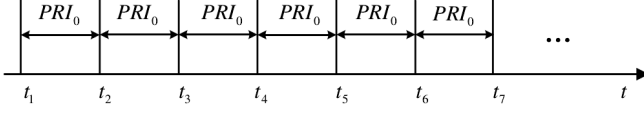


Figure 1. Schematic diagram of PRI fixed modulation

2) The PRI is uneven:

Radar with PRI staggered is mainly used to eliminate blind velocity or range ambiguity. The pulse sequence of PRI staggered radar can be regarded as the interleaving of multiple PRI pulse sequences. There are m PRI values in the PRI staggered modulation mode, $PRI_k (k = 1, 2, \dots, m)$ is the subperiod, and the initial pulse is the head, so that the interval of the pulse is $PRI_1, PRI_2, \dots, PRI_m$, and the values are reciprocated in this order. m pulses are taken as a complete unit, and the subperiod is cycled once. The PRI value of the i th pulse is expressed as follows:

$$PRI_i = PRI_k \quad k = i \bmod m \quad (2)$$

Here, $\bmod$ is the mod function. Figure 2 shows the PRI values of the PRI staggered modulation mode.

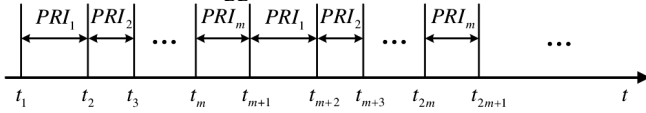


Figure 2. Schematic diagram of PRI staggered modulation

3) PRI jitter:

PRI jitter modulation means that PRI varies randomly around a fixed value (mean value), and this variation is called jitter, which generally follows Gaussian or uniform distribution. Typical PRI jitter modulation methods include pseudo-random sequence modulation, sine modulation, etc. The mathematical model of PRI value of PRI jitter modulation mode is as follows.

$$PRI_i = PRI_0 + \delta_i \quad i = 1, 2, 3, \dots \quad (3)$$

Where PRI_0 is a constant central value, δ ranges from $[-T, T]$, and the maximum jitter γ is expressed as follows:

$$\gamma = \pm \frac{\Delta T}{PRI_0} \quad (4)$$

The γ usually ranges from $[\pm 1\%, \pm 30\%]$, and the variation pattern of PRI jitter modulation is shown in Figure 3.

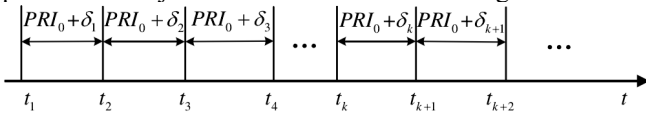


Figure 3. Schematic diagram of PRI jitter modulation

B. Mathematical model of radar signal blind source separation

Suppose that K far-field radar signals are incident on a one-dimensional uniform linear array composed of M sensors from

directions $\theta_1, \dots, \theta_K$. The array observed signal at sampling time t is $\mathbf{X}(t)$, which is expressed as [11]:

$$\mathbf{X}(t) = \sum_{k=1}^K a(\theta_k) s_k(t) + \mathbf{w}(t) = \mathbf{A}(\boldsymbol{\theta}) \mathbf{s}(t) + \mathbf{w}(t) \quad (5)$$

Where $\mathbf{X}(t) = [\mathbf{X}_1(t), \mathbf{X}_2(t), \dots, \mathbf{X}_M(t)]^T$ (superscript T transposed) as the vector array observation signal, $a(\theta_k)$ is the array orientation vector, $\mathbf{A}(\boldsymbol{\theta}) = [a(\theta_1), a(\theta_2), \dots, a(\theta_K)]$ is a matrix of direction vectors, $\boldsymbol{\theta} = [\theta_1, \dots, \theta_K]^T$ is the signal Angle parameter vector, $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$ for the incoming signal vector, $\mathbf{w}(t) = [w_1(t), w_2(t), \dots, w_M(t)]^T$ is an additive noise vector, sampling time $t = 1, 2, \dots, N$, N is the number of signal sampling time, that is, the number of signal sampling points. The basic assumptions of the array observation signal model shown in Equation (9) are as follows:

- (1) The incident signals are independent narrowband stationary signals with mean $E\{\mathbf{s}(t)\} = 0$, covariance matrix $E\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \text{diag}\{p_{s_1}, p_{s_2}, \dots, p_{s_K}\} \triangleq \mathbf{P}_s \in \mathbb{R}^{K \times K}$, p_{s_k} for the first k signal power;
- (2) The additive noise in the array observation signal vector is additive noise and independent of the signal;
- (3) The number of incident signals is less than or equal to the number of sensors, and the number of signals and sensors is far less than the number of samples, that is, $K \leq M$, K and $M \ll N$;
- (4) The signal propagates in ideal space, and the array sensor has omnidirectional consistency.

III. THE PROPOSED PRI ESTIMATION METHOD

The proposed PRI estimation method based on BSS utilizes a one-dimensional uniform linear antenna array. Multi-channel signals are separated using blind source separation to extract single radar emitter signals. These separated signals undergo Hilbert transform to become complex signals, from which the signal envelope is extracted and shaped into a rectangular pulse train by applying a threshold. Finally, pulse arrival and end times are estimated to calculate the PRI.

The specific steps to implement the proposed method are as follows:

Step 1: For the array received signal matrix $\mathbf{X}(t)$, shown in Equation (5), the blind source separation method is adopted to process the signal blind source separation. The signal matrix obtained by separation is expressed as follows: $\hat{\mathbf{s}}(t) = [\hat{s}_1(t), \hat{s}_2(t), \dots, \hat{s}_K(t)]^T$, $t = 1, 2, \dots, N$ is the number of signal sampling points, and K is the number of signals.

Step 2: Apply the Hilbert transform to the separated signals $\hat{s}_1(t), \hat{s}_2(t), \dots, \hat{s}_K(t)$, respectively, followed by subsequent processing. In this and the following steps, $\hat{s}_1(t)$ is taken as an example for the convenience of illustration without loss of generality. The Hilbert transform is applied to $\hat{s}_1(t)$, which is expressed as $\hat{s}_1(t)$ after transformation, and it is a complex signal.

Step 3: For $\hat{s}_1(t)$, the signal envelope extraction is carried out, and the envelope extraction method used is to take the absolute value of the complex signal. The envelope extracted for $\hat{s}_{1h}(t)$ is denoted as $\text{abs}[\hat{s}_{1h}(t)]$.

Step 4: For the signal envelope $\text{abs}[\hat{s}_{1h}(t)]$, the threshold is set according to 20% of the envelope amplitude of each pulse

in the signal, and its threshold is set as TH_{1h} . It can be seen that TH_{1h} is a set of vectors.

Step 5: Shape the signal envelope $\text{abs}[\hat{s}_{1h}(t)]$ according to the threshold TH_{1h} . The shaping method is that if the amplitude of a pulse of the signal envelope $\text{abs}[\hat{s}_{1h}(t)]$ is greater than or equal to its corresponding threshold, that is, one of the values in TH_1 , then the pulse amplitude is set to 1, otherwise the pulse amplitude is set to 0, so that a string of rectangular pulses corresponding to $\text{abs}[\hat{s}_1(t)]$ can be obtained. It is denoted as $\text{Tr}\{\text{abs}[\hat{s}_1(t)]\}$.

Step 6: For $\text{Tr}\{\text{abs}[\hat{s}_{1h}(t)]\}$, the threshold is set to 0.5. According to the signal sampling frequency and threshold, the pulse arrival time, pulse cutoff time, pulse width are estimated, and then the PRI is estimated. Where the signal sampling frequency is the sampling frequency set when the array antenna is used to receive the signal.

IV. EXPERIMENTAL VERIFICATION AND ANALYSIS

The experimental verification of the proposed method is carried out under the simulation conditions of DELL9020MT personal computer, Intel(R) Core(TM) i7-4770 CPU @3.40 GHz, 64-bit Windows operating system, and MATLAB R2010a is used as the simulation software. Conduct experimental tests.

Experiment: All five radar emitter signals adopt the interpulse modulation style of PRI jagged jitter. The PRI jagged dither radar signal is formed by adding a dither quantity to the PRI jagged signal. There are four reference PRI values in each cycle, which are $100\ \mu\text{s}$, $120\ \mu\text{s}$, $140\ \mu\text{s}$, $180\ \mu\text{s}$, respectively, and the jitter percentage of each PRI is 5%. The reference value plus the jitter amount forms the actual PRI value of the radar signal.

The intra-pulse modulation methods of radar signal are two-phase coding (BPSK), single frequency point (CW), linear frequency modulation (LFM), polyphase coding (MPSK) and nonlinear frequency modulation (NLFM), and their parameters are as follows:

BPSK: pulse width is $10/31\ \mu\text{s}$, encoding for $[0,0,1,1,1,0,1,1,0,1,1,0,1,0,1,0,0,0,0,0,1,1,1,0,0,1,1,0]$, the initial phase of 0, the signal carrier frequency of 30 MHz.

CW: The pulse width is $20\ \mu\text{s}$, the carrier frequency is 30MHz, and the initial phase is 0.

LFM: The pulse width is $20\ \mu\text{s}$, the bandwidth is 10MHz, the starting frequency is 30 MHz, and the initial phase is 0.

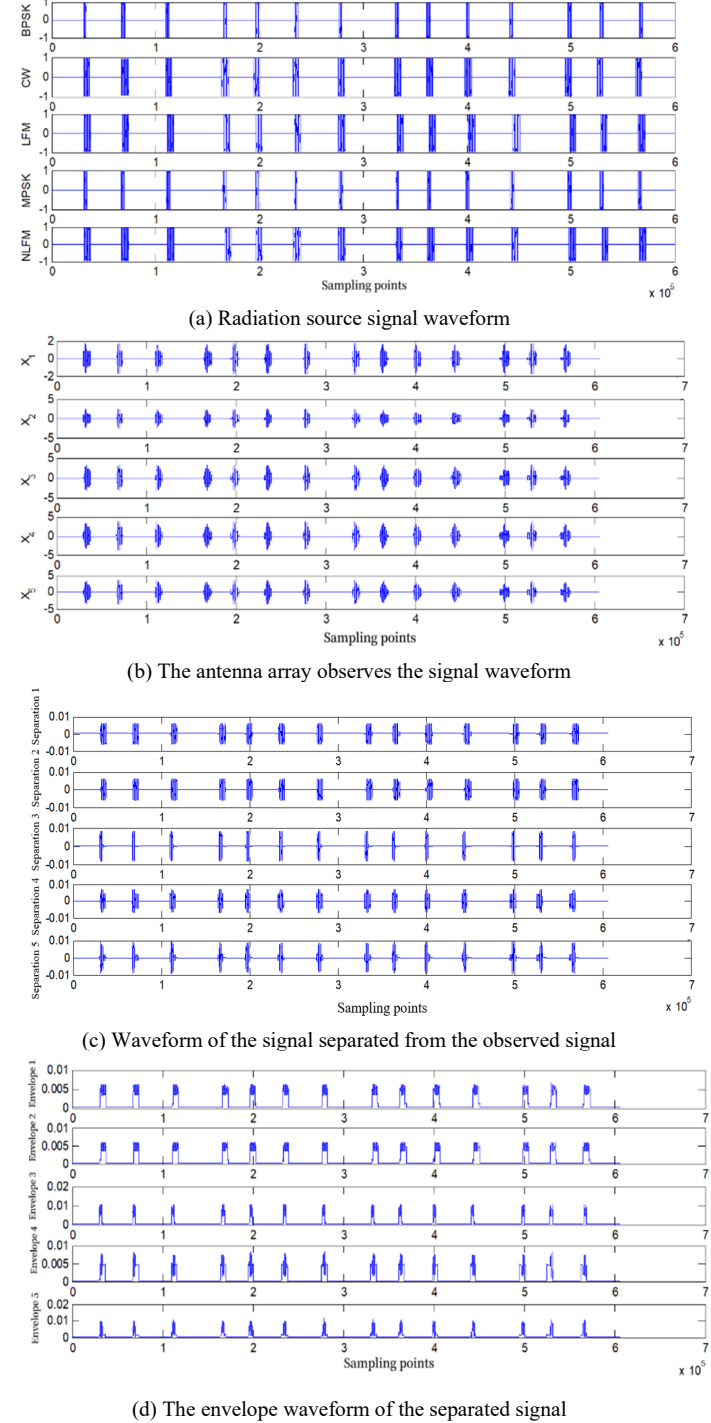
MPSK: The subpulse width is $0.4\ \mu\text{s}$, Frank coding is used, the number of code elements is 5, the carrier frequency is 30 MHz, and the initial phase is 0.

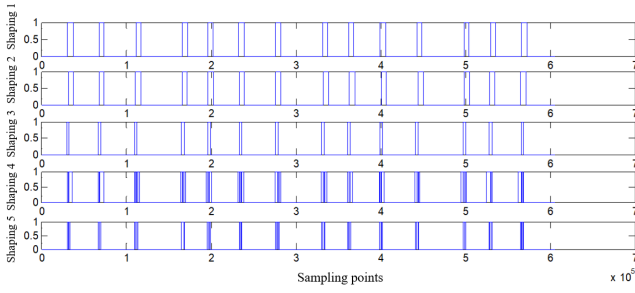
NLFM: The asymmetric NLFM signal shown on Radar Manual is used, the frequential-time function and coefficient value are the same as in the literature[12], the pulse width is $20\ \mu\text{s}$, the bandwidth is 10 MHz, the starting frequency is 30 MHz, and the initial phase is 0.

In the simulation experiment, the number of antenna array elements $M = 5$, the number of radars $K = 5$, the signal

amplitude is 1, the sampling frequency $F_s = 300\ \text{MHz}$, the number of signal sampling points $N = 605999$, the signal-to-noise ratio of the observation signal after adding noise is 30 dB, and the blind source separation method adopts the SOBI method[13].

The waveform of each key link signal is shown in Figure 4. The similarity coefficients of the separated signals and radar emitter signals are shown in Table 1. The PRI parameters of the five radar emitter signals and the PRI parameter results calculated by separating the five signals are shown in Figure 5.





(e) Pulse train waveform after envelope shaping

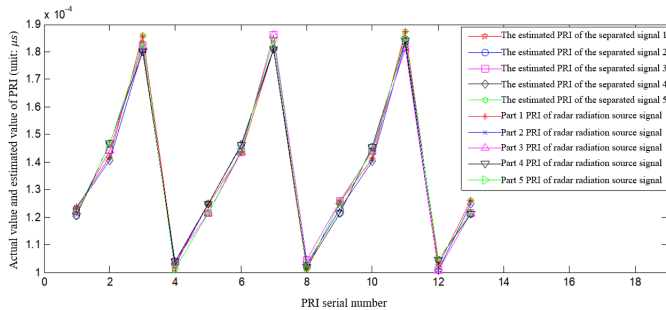
Figure 4. Signal waveforms of the key step of the experiment

It can be intuitively seen from the figure that after the processing of the proposed method, the rectangular pulse train waveforms after shaping are obtained for the separated signals, and the rising and falling edges are very steep, which can undoubtedly lay a solid foundation for the extraction of pulse arrival time, end time and PRI value.

TABLE 1. SIMILARITY COEFFICIENT BETWEEN THE SEPARATED SIGNAL AND THE RADAR EMITTER SIGNAL

Separate signal	Radar emitter signal				
	s_1	s_2	s_3	s_4	s_5
Separate signal 1	0.0022	-0.0001	0.2480	-0.0016	0.9676
Separate signal 2	-0.0039	-0.0000	-0.9686	-0.0004	0.2510
Separate signal 3	0.9829	-0.0106	0.0018	-0.1837	-0.0080
Separate signal 4	-0.0483	-0.9320	0.0122	-0.3347	0.0235
Separate signal 5	0.1775	-0.3622	-0.0103	0.9242	-0.0149

In Table 1, the underlined values represent the similarity coefficients between separated signals and radar emitter signals. The coefficient's sign does not impact the signal information; only its absolute value matters, with values closer to 1 indicating better results. The table shows high similarity between separated signals and their corresponding radar emitter signals, such as 0.9676 for separation signal 1 and the fifth emitter, -0.9686 for signal 2 and the third emitter, and 0.9829 for signal 3 and the first emitter, demonstrating excellent separation performance.

**Figure 5.** Comparison of PRI values of radar emitter signals and estimated PRI values of separated signals

As can be seen from Figure 5, after the processing steps of the proposed method, the PRI value estimated from the separated signals is very close to the real PRI value of the radar emitter signal, which can prove the effectiveness of the proposed method.

V. CONCLUSIONS

The radar signals often produce aliasing in complex electromagnetic environment, which brings serious challenges to PRI parameter estimation. Firstly, the blind source separation method is used to separate the signals received by one-dimensional antenna array, and the pulse flow belongs to a single radar signal, which effectively solves the challenges brought by signal aliasing to radar signal parameter estimation and identification analysis. For the separated radar signal pulse stream, the rectangular pulse train corresponding to the radar signal pulse stream is obtained by using the waveform shaping method in the proposed method, which lays a solid foundation for the accurate acquisition of pulse arrival time and pulse ending time. For the rectangular pulse train of the radar signal pulse stream, the pulse arrival time and the pulse ending time can be accurately obtained by using the calculation method of the pulse arrival time and the pulse ending time in the proposed method, and the pulse repetition interval can be accurately calculated.

Aiming at the problem of aliasing extraction of pulse radar signals in complex environment, this paper uses SOBI method to separate the aliased signals from one-dimensional uniform antenna array to form a single radar pulse stream signal. The extracted signals are simulated with five signal systems and five radars' pulse repetition intervals, which effectively solves the challenge of signal aliasing for radar signal parameter estimation and identification analysis. For the separated radar signal pulse stream, this paper proposes an innovative waveform shaping method based on the floating threshold of Hilbert envelope, and obtains the rectangular pulse train corresponding to the radar signal pulse stream, which lays a solid foundation for the accurate acquisition of the pulse arrival time and the pulse ending time. Through simulation verification, the similarity between the time and position characteristics of the separated pulse flow signal and the radar transmitted pulse is more than 0.999, so that the pulse repetition interval value can be accurately calculated. This new method greatly improves the system performance. Further study will focus on other innovative waveform shaping methods such as cognitive radar waveform design with adaptive waveform optimization and machine learning for waveform selection.

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